B- COURSE OF MATHEMATICS   TIME ALLOWED: 3 hours

PAPER: A   MAX MARKS: 100

Attempt SIX questions by selecting ONE question from section (I), TWO
from section (II) TWO from section (III) and ONE from section (IV).

SECTION – I

1.
   a) Find a unit vector which makes an angle of $45°$ with $2\hat{i} + 2\hat{j} - \hat{k}$ and an angle of $60°$ with $\hat{j} - \hat{k}$.   8

   b) Prove that the medians of triangle are concurrent, and their point of intersection divides each median in the ratio 2 : 1 by vector method.   8

2.
   a) Solve $\vec{a} \times \dfrac{d^2\vec{v}}{dt^2} = \vec{b}$ where $\vec{a}$ and $\vec{b}$ are constant   8

   vectors and $\vec{v}$ is a vector function of t.

   b) Prove that
grad $(\vec{A} \cdot \vec{B}) = \vec{B} \times$ curl $\vec{A} + \vec{A} \times$ Curl $\vec{B} + (\vec{B} \cdot \nabla)\vec{A} + (\vec{A} \cdot \nabla)\vec{B}$.   8

SECTION – II

3.
   a) What is couple. Prove that the effect of a couple upon a rigid body is unaltered if it is replaced by any other couple of the same moment lying in the same plane.   9

   b) Forces P, Q, R act along the sides BC, CA, AB of a triangle ABC taken in order. If there resultant touches the incribed circle, prove that.   8
$$\frac{\cos^2 \frac{A}{2}}{P} + \frac{\cos^2 \frac{B}{2}}{Q} + \frac{\cos^2 \frac{C}{2}}{R} = 0$$

4.
   a) A heavy uniform rod, of length ℓ, rests partly within and partly without a fixed smooth hamispherical bowl of radius r, the rim of the bowl is horizontal and one point of the rod is in contact with the rim. If Q be the inclination of the rod to the horizon. Show the $4r \cos2\theta = \ell \cos\theta$.   9

   b) A light thin rod 12 feet long can turn in a verticle plane about one of its points which is attached to a pivot. If weights of 3lb and 4lb are suspended from its ends, it rests in a horizontal position. Find the position of the pivot and its reaction on the rod.   8

5.
   a) A radius of the faces of a frustum of a solid cone are 2ft and 3ft and the height of the frustum is 4ft. Find the distance of the centre of gravity from the larger face.   9

   b) Find the force necessary to support a heavy particle on an inclined plane of inclination α.   8

6.
   a) A rod 4 feet long, rests on a rough floor against the smooth edge of a table of height 3 feet. If the rod is on the point of slipping when inclined at an angle of $60°$ to the horizon. Find the coefficient of friction.   9

   b) Find centre of gravity of the area bounded by the parabola $y^2 = 4ax$, the axis of x and the letus rectum.   8

SECTION – III

7. a) A particle is moving with uniform speed V along with curve 9
    $$x^2 y = a\left(x^2 + \frac{a^2}{\sqrt5}\right)$$ show that is acceleration has the
    maximum value $\dfrac{10}{9}\dfrac{v^2}{a}$ 8

    b) What is simple Harmonic motion. Prove that time-period is
    independent of the amplitude.

8. a) A body starts with velocity u and moves in a straight line with 9
    constant acceleration a. When the velocity has become 5u, the
    acceleration is reversed, its magnitude being unaltered. Find the
    velocity with which the body reaches the starting point.

    b) A shell of mass M is moving with the speed V. An internal 8
    explosion generates an amount of energy E and beaks the shell
    into portion whose masses are in the ratio $m_1 : m_2$. The fragments
    continue to move in the original line of motion of the shell. Show
    that their speed are $$V + \sqrt{\frac{2m_2 E}{m_1 M}} \;and\; V - \sqrt{\frac{2m_1 E}{m_2 M}}$$

9. a) Discuss general motion on a plane curve under gravity in a 9
    vertical plane.

    b) A canon has its maximum rang R. Prove that 8
    i) The height reached is ¼ R and
    ii) The time of flight is $\sqrt{\dfrac{2R}{g}}$

10. a) What are Apse and Apsidal distance. Find the law of force to the 9
    pole when path is the cardioid $r = a(1-\cos\theta)$ and prove that if F be
    the force at the apse and V the velocity, $3V^2 = 4a\,F$.

    b) Find the law of force for the orbit $\quad = a^2 \cos2\theta$, the pole being the 8
    centre of force.

SECTION – IV

11. a) Prove that the time rate of change of angular momentum of a 8
    system S of particles about 0 is equal to the external torque of S
    about 0

    b) Two smooth equal spheres, moving with velocity u_1 and u_2 in the 8
    direction of the line joining their centres, impinge directly. If the
    coefficient of restitution be ½. Show that exactly half the energy is
    lost in collision if u_1 and u_2 are in the ratio. $1+\sqrt2 : 1-\sqrt2$

12. a) Discuss the impact of two smooth elastic spheres of masses m_1 8
    and m_2 moving with velocities u_1 and u_2 collide so that their
    direction of motion just before impact make angles α_1 and α_2 with
    the line of centres, where e be the coefficient of restitution.

    b) A heaving ball drops force the ceiling of a room and after 8
    rebounding twice from the floor reaches a height
    equal to one half of the ceiling. Show that coefficient of
    restitution is equal to $\left(\dfrac{1}{2}\right)^{\frac14}$