2**A/2004**BSc

A- COURSE OF MATHEMATICS

TIME ALLOWED: 3 hours

PAPER: **B**

MAX. MARKS: 100

Attempt **SIX** questions by selecting TWO questions from Sections-I, ONE from Section-II, ONE from Section-III and TWO from Section-IV.

SECTION – I

1. a) Prove that the product of Matrices 9

$$\begin{bmatrix} Cos^2\theta & Sin\theta Cos\theta \\ Cos\theta\, Sin\theta & Sin^2\theta \end{bmatrix} \text{and} \begin{bmatrix} Cos^2\phi & Cos\phi\, Sin\phi \\ Cos\phi\, Sin\phi & Sin^2\phi \end{bmatrix}$$ is the zero

Matrix when θ and ϕ differ by an odd Multiple of π/2.

b) Evaluate determinant $\begin{vmatrix} x^3 & x^2 & c & 1 \\ \alpha^3 & \alpha^2 & \alpha & 1 \\ \beta^3 & \beta^2 & \beta & 1 \\ v^3 & v^2 & v & 1 \end{vmatrix}$. 8

2. a) For what value of λ the equations 9

$$(5-\lambda)x_1 + 4x_2 + 2x_3 = 0$$
$$4x_1 + (5-\lambda)x_2 + 2x_3 = 0$$
$$2x_1 + 2x_2 + (2-\lambda)x_3 = 0$$

have non trivial solutions. Find the solutions.

b) If A and B are symmetric matrices then prove that AB is symmetric if and only of A and B commute. 8

3. a) Let V be a vector space over a field F and W be a non empty subset of V then W is a sub space of V if and only if 9

I $w_1, w_2 \in W \Rightarrow w_1 + w_2 \in W$

II $w \in W, a \in F \Rightarrow aw \in W$

b) Find K so that the vectors (1, – 1, K – 1), (2, K, – 4), (0, 2 + K, – 8) in R^3 are linearly dependent. 8

4. a) Prove that a one – one linear transformation preserves basis and dimension. 9

b) Let $V_1 = (1,1,1)$, $V_2 = (1,1,0)$ and $V_3 = (1,0,0)$ be a basis for R^3. Find a linear transformation $T : R^3 \to R^2$ such that $T(V_1) = (1, 0)$, $T(V_2) = (2, -1)$ and $T(V_3) = (4, 3)$. 8

SECTION – II

5. a) A is a square matrix and A is orthogonal then the rows of A ferm an orthonormal set. 8

b) Let $u, v \in R^2$, $u = (x_1, x_2)$, $v\ (y_1, y_2)$ 8

then $<u, v> = x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$ is an inner product on R^2.

6. a) Find Eigen values, Eigen vectors and an orthogonal matrix P for which P^tAP is diagonal where $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Find also diagonal matrix. 8

b) Prove that Eigen values of a symmetric matrix are all real. 8

P.T.O.

SECTION – III

7. a) Solve: $(x+2y^3)\frac{dy}{dx}=y$. 8

b) Solve: $y + px = p^2x^4$. 8

8. a) Solve: $\frac{dy}{dx}=\frac{3x-4y-2}{3x-4y-3}$. 8

b) Solve: $xp^2 - 2yp + 4x = 0$ and find the singular solution. 8

SECTION – IV

9. a) Solve: $(D^2 + 7D + 12)\,y = e^{2x}(x^3 - 5x^2)$. 9

b) Solve: $(2x+1)^2\frac{d^2y}{dx^2}-6(2x+1)\frac{dy}{dx}+16y=8(2x+1)^2$. 8

10. a) Solve: $\frac{d^2y}{dx^2}+y=\tan x\sec x$. 9

b) Solve: $(x^2+1)\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=2\cos x-2x$. 8

11. a) Find the power series solution of $y'' - 4xy' - 4y = 4 + 6x$, around $x = 0$. 9

b) Compute: i) $\mathcal{L}\,(t^3e^{-t})$, ii) $\mathcal{L}^{-1}\left(\frac{s}{(s+a)^2+b^2}\right)$ 8

12. a) Use Laplace Transform method to solve 9

$$\frac{d^2y}{dt^2}-2\frac{dy}{dt}=20\,e^{-t}\cos t$$

$$y(0)=0=y'(0)$$

b) Use Laplace Transform to find the solution $(x(t), y(t))$ of the system. 8

$$\frac{dx}{dt}-4x-5y=e^{-4t},\; x(0)=0$$

$$\frac{dy}{dt}+4x+4y=e^{4t},\; y(0)=0$$