

Q(1) EVALUATE THE INTEGRALS BY DEFINITION

$$1) \int_a^b \frac{1}{X} dX \quad (8)$$

$$2) \int_a^b \sin^2 x dx \quad (8)$$

Q(2) EVALUATE THE INTEGRALS (7+7+6)

$$(1) \int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$$

$$(2) \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin x \cos x} dx$$

$$(3) \int_1^{\infty} \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$$

Q(3) IF $I_n = \int x^n (a^2 - x^2)^{\frac{1}{2}} dx$ then prove that (8+6)

$$I_n = -\frac{x^{n-1}(a^2 - x^2)^{\frac{3}{2}}}{n+2} + \frac{n-1}{n+2} a^2 I_{n-2}$$

$$(b) \text{evaluate } \int_0^{\frac{\pi}{8}} \sin^5 4x \cos^4 4x dx$$

Q(4)(a) if w is cube root of unity then evaluate (8+12)

$$\begin{pmatrix} w & 1 & w^2 \\ 1 & w & w^3 \\ 1 & w & w^5 \end{pmatrix}^3$$

b) Solve the system of equations (if possible) by three different methods

$$2x_1 + 3x_2 + 4x_3 + 5x_4 = 14$$

$$2x_1 - 3x_2 - x_3 + 8x_4 = 6$$

$$-2x_1 - 3x_2 - x_3 + 8x_4 = 2$$

$$-2x_1 - 3x_2 - x_3 - x_4 = -8$$