UNIVERSITY OF THE PUNJAB

A/2010
Examination:- B.A./B.Sc.

Roll No.

Subject: Mathematics-A Course
PAPER: A

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Attempt any SIX questions by selecting TWO from each Section I & II and ONE from each Section III & IV.

Section-I

Q.1. a) Given that $\lim_{x\to 0}(1+x)^{1/x} = e$, show that $\lim_{y\to 0}\frac{a^y-1}{y} = \ln a\,,\ (a>1)$ 9,8

b) If $f(x,y) = arc\sin\left(\frac{x}{y}\right)$, verify that $f_{xy} = f_{yx}$

Q.2. a) Find k such that 9,8

$$f(x) = \begin{cases} \frac{1-\sqrt{x}}{x-1} & , \quad if \quad 0 \le x < 1 \\ k & , \quad if \quad x = 1 \end{cases}$$

is CONTINUOUS for all $x\epsilon[0,1]$

b) Using NEWTON-RAPHSON Method, find a root of

$f(x) = x^3 - 2x - 5 = 0$

Q.3. a) If $y = arc\,(\tan x)$, show that $(1+x^2)y'' + 2xy' = 0$, Hence find $y^{(n)}(0)$ 9,8

b) Give statement of Rolle's Theorem. Is it valid for $f(x) = x^2 - 3x + 2$ on $[1\,,2]$? If valid, find $c\epsilon]1\,,2[$ such that $f'^{(c)} = 0$

Q.4. a) Prove that $f(x) = \frac{\ln(x+1)}{x}$ decreases on $]0\,,\infty[$ 9,8

b) Show that $\lim_{x\to 0}(\cot x)^{\sin 2x} = 1$

Section-II

Q.5. a) Show that the point $(at^2\,,2at)$always lies on the parabola $y^2 = 4ax$. Show that its tangent at the above point is $x - yt + at^2 = 0$ 9,8

b) Examine whether the given equation $6x^2 - 15y^2 - xy + 16x + 24y = 0$ represents two straight lines? If so, find equation of each of the straight line.

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Q.6. a) Express $3y^2 - 16y - x^2 + 16 = 0$ in POLAR form, find the eccentricity and equation of the directrix. Also, identify the curve. 9,8

b) Find the measure of the angle of intersection of the curves $r = a\theta$, $r\theta = a$

Q.7. a) Find the equation of the perpendicular from the point P $(1,6,3)$ to the straight line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ 9,8

b) Find the distance of the point $A(3,-1,2)$ to the plane $2x + y - z - 4 = 0$. Also, reduce the equation of the plane in the NORMAL FORM and find its distance from the origin.

Q.8. a) Find equation of the plane through the points $(1,0,1)$ & $(2,2,1)$ and perpendicular to the plane $x - y - z + 4 = 0$. 9,8

b) Show that the SHORTEST DISTANCE between the straight lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ and equation of the line perpendicular to both lines is $11x + 2y - 7z + 6 = 0 = 7x + y - 5z + 7$

Section-III

Q.9. a) Find the intervals in which the curve $y = (x^2 + 4x + 5)e^{-x}$ faces up or down. Also, find its points of inflection if any. 8,8

b) Find equations of ASYMPTOTES of the curve $y(x - y)^2 = x + y$

Q.10. a) Find the entire length of the CARDIOID $r = a(1 - \cos\theta)$, $a > 0$ 8,8

b) Prove that for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $\rho = \frac{a^2b^2}{p^3}$ where p is the length of the perpendicular from the centre $C(0,0)$ to the tangent to the ellipse at any point $p(x, y)$.

Section-IV

Q.11. Integrate the following 8,8

i) $\int \frac{1}{\sin(x-a)\sin(x-b)} dx$ ii) $\int e^x \frac{(1-\sin x)}{(1-\cos x)} dx$

Q.12. a) Evaluate $\int_{-3}^{3} |x| dx$ 8,8

b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^4 x \, dx$