UNIVERSITY OF THE PUNJAB

A/2010
Examination:- B.A./B.Sc.

Roll No. .....

Subject: Mathematics-B Course **TIME ALLOWED: 3 hrs.**
PAPER: A **MAX. MARKS: 100**

Attempt SIX questions by selecting ONE question from Section-I, TWO questions from Section-II, TWO questions from Section-III, and ONE question from Section-IV.

Section-I

Q.1. a) Prove that $\frac{|\vec{b}|\vec{a}+|\vec{a}|\vec{b}}{|\vec{a}|+|\vec{b}|}$ is equally inclined with $\vec{a}$ and $\vec{b}$. 8,8

b) The necessary and sufficient condition for the vector $\vec{a}$ to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$

Q.2. a) If $\vec{F}$ and $\vec{G}$ are vector point function, then prove that 8,8
$$grad\,(\vec{F}\cdot\vec{G}) = \vec{F}\times(\nabla\times\vec{G}) + \vec{G}\times(\nabla\times\vec{F}) + \vec{F}\cdot(\nabla\vec{G}) + \vec{G}\cdot(\nabla\vec{G})$$

b) If $\vec{\nabla}\varphi = 2r^4\vec{r}$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then find the scalar function 'φ'

Section-II

Q.3. a) Three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ act along the sides BC, CA and AB respectively of a triangle ABC. Prove that 9,8
if $P\sec A + Q\sec B + R\sec C = 0$
then the line of action of the resultant passes through the orthocentre of the triangle.

b) If forces $p\overrightarrow{AB}, q\overrightarrow{CB}, r\overrightarrow{CD}, s\overrightarrow{AD}$ acting along the sides of plane quadrilateral are in equilibrium, show that $pr = qs$.

Q.4. a) The resolved part of the resultant '$\vec{R}$' of two coplaner forces '$\vec{P}$' and '$\vec{Q}$' acting at a point 'O' in the direction of '$\vec{P}$' is of magnitude, Q. Show that the angle between the forces is $2\sin^{-1}\sqrt{\frac{P}{2Q}}$. 9,8

b) Two equal beams AB, AC each of weight 'W' connected by a hinge at 'A' are placed in a vertical plane with their extremities 'B', 'C' resting on a horizontal plane; they are kept from falling by strings connecting 'B' and 'C' with the middle points of the opposite sides; show that the tension of either string is $\frac{w}{8}\sqrt{1+9\cot^2\theta}$ where 'θ' is the inclination of either beam to the horizon.

Q.5. a) Find the centroid of the arc of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ lying in the first quadrant. 9,8

b) Find the $C\cdot G\cdot$ of a uniform wire in the shape of the parabolic $arc\ y^2 = 4ax$ with ends as the extremities of the latus rectum.

Q.6. a) The least force which will move a weight up an inclined plane is of magnitude 'P'. Show that the least force acting parallel to the plane, which will move the weight upwards is $P\sqrt{1+\mu^2}$ 9,8
Where 'μ' is the co-efficient of friction.

PTO

b) A uniform rod of length '$2a$' rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance 'b' from the wall. Show that in the position of equilibrium, the beam is inclined to the wall at an angle $\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$

Section-III

Q.7. a) A particle is moving with uniform speed 'v' along the curve $x^2y = a\left(x^2 + \frac{a^2}{\sqrt{5}}\right)$. Show that its acceleration has the maximum value $10\frac{v^2}{9a}$. 9,8

b) Find the radial and transverse components of the acceleration of a particle moving along the curve $x^2 + y^2 = a^2$ with constant angular velocity, w

Q.8. a) A particle moving along a straight line starts from rest and is accelerated uniformly till it attains a velocity 'v'. The motion is then retarded and the particle comes to rest after traversing a total distance 'x'. If the acceleration is 'f' find the retardation and the total time taken by the particle from rest to rest. 9,8

b) Prove that the least velocity with which a particle must be projected so that it passes through the points 'P' and 'Q' at the heights h_p and h_q respectively from the ground is given by

$$\sqrt{g(h_P + h_q + PQ)}$$

Q.9. a) A particle of mass 'm' is moving under the action of the forces '$F_1 = -mw^2x$' '$F_2 = mF_0t$' and '$F_3 = -2m\mu x$'. Assumping that damping is small, st up and solve the equation of motion. 9,8

b) A point describes $S \cdot H \cdot M \cdot$ in such a way that its velocity and acceleration at a point 'P' are 'u' and 'f' respectively and the corresponding quantitites at another point 'Q' are 'v' and 'g'. Find the distance 'PQ'.

Q.10. a) Show that the law of force towards the pole of a particle describing the curve $r^n = a^n \cos\theta$ is given by 9,8

$$f = \frac{(n+1)h^2a^{2n}}{r^{2n+3}}$$

b) Prove that when a particle moves under a central force, the areal velocity is constant.

Section-IV

Q.11. a) Three perfectly elastic balls of masses 'm' '$2m$' and '$3m$' are placed in a line. The first ball impinges directly on the second with velocity 'u' and the second ball impinges on the third ball. Find the velocity of the third ball after impact. 8,8

b) Two elastic spheres each of mass 'm' collide directly. Show that the energy lost during the impact is $\frac{1}{4}m(u^2 - v^2)$. Where 'u' and 'v' are the relative velocities before and after impact.

Q.12. a) An imperfectly elastic smooth sphere whose elasticity is equal to $\frac{1}{\sqrt{3}}$ impinges upon a smooth plane with a velocity such that the velocity after impact is equal to $\frac{1}{\sqrt{2}}$ times the velocity before impact. Find the angles of incidence and reflection. 8,8

b) Two spheres of given masses, moving with given velocities impinge directly. Show that there is always loss of Kinetic Energy unless the elasticity is perfect.