A/2001BABSC

NEW COURSE

A- COURSE OF MATHEMATICS **TIME ALLOWED: 3 hours**

PAPER: A **MAX MARKS: 100**

Attempt **SIX** questions by selecting TWO questions from Sections (I), TWO from Section (II) ONE from Section (III) and ONE from Section (IV)

SECTION – I

1. a) Evaluate $\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^x$. 8

b) Let f (x) = x^2 and $g(x)\begin{cases}-4 & \text{if } x\le 0\\ |x-4| & \text{if } x>0\end{cases}$ 8½

Determine whether fog and gof are continuous at x = 0.

2. a) Differentiate arctan $\left(\frac{1+2x}{2-x}\right)$ w.r.t.x. 8

b) Use the Newton-Raphson to approximate upto four places of decimal root of f (x) = x^3-2x-5=0. 8½

3. a) If y = arctan x, show $(1+x^2)\,y\ \ +2xy = 0$. Hence find the values of all derivatives of y when x = 0. 8

b) Use Hospital's Rule to prove that 8½

$$\lim_{x\to\infty}\left\{\frac{a^{\frac{1}{x}}+b^{\frac{1}{x}}}{2}\right\}^x=\sqrt{ab};\ a>0, b>0$$

4. a) Show that $\frac{d^n}{d^{xn}}\left(\frac{\ln^x}{x}\right)=\frac{(-1)^n\,n!}{x^{n+1}}\left\{\ln x-1-\frac{1}{2}-\frac{1}{3}\cdots-\frac{1}{n}\right\}$ 8

b) If f is thrice differentiable, prove by L' Hospital's rule that: 8½

$$\lim_{x\to\infty}\frac{f(x+h)-f(x)-hf'(x)-\frac{h^2}{2}f''(x)}{h^3}=\frac{f'''(x)}{6}$$

SECTION – II

5. a) What is an Epicycloid? Derive its parametric equations. 2, 6

b) Show that the Pedal equation of astroid $x=a\cos^3\theta$, $y=a\sin^3\theta$ is $r^2 = a^2 - 3p^2$. 8½

6. a) What are conjugate diameters? Prove that the condition for two diameters y = m1x & y = m2x to be conjugate is 2, 6

$$m_1m_2 - \frac{b^2}{a^2}$$

b) Find an equation of a normal to the hyperbola 4, 4½

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \ \textit{in the form}\ \frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$$

prove that the normal is external bisector of the angle between the focal distances of its foot.

P.T.O.

7. a) Show that the shortest distance between any two opposite edges of a tetrahedron formed by the planes y+z = 0, x+z=0, x+y=0, x+y+z=a is $\frac{2a}{\sqrt{6}}$ and that the three straight lines of the shortest distances intersect at the point (-a, -a, -a). 6, 2

b) Find the equation of two plans whose distances from the origin are 3 units each and which are perpendicular to the line through the points A(7,3,1), B(6,4,-1). 8½

8. a) Find the equation of the torus obtained by revolving about y-axis the circle in they xy-plane with centre at (a,0,0) and radius b where 0<b<a. 8

b) Identify the surface defined by the equation $x^2 - 9y^2 - 4z^2 - 6x + 18y + 16z + 20 = 0$. 8½

SECTION – III

9. a) The radius of a circle increases from x = 10cm to x = 10.1cm. Find approximate in the area of the circle. Also find the percentage change in the area. 8

b) Prove that the last perimeter of an isoceles traingle in which a circle of radius r can be inscribed is $6\sqrt{3}\ r$. 8½

10. a) The cardoid r = a(1+cosθ) is divided by the line 4r cosθ = 3a into two parts. Find the ratio of the lengths of the arcs on the two sides of this line. 8

b) Find the point on the curve y = ln x where the curvature κ is maximum. 8½

SECTION - IV

11. a) Find a reduction formula for $\int x^m (\ln x)^n\, dx, m \neq -1$ and x is an integer greater than 1 and apply it to evaluate $\int x^3 (\ln x)^2\, dx$ 3, 5

b) Evaluate $\int \frac{dx}{\sin(n-a)\sin(x-b)}$ 8½

12. a) Evaluate $\int_a^b \frac{1}{x}\, dx$ by definition 8

b) Prove that $\int_0^{2a} f(x)\, dx = \int_0^a f(x)\, dx + \int_0^a f(2a-x)\, dx$ and hence evaluate $\int_a^{2\pi} \frac{dx}{5 + 3\cos x}$ 4, 4½