UNIVERSITY OF THE PUNJAB

A/2011
Examination:- B.A./B.Sc.

Roll No. 023940....

Subject: A Course of Mathematics
PAPER: B

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Attempt SIX questions by selecting TWO questions from Section-I, ONE question from Section-II, ONE question from Section-III and TWO questions from Section-IV.

Section-I

Q.1. a) Show that every square matrix over C can be expressed as unique way as $P + iQ$ where P and Q are Hermitian 9,8

b) Show that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

Q.2. a) For what values of λ the equations 9,8

$$(5-\lambda)x_1 + 4x_2 + 2x_3 = 0$$
$$4x_1 + (5-\lambda)x_2 + 2x_3 = 0$$
$$2x_1 + 2x_2 + (2-\lambda)x_3 = 0$$

Have non-Trivial solutions, find these solutions.

b) By using elementary row operations find the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 4 & 1 \\ 1 & 3 & 0 \end{bmatrix}$$

Q.3. a) Let $S = \{u_1, u_2, \cdots u_n\}$ be basis of n-dimensional vector space V over field F. Then every set with more than n vectors is linearly dependent. 9,8

b) For what value of K will be the vector $(1, -2, K)$ in R^3 be linear combination of the vectors $(3, 0, -2)$ and $(2, -1, -5)$

Q.4. a) A linear transformation $T: U \to V$ is one-to-one if and only if $N(T) = 0$ 9,8

b) Show that linear transformation preserves the linear dependence.

Section-II

Q.5. a) Let V be a vector space of real valued continuous functions on the interval $a \leq x \leq b$ 8,8

Show for $f, g \in V$

$\langle f, g\rangle = \int_a^b f(x)\, g(x)dx$ is an inner product in V

b) Find an orthogonal matrix whose first row is $\left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

Q.6. a) Find eigen values and corresponding eigen vectors for matrix 8,8

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$$

b) For symmetric matrix $A = \begin{bmatrix} 5 & 4 \\ 4 & -1 \end{bmatrix}$ find an orthogoanal matrix P for which $P^T AP$ is diagonal.

Section-III

Q.7. a) Solve the D.E. $\frac{dy}{dx} = \frac{x+3y-5}{x-y-1}$ 8,8

b) Solve the linear D.E. $x\frac{dy}{dx} + (1 + x\cot x)y = x$

Q.8. a) Find the orthogonal trajectories of the family of cardioids $r = a(1 + \cos\theta)$ 8,8

b) Find singular solution of $x^3p^2 + x^2yp + a^3 = 0$

Section-IV

Q.9. a) Solve the D.E $(D^3 - 7D - 6)y = e^{2x}(1 + x)$ 9,8

b) Solve by any method $y'' - 4y' + 4y = e^{2x}$

Q.10. a) Solve $\frac{d^2y}{dx^2} + y = \sec^3 x$ 9,8

b) Find the particular solution of the D.E

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}\ln x$

Q.11. Evaluate 9,8

i) $\mathcal{L}\,[te^{-3t}\,\mathrm{Sin}\,a\,t]$ ii) $\mathcal{L}^{-1}\left[arc\ \mathrm{Tan}\frac{a}{s}\right]$

Q.12. a) Use Laplace Transform to solve the D.E 9,8

$\frac{d^2y}{dt^2} + y = \cos t \qquad y(0) = 0 = y'(0)$

b) Use power series method to solve the D.E

$y' = -2y$