UNIVERSITY OF THE PUNJAB

A/2009
Examination:- B.A./B.Sc.

Roll No. .....

Subject: Mathematics-B Course
PAPER: A

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Attempt SIX questions by selecting ONE question from Section I, TWO questions from Section II, TWO questions from Section III and ONE question from Section IV.

Section-I

Q.1. a) If $\bar{a}+\bar{b}+\bar{c}=0$ then prove that $\bar{a}\times\bar{b}=\bar{b}\times\bar{c}=\bar{c}\times\bar{a}$ 4+6+6

b) If $\bar{a},\bar{b},\bar{c}$ are any vectors then show that $\bar{a}-\bar{b},\bar{b}-\bar{c},\bar{c}-\bar{a}$ are coplanar.

c) If $\bar{a},\bar{b},\bar{c}$ and $\bar{a}'+\bar{b}'+\bar{c}'$ are reciprocal system of vectors then show that $\bar{a}\cdot\bar{a}'+\bar{b}\cdot\bar{b}'+\bar{c}\cdot\bar{c}'=3$

Q.2 a) Show that component from a unit tangent to a cricle $x^2+y^2=a^2$ is given by $\pm\frac{1}{a}(-y\bar{i}+x\bar{j})$ 4+6+6

b) If $\frac{d^2\bar{r}}{dt^2}=\mu\bar{r}$ then prove that $\left(\frac{dr}{dt}\right)^2=\mu r^2+c$

c) Show that $\nabla\cdot\left[r\,\nabla\left(\frac{1}{r^3}\right)\right]=3r^{-4}$

Section-II

Q.3. a) Forces $\bar{P},\bar{Q}$ act at a point and their resultant is $\bar{R}$. If any transversal cut there lines of action of the forces in the points A,B,C respectively prove that $\frac{P}{OA}+\frac{Q}{OB}=\frac{R}{OC}$ 8+9

b) Forces $X, P+X, Q+X$ act at a point in the directions of the sides an equilateral triangle taking one way round. Show that they are equivalent to two forces P and Q acting at an angle of 120^o

Q.4. a) State and prove tammy's theorem. 8+9

b) A regular octahedron form of twelve equal rods each of weight w freely jointed together is suspended from the e corner. Show that the thrust with each horizontal rod is $\frac{3\sqrt{2}w}{2}$

Q.5. a) A uniform ladder rest in limiting equilibrium with one end on a rough horizontal plane and other against a smooth vertical wall. A man ascends the ladder. Show that he can not go more than half the way up. 8+9

b) A uniform rectangular block of height h whose base is a square of side 'a' rest on a rough horizontal plane. The plane is gradulaly tilted about a line $\|$ to the edges of a base. Show that the block will slide or topple over according as $a \gtrless \mu h$ where μ is the coefficient of friction.

Q.6. a) Show that the $C\cdot G$ of the lamina bounded by a loop of leminscate's $r^2=a^2\cos 2\theta$ is on the initial line at a distance $\frac{\pi a}{4\sqrt{2}}$ from the pole. 8+9

b) A lamina is in the shape of a square described on the base of an issoceles triangle. Find the tangent of the semi-vertical angle of the triangle of the centre of mass of the whole lamina is at the middle point of the base.

Section-III

Q.7. a) A particle describes a cycloid $S = 4a\sin\Psi$ with uniform speed 'V'. Determine its acceleration at any point in terms of V, a and S. 8+9

b) Prove that the force field $\overline{F} = (y^2 - 2xyz^3)\bar{i} + (3 + 2xy - x^2z^3)\bar{j} + (6z^3 - 3x^2yz^2)\bar{k}$ conservative. Determine its potiential.

Q.8. a) Prove that the speed required to project a particle from a height h to fall a horizontal distance 'a' from the point of projection is at least $\sqrt{g(\sqrt{a^2+h^2}-h)}$ 8+9

b) A projectile having horizontal range R reaches a maximum height H. Prove that it must have been launched with at an angle with the horizontal given by $\sin^{-1}\left(\frac{4H}{\sqrt{R^2+16H^2}}\right)$

Q.9. a) A particle describes $S \cdot H \cdot M$ with frequency 'N'. If the greatest velocity is v. Find the amplitude and Maximum value of the acceleration of the particle. 8+9

b) A particle is projected vertically upwards. After a time 't' another particle is sent up from the same point with the same velocity and meets the first at height 'h' during the downward flight of the first. Find the velocity of projection.

Q.10. a) Prove that the orbit described under a central attractive force varying directly as distance is an ellipse having the centre at the centre of the force. 8+9

b) A particle describes the curve $\frac{a}{r} = e^{n\theta}$ under a force F to the pole, show that the force is stated as $F \propto \frac{1}{r^3}$

Section-IV

Q.11. a) A ball impinges directly upon another ball at rest and is itself reduced to rest by impact. If half of the initial $K \cdot E$ is destroyed in the collision. Find the coefficient of restitution. 8+8

b) A small sphere of mass m travelling with velocity u impinges obliquely on a smooth sphere of mass M at rest . Its original line of motion makig an angle α with the line of centre at a moment of impact. Show that the sphere of mass m will be deflected a right angle if $\tan^2\alpha = \frac{eM-m}{M+m}$

Q.12. a) If the mass of the balls be 2: 1 and their respective velocities before impact be as 1: 2 in opposite direction. Show that if the coefficient of restitution be $\frac{5}{6}\cdot$ Each ball moves back after impact with $\frac{5}{6}th$ of its original velocity. 8+8

b) A heavy elastic ball is dropped upon a horizontal floor from a height of 20 feet and after rebound twice it is observed to attain a height of 10 feet. Find the coefficient of restitution.