UNIVERSITY OF THE PUNJAB

A/2010
Examination:- B.A./B.Sc.

Roll No. ....

Subject: Mathematics-A Course **TIME ALLOWED: 3 hrs.**
PAPER: B **MAX. MARKS: 100**

Attempt SIX questions by selecting TWO questions from Section-I, ONE question from Section-II, ONE question from Section-III and TWO questions from Section-IV.

Section-I

Q.1. a) Show every square matrix over C can be expressed in a unique way as $P + iQ$ where P and Q are Hermitian 9,8

b) Prove that $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = 4abc$

Q.2. a) Show that the system 9,8

$$2x_1 - x_2 + 3x_3 = a$$
$$3x_1 + x_2 - 5x_3 = b$$
$$-5x_1 - 5x_2 + 21x_3 = c$$

is in consistent if $c \neq 2a - 3b$

b) By using elementary row operations,find inverse of matrix

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 4 & 1 \\ 1 & 3 & 0 \end{bmatrix}$$

Q.3. a) Let V be a vector space over a field F and let W be a non empty sub set of V then W is sub space of V 9,8

If and only if

i) $w_1, w_2 \in W \Rightarrow w_1 + w_2 \in W$

ii) $w \in W, a \in F \Rightarrow aw \in W$

b) Let U and W be 2-dimensional subspaces of R^3

Show that $U \cap W \neq \{0\}$

Q.4. a) A linear transformation $T: R^2 \to R^3$ maps the vectors $(1, 1)$ into $(0, 1, 2)$ and the the vectors $(-1, 1)$ into $(2, 1, 0)$ what matrix does T represent with respect to the standard bases for R^2 and R^3. 9,8

b) A linear transformation $T : U \to V$ is one to one if and only if $N(T) = 0$

Section-II

Q.5. a) The norm is an inner product space satisfies the following axioms for all $u, v \in V$ and $k \in R$ 8,8

i) $\|v\| \geq 0$ and $\|v\| = 0$ if and only if $v = 0$

ii) $\|kv\| = |k|\|v\|$

iii) $\|u + v\| \leq \|u\| + \|v\|$

b) Find an orthogonal matrix A whose first row is $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

Q.6. a) Find eigen values and corresponding eigen vectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. 8,8

b) Find a real orthogoanal matrix P for which $P^T AP$ is diagonal where $A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$

Section-III

Q.7. a) Solve $\frac{dy}{dx} = \frac{x+3y-5}{x-y-1}$

8,8

b) Show that given differential equation is an exat and hence solve it $\frac{dy}{dx} = -\frac{ax+hy}{hx+by}$

Q.8. a) Solve the differential equation 8,8

$x\frac{dy}{dx} + 3y = x^3y^2 \qquad y(1) = 2$

b) Solve $\frac{dy}{dx} - y^2 = -1$ given that $y_1 = 1$ is a particular solution of the given equation.

Section-IV

Q.9. a) Find general solution of the equation

$$(D^3 - 7D - 6)y = e^{2x}(1 + x)$$

9,8

b) Solve the differential equation

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\ln x)$$

Q.10. a) Solve $\frac{d^2y}{dx^2} + 4y = 4\tan x$ 9,8

b) Find a particular solution of the following

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = e^{-2x} \sec x$

Q.11. Evaluate

9,8

i) $\mathcal{L}\left[\frac{\sin at}{t}\right]$ ii) $\mathcal{L}^{-1}\left[\ln\left(\frac{S^2+1}{(S-1)^2}\right)\right]$

Q.12. a) Apply power series method to solve the differential equation 9,8

$$y' = y\left(1 + \frac{1}{x}\right)$$

b) Use Laplace Transform method to solve the following initial value problem

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} = 20e^{-t}\cos t$$

$$y(0) = 0 = y'(0)$$