A/2001BABSC

NEW COURSE

A- COURSE OF MATHEMATICS
PAPER: B

TIME ALLOWED: 3 hours
MAX MARKS: 100

Attempt SIX questions by selecting TWO questions from sections (I), ONE from section (II), ONE from section (III) and TWO from section (IV)

SECTION – I

1. a) Reduce the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$ **9**

to the normal form. Also find the nonsingular matrices P and Q such that PAQ is in the normal form.

b) Show that $\begin{bmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{bmatrix} = abcd\ (1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}).$ **8**

2. a) Solve the following system of linear equations, the field of scalars being R. **9**

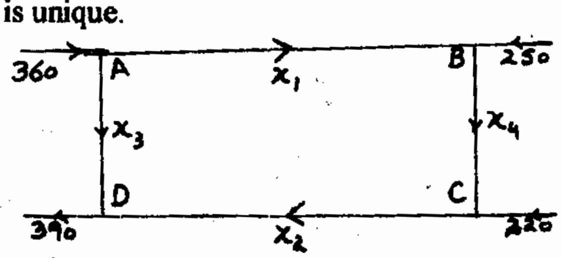
$$2x_1 + x_3 = 1$$
$$2x_1 + 4x_2 - x_3 = -2$$
$$x_1 - 8x_2 - 3x_3 = 2$$

b) A part of Lahore's road network for vehicular traffic is as shown by arrows in the following diagram: **8**

i) Write the equations indicating the traffic flow given in the diagram.

ii) Show that the traffic flow along AB, CD can be expressed in terms of the traffic flow along AD.

iii) If the stretch AD is closed, then show that the solution to the problem is unique.



3. a) Give the definitions of a vector space and a subspace of a vector space. Prove that a nonempty subset W of a vector space V(F) is a subspace of V(F) if and only if, w_1, w_2 $\in$ W and a, b $\in$ F imply $aw_1 + bw_2 \in W$. **9**

b) Show that the vectors $(3 + \sqrt{2}, 1 + \sqrt{2})$ and $(7, 1 + 2\sqrt{2})$ in R^2 are linearly dependent over R but linearly independent over Q. **8**

4. a) Let $v_1, v_2 \ldots, v_n$ be linearly independent in a vector space V over a field F. If v is any nonzero vector in V, then prove that the set $\{v_1, v_2 \ldots, v_n, v\}$ is linearly independent if and only if v is not in the linear span $<v_1, v_2 \ldots, v_n>$. **9**

b) The matrix of a linear transformation $T : R^2 \to R^3$ is **8**

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$$

Find T in terms of co-ordinates and its matrix with respect to the basis $v_1 = (0,1,2)$, $v_2 = (1,1,1)$, $v_3 = (1,0,-2)$.

P.T.O.

SECTION – II

5. a) Prove that the norm in an inner product space V satisfies the following axioms: 8

$\|v\| \geq 0$ and $\|v\| = 0$ if and only if $v=0$, $v \in V$

$\|Kv\| = |K|\,\|v\|$ for all $v \in V$ and $K \in R$

$\|u+v\| \leq \|u\| + \|v\|$ for all $u, v \in V$

b) Find an orthogonal matrix whose first row is $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$ 8

6. a) Prove that nonzero eigenvectors of a matrix A corresponding to distinct eigenvalues are linearly independent. 8

b) Find the eigenvalues and eigenvectors and an orthogonal matrix P for which P^tAP is diagonal where $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$. Find also the diagonal matrix. 8

SECTION – III

7. a) Solve: $(y+\sqrt{x^2+y^2})\,dx - xdy = 0,\ y(1)=0$ 8

b) Find orthogonal trajectories of the family of curves 8

$$r = \frac{a}{2+Cos\theta}$$

8. a) Solve: $p^3 - (x^2+xy+y^2)\,p + xy^2 + x^2y = 0$. 8

b) Solve and find the singular solution, if it exists $p^2 + 2px^3 - 4x^2y = 0$. 8

SECTION – IV

~~**9.** a) Solve: $(D^2 - 7D + 12)y = e^{2x}(x^3 - 5x^2)$.~~ 9

b) Solve: $(x+1)^2\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = 4[Cos\ \ln(x+1)]^2$ 8

10. a) Find the general solution of 9

$$x^2\frac{d^2y}{dx^2} - x(x+2)\frac{dy}{dx} + (x+2)\,y = x^3$$

by the method of variation of parameters given that $y = xe^x$ is a solution of the associated homogeneous equation.

b) Solve $\frac{d^2y}{dx^2} - 2\tan x\frac{dy}{dx} + 3y = 2Sec\ x$, given that $y = \text{Sin}\,x$ is a solution of the associated homogeneous equation. 8

11. a) Find the power series solution of 9

$"y - 4xy' - 4y = 4 + 6x$

around the ordinary point $x = 0$.

b) Compute: I) $\mathcal{L}(t^2\ Sin\ at)$ II) $\mathcal{L}^{-1}\left(\frac{1}{(s^2+a^2)(s^2+b^2)}\right)$ 8

12. a) Use the Laplace Transform method to solve 8

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = e^t$$

$$y(0) = ,\ y'(0) = 0$$

b) Find the solution (x(t), y(t)) of the system 9

$$\frac{dx}{dt} - x + y = 2e^t,\ x(0) = 0$$

$$\frac{dx}{dt} + x - y = e^t,\ y(0) = 0$$