



# UNIVERSITY OF THE PUNJAB

A/2009

Examination:- B.A./B.Sc.

Roll No.

Subject: Mathematics-A Course  
PAPER: B

TIME ALLOWED: 3 hrs.  
MAX. MARKS: 100

Attempt SIX questions by selecting TWO questions from Section I, ONE question from Section II, ONE question from Section III and TWO question from Section IV.

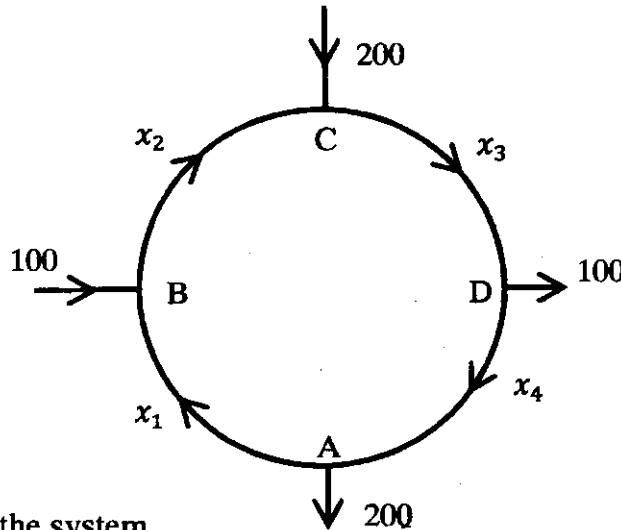
## Section-I

Q.1. a) If the matrices  $A, B$ , are conformable for product then prove that  $(AB)^t = B^t A^t$  9,8

b) With out expanding prove that

$$\begin{vmatrix} a^2 & b^2 \sin \alpha & c \sin \alpha \\ b \sin \alpha & 1 & \cos \alpha \\ c \sin \alpha & \cos \alpha & 1 \end{vmatrix} = 0$$

Q.2. a) The traffic flow of Kalma Chowk Lahore is shown below 9,8



i) Solve the system

ii) Find the traffic flow when  $x_4 = 300$

b) Show that every square matrix can be expressed in a unique way as  $P + iQ$  where  $P$  and  $Q$  are Hermitian.

Q.3. a) Suppose  $u, v, w$  are linearly independent vectors. Prove that  $u + v - 2w$ ,  $u - v - w$ ,  $u + w$  are linearly independent. 9,8

b) Prove that intersection of any number of sub-spaces of a vector space is also a sub-space.

Q.4. a) Show that linear transformation preserves linear dependence. 9,8

b) Find an equation defining the sub-space  $w$  of  $\mathbb{R}^3$  spanned by  $v_1 = (1, -3, 2)$ ,  $v_2 = (-2, 1, 2)$ ,  $v_3 = (-3, -1, 6)$

## Section-II

Q.5. a) Let  $V$  be a vector space of real valued continuous functions on the interval  $a \leq x \leq b$  8,8

Show that for  $f, g \in V$

$$\langle f, g \rangle = \int_a^b f(x) g(x) \text{ is an inner product in } V$$

b) Find an orthogonal matrix  $A$  whose first row is  $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

Q.6. a) For the matrix  $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$  find eigen values and bases of eigen spaces 8,8

b) Prove that eigen vectors of a symmetric matrix  $A$  corresponding to distinct eigen values are orthogonal

### Section-III

Q.7. a) Solve  $y(xy - 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$  by find I.F (Integrating Factor) 8,8

b) Solve  $x^3p^2 + x^2yp + 1 = 0$

Q.8. a) Find an equation of a orthogonal trajectories of the curve  $r = \frac{a}{2+\cos\theta}$  8,8

b) Find differential equation of all conics whose axes coincide with the axis of co-ordinates.

### Section-IV

Q.9. a) Solve  $(D^2 - 7D + 12)y = e^{2x}(x^3 - 5x)$  9,8

b) Solve the initial value problem

$$x^2y'' - 2xy' + 2y = x \ln x$$

$$y(1) = 1$$

$$y'(0) = 0$$

Q.10. a) Solve  $\frac{d^2y}{dx^2} - 2 \tan x \frac{dy}{dx} + 3y = 2 \sec x$  9,8

given that  $y = \sin x$  is a solution of the associated homogenous equation.

b) Find a particular solution of

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 2e^{-x} \tan^2 x$$

Q.11. a) Compute i) Laplace  $\mathcal{L}\{\sin at\}$  by definition 9,8

ii)  $\mathcal{L}^{-1}\left\{\frac{7s+5}{(3s-8)^2}\right\}$

Q.12. a) Use the Laplace transformation method to solve the initial value problem 9,8

$$\frac{d^3y}{dx^3} - 4 \frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = te^t$$

$$y(0) = y'(0) = 0$$

$$y''(0) = 1$$

b) Use Laplace transformation to find  $(x(t), y(t))$  of the system

$$\frac{dx}{dt} - 4x - 5y = e^{-4t}$$

$$x(0) = 0$$

$$\frac{dy}{dt} + 4x + 4y = e^{4t}$$

$$y(0) = 0$$