A- COURSE OF MATHEMATICS TIME ALLOWED: 3 hours

PAPER: **A** MAX. MARKS: 100

Attempt **SIX** questions by selecting TWO questions from Sections - I, TWO from Section – II, ONE from Section - III and ONE from Section - IV

SECTION – I

1. a) Find the values of 'a' and 'b' so that f is continuous and differentiable at x = 1 where $f(x)=\begin{cases} x^3 \text{ if } x<1 \\ ax+b \text{ if } x\ge 1 \end{cases}$ 9

b) Evaluate $\lim_{x\to 0} x\left[\frac{1}{x}\right]$, $[\cdots]$ being the bracket function.

2. a) Differentiate $\log_{10}^{\left(\frac{x+1}{x}\right)}$ with respect to x. 9

b) Use the Newton – Raphson method to approximate upto four places of decimal, root of $x^3 - 5x + 3 = 0$ with $x_0 = 0$. 8

3. a) Show that $\frac{d^n}{dx^n}\left(\frac{\ln x}{x}\right)=\frac{(-1)^n\, n!}{x^{n+1}}\left[\ln x-1-\frac{1}{2}-\frac{1}{3}\cdots-\frac{1}{n}\right]$ 9

b) If f is a thrice differentiable function, prove by L' Hospital's Rule that 8

$$\lim_{h\to 0}\frac{f(x+h)-f(x)-hf'(x)-\frac{h^2}{2}f''(x)}{h^3}=\frac{f'''(x)}{6}.$$

4. a) If $x>0$ prove that $x-\ell n(1+x)>\frac{x^2}{2(1+x)}$ 9

b) Prove that under certain conditions (to be stated) 8
$f(a+h)=f(a)+hf'(a+\theta h)$, where $0<\theta<1$. Prove also that the limiting value of θ, when h decreases infinitely, is $\frac{1}{2}$.

SECTION – II

5. a) If a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, with centre "C" meets the major and minor axes in T and t, prove that $\frac{a^2}{CT^2}+\frac{b^2}{ct^2}=1$. 9

b) Show that the curves $r^m = a^m \cos m\theta$ and $r^m = a^m \sin m\theta$ cut each other orthogonally. 8

6. a) Show that the locus of the middle points of a system of parallel chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $y=\frac{-b^2}{a^2m}x$ where m is slope of the chords. 9

b) Examine whether the equation $2x^2 - xy + 5x - 2y + 2 = 0$ represents two straight lines. If so find equation of each straight line. 8

7. a) Find the equation of the perpendicular from the point P (1, 6, 3) to the straight line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also obtain its length and coordinates of the foot of the perpendicular. 9

b) Prove that the straight lines $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ and $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ intersect. Also find the point of intersection and the plane through them. 8

 P.T.O.

8. a) Find the shortest distance between the straight line joining the points A(3, 2, −4) and B(1, 6, −6) and the straight line joining the points C(−1, 1, −2) and D(−3, 1, −6). Also find equations of the line of shortest distance and coordinates of the feet of the common perpendicular. 9

b) Show that an equation of the cylinder whose generators are parallel to z-axis and which passes through the curve $x^2 + y^2 + z^2 = 1, x + y + z = 1$ is $x^2 + y^2 + xy - x - y = 0$. 8

SECTION – III

a) A dome is in the shape of a hemisphere with radius 60 ft. The dome is to be painted with a layer of 0.01 inch thickness. Use differentials to estimate the amount of the paint required. 8

b) Prove that the intrinsic equation of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $S = 4a \sin\psi$. 8

10. a) Prove that the least perimeter of an isosceles triangle in which a circle of radius 'r' can be inscribed is 6r v3. 8

b) Find the point on the curve $y = \ln x$ where the curvature k is maximum. 8

SECTION – IV

11. Evaluate $\int \frac{dx}{(x-1)\sqrt{x^2+1}}$ 8

b) Calculate $\int_0^{\pi/2} \ell n(\sin x)dx$ 8

12. a) Evaluate $\int_a^b x^2 dx$ by definition. 8

b) Evaluate $\int_0^{\pi/3} \sin^2 6x \cos^4 3x dx$. 8