UNIVERSITY OF THE PUNJAB

A/2009
Examination:- B.A./B.Sc.

Roll No.

Subject: Mathematics-A Course — **TIME ALLOWED: 3 hrs.**
PAPER: A — **MAX. MARKS: 100**

Attempt SIX questions by selecting TWO questions from Section I, TWO from Section II, ONE question from Section III and ONE question from Section IV.

Section-I

Q.1. a) Find $\frac{dy}{dx}$ when $f(x) = arc\sec(\operatorname{cosec} x + \sqrt{x})$ 8+9

b) The cost function $C(x)$ and revenue function $R(x)$ for producing x units of certain product are given by $C(x) = 5x + 350$, $R(x) = 50 - x^2$. Find values of x that yield a profit.

Q.2. a) Find the values of a and b so that the function f is is continuous and differentiable at $x = 1$, where $f(x) = \begin{cases} x^3 & if \quad x < 1 \\ ax + b & if \quad x \geq 1 \end{cases}$ 8+9

b) Find a root of equation $x\sin x + \cos x = 0$ with $x_0 = \pi = 3.1416$, up to four places of decimal by using Newton-Rophson method.

Q.3. a) Evaluate $\lim_{x\to\frac{\pi}{2}} (\cos x)^{\frac{\pi}{2}-x}$ 8+9

b) If $y = (arc\sin x)^2$, prove that $(1 - x^2)y'' - xy' - 2 = 0$. Differentiate this equation n times and find the value of $y^{(n)}$ at $x = 0$.

Q.4. a) Use the Mean value Theorem to show that $|\sin x - \sin y| \leq |x - y|$ for any real number x, y. 8+9

b) State and prove Taylor's Theorem as Lagrange's Form of Remainder.

Section-II

Q.5. a) Find the points at which $r = 1 + \cos\theta$ has horizontal and vertical tangents. 8+9

b) Find an equation of the straight line joining two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentric angles are given. Hence find equations of tangent and normal at any point 'θ' on the ellipse.

Q.6. a) Find the measure of the angle of intersection of the given curves $r = \frac{a\theta}{1+\theta}$ and $r = \frac{a}{1+\theta^2}$ 8+9

b) Express $3y^2 - 16y - x^2 + 16 = 0$ in polar form and find the eccentricity and equation of directrix. Also identify the curve.

Q.7. a) Find equations to the planes through the points $(4,-5,3),(2,3,1)$ and parallel to y-axis. 8+9

b) Show that $L:\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $M:\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ are skew lines. Find co-ordinates of the feet of the common perpendicular to the two lines and equations and length of common perpendicular.

Q.8. a) Find an equation of the sphere passing through the points $(0,0,0)$, $(0,1,-1)$, $(-1,2,0)$ and $(1,2,3)$. Also find its centre and radius. 8+9

b) Show that the shortest distance between the straight lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ and equations of the straight line perpendicular to both are $11x+2y-7z+6=0=7x+y-5z+7$.

Section-III

Q.9. a) Sketch the graph of the curve $r=a(1-\sin\theta)$,$a>0$. 8+8

b) Find the asymptotes of the curve $r^2\sin\theta=a^2\cos 2\theta$.

Q.10. a) Find a and b so that the function $f(x)=ax^3+bx^2$ has $(1,6)$ as the point of inflection. 8+8

b) Show that the intrinsic equation of the astroid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ is $s=\frac{3a}{2}\sin^2\alpha$

Section-IV

Q.11. Integrate the following. 5+5+6

i) $\int arc\tan\sqrt{\frac{1-x}{1+x}}\cdot dx$ ii) $\int\frac{x^3}{1+x^2}dx$ iii) $\int\frac{1}{x^4+1}dx$

Q.12. a) Prove that $\int_0^{\pi/2}\frac{\cos x}{\sin x+\cos x}dx=\frac{\pi}{4}$ 8+8

b) Prove that $\int_0^{\pi/2}\cos^m x\cdot\sin nx\,dx=\frac{1}{m+n}+\frac{m}{m+n}\int_0^{\pi/2}\cos^{m-1}x\cdot\sin(n-1)x\,dx$

Hence evaluate $\int_0^{\pi/2}\cos^6 x\cdot\sin 3x\,dx$