B- COURSE OF MATHEMATICS TIME ALLOWED: 3 hours

PAPER: **A** MAX. MARKS: 100

Attempt **SIX** questions by selecting ONE question from section-I, TWO from section-II, TWO from section-III and ONE from section-IV.

SECTION – I

1. a) Show that: $|\bar{\upsilon}.\bar{V}| \leq |\bar{\upsilon}||\bar{V}|$ 8, 8

b) Show that vectors $\vec{a}\times(\vec{b}\times\vec{c}), \vec{b}\times(\vec{c}\times\vec{a}), \vec{c}\times(\vec{a}\times\vec{b})$ are co-planar.

2. a) Evaluate $\nabla\cdot\left[r\nabla(\frac{1}{r^3})\right]$ where $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 8, 8

b) Prove that $\nabla\times(\nabla\times\vec{V}) = -\nabla^2\vec{V} + \nabla(\nabla\cdot\vec{V})$

SECTION – II

3. a) If two forces P_1 and P_2 are acting at a point 0 and the angle between them is α. Prove that their resultant R is $\sqrt{P_1^2 + P_2^2 - 2P_1P_2\,Cos\alpha}$. 8, 9

b) If forces $p\overrightarrow{AB}$, $q\overrightarrow{CB}$, $r\overrightarrow{CD}$, $s\overrightarrow{AD}$ acting along the sides of a plane quadrilateral are in equilibrium show that pr = qs.

4. a) Forces X, P+X, Q+X act at a point in the direction of the sides of an equilateral triangle taken one way round. Show that they are equivalent to two forces P, Q acting at an angle of 120°. 8, 9

b) A circular disc of weight W and radius a is suspended horizontally by two vertical strings each of length ℓ attached to the ends of the diameter. A horizontal couple applied to the disc turns it through an angle θ. Find the moment of couple.

5. a) Find c.m. of right circular solid cone. 8, 9

b) The radii of the faces of a solid frustrum of a solid cone are 2ft. and 3ft. and its height is 4ft. Find the distance of c.g. from larger face.

6. a) Find force P necessary just to support a heavy particle on an inclined plane of an inclination α (α>λ) where λ is angle of friction and μ, the co-efficient of friction. 8, 9

b) A uniform ladder rests in limiting equilibrium with one end on a rough horizontal plane and other against a smooth vertical wall. A man accends the ladder. Show he cannot go half way up.

SECTION – III

7. a) Find the radial and transverse components of velocity and acceleration. 8, 9

b) Find the radial and transverse components of the velocity of a particles moving along the curve $ax^2 + by^2 = 1$ at any time if the polar angle $\theta = ct^2$.

8. a) Explain the Simple Hormonic Motion by finding its equation. 8, 9

b) A particle is projected upward with a velocity $\sqrt{2gh}$ and another is left fall from height of the point. Where they meet each other.

9. a) Prove that the work done by the forces on a system of particles in moving from one configuration at time t_1 to configuration at time t_2, is equal to increase $T_2 - T_1$ in kinetic Energy (T_1, T_2 are kinetic energies at t_1 and t_2). 8, 9

b) A Gun of mass M fires a shell of mass m horizontally and energy of explosion is such that it would be sufficient to project the shell vertically to a height h. Show that the velocity of recoil of gun has magnitude $\sqrt{\frac{2m^2gh}{M(m+M)}}$.

10. a) A shell bursts on contact with the ground and pieces from it fly in all directions with all speeds up to 80ft/sec. Prove that a man 100ft. away is in danger for $\frac{5}{\sqrt{2}}$ seconds. 8, 9

b) With what speed an object be thrown up a small plane of length ℓ inclined at α with the horizontal so as to just reach at the top. Find also the time taken.

SECTION – IV

11. a) A ball is dropped from height h on a floor. If the co-efficient of restitution is e. Find the height of the ball at the top of nth rebound. 8, 8

b) A heavy elastic ball is dropped upon a horizontal floor from a height of 20ft. After rebounding twice it is observed to attain a height of 10ft. Find the co-efficient of restitution.

12. a) A uniform rod of mass m and length 3a hangs from a pin passing through it at a distance a from upper end. Find in terms of a, m, g, the magnitude of the smallest blow struck at the lower end of the rod which will make the rod a complete revolution. 8, 8

b) Assuming that in a cannon, the force on a ball depends only on the volume of the gas generated by the gunpowder. Show that the final velocity of ball when the gun is free to recoil to its velocity when the gun is fixed, is $\sqrt{\frac{M}{M+m}}$, M is mass of cannon, m is mass of ball.