UNIVERSITY OF THE PUNJAB

A/2011
Examination:- B.A./B.Sc.

Roll No. a23946a.....

Subject: A Course of Mathematics **TIME ALLOWED: 3 hrs.**
PAPER: A **MAX. MARKS: 100**

Attempt any SIX questions by selecting TWO questions from each Section I & II and ONE from each Section III & IV.

Section-I

Q.1. a) Let $f(x)=\begin{cases} x+2 & , \quad if \quad x \le -1 \\ kx^2 & , \quad if \quad x > -1 \end{cases}$ 8,9

Find k so that $\lim_{x\to -1} f(x)$ exists.

b) Define continuity of a function at a point $x = a$. Find the points of discontinuty of a function $f(x)$ defined by

$$f(x)=\begin{cases} x+4, & if \quad -6 \le x < -2 \\ x, & if \quad -2 \le x < 2 \\ x-4, & if \quad 2 \le x \le 6 \end{cases}$$

Q.2. a) If $x=\frac{3at}{(1+t^2)}$, $y=\frac{3at^2}{(1+t^2)}$ show that $\frac{dy}{dx}-x\frac{dt}{dx}=t$ 8,9

b) If $f(x)=x^3-2x-5=0$, show that a root of $f(x)=0$ lies between $x=2$ & $x=3$

Hence find a root by taking $x_o = 2$

Q.3. a) If $y=\cos(\ln x)+\sin(\ln x)$ then by showing $x^2y''+xy'+y=0$, 8,9
prove that $x^2y^{(n+2)}+(2n+1)xy^{(n+1)}+(n^2+1)y^{(n)}=0$

b) Let $f(x)$ be a function satisfying the following conditions:

i) $f(x)$ is continuous for all $x \in [a, b]$

ii) $f(x)$ is differentiable for all $x \in]a, b[$

iii) $f(a) = f(b)$

Then show that there exists at least one point $c \in]a, b[$ such that $f'(c) = 0$

Q.4. a) Determine the intervals on which $f(x)=2x^3-15x^2+36x+1$ is 8,9
decreasing or increasing.

b) If $\lim_{x\to 0}\frac{\sin 2x+k\sin x}{x^3}$ be finite, find k and find the limit.

Section-II

Q.5. a) Analyze & graph the conic represented by $\sqrt{x}+\sqrt{y}=1$ 8,9

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b) Find the polar equation of the conic in the form $\frac{l}{r} = 1 + e\cos\theta$; identify when this equation will represent.

i) a parabola ii) an ellipse iii) a hyperbola

Q.6. a) Express $3y^2 - 16y - x^2 + 16 = 0$ into polar form, find the eccentricity & equation of the directrix. 9,8

b) Find the parametric equations for the cardioid $r = 1 + \sin\theta$; hence find the points at which it has vertical tangents.

Q.7. a) Find the equation of the line through the point $\left(5, \frac{7}{2}, 5\right)$ and intersecting at right angle an other line with equation $x = 4 + 3t, y = 1 + t, z = -3t$ 8,9

b) Find an equation of the plane passing through $A(1,0,1), B(2,2,1)$ and perpendicular to the plane $x - y - z + 4 = 0$.

Q.8. a) Show that the straight line $x = z - 4,\ y = 2z - 3$ lies in the plane $2x - 3y + 4z - 1 = 0$ 8,9

b) Find the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y + z = 0$, $z + x = 0$, $x + y = 0$, $x + y + z = a$

Section-III

Q.9. a) Find the asymptotes of the curve $x^2 - y^2 = 1$ 8,8

b) An open rectangular box is to be made from a sheet of cardboard 8 dm by 5 dm by cutting equal squares from each corner and turning up the sides. Find the edge of the square which gives maximum volume.

Q.10. a) Write down equation of a circle having centre at origin and radius k, where k is any positive constant. Using integration techniques, show that area of this circle is πk^2 8,8

b) Show that redius of curvature at any pont $x = a\cos^3\theta$, $y = a\sin^3\theta$ of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is given by $\rho = 3(axy)^{1/3}$

Section-IV

Q.11. Integrate the following 5,6,5

i) $\int \frac{1}{2\sin^2 x + 3\cos^2 x}\,dx$ ii) $\int \frac{dx}{x^4+1}$ iii) $\int \frac{1-\sin x}{1+\cos x}\,dx$

Q.12. a) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ 8,8

b) Use trapezoidal rule, approximate the integral $\int_0^2 e^{-x^2}\,dx$ with $n = 4$