# Test Mathematics
## B.Sc.
## Method (Chap 3,4), Calculus (Chap 1, Ex. 2.1,2.2,2.3.2.5)

*You can skip definitions; these are to make strong concepts.*

**Total Marks: 100** **Time Allowed: 3 Hours**

**Q. 1(i)** Define the greatest integer function.
Evaluate $\lim_{x\to 0} x\left[\frac{1}{x}\right]$, [...] denotes the greatest integer function.
**(ii)** Define the continuity of a function at a point.
Find constants $a$ and $b$ such that the function $f$ defined by

$$f(x)=\begin{cases} x^3 & \text{if } x<-1 \\ ax+b & \text{if } -1\le x<1 \\ x^2+2 & \text{if } x\ge 1 \end{cases}$$

is continuous for all $x$.

**Q. 2(i)** Prove that every differentiable function is continuous. What can you say about the converse. Justify?
**(ii)** Show that

$$\frac{d^n}{dx^n}\left(\frac{\ln x}{x}\right)=\frac{(-1)^n n!}{x^{n+1}}\left[\ln x-1-\frac{1}{2}-\frac{1}{3}-\ldots-\frac{1}{n}\right]$$

**Q. 3(i)** What is a differentiable function.
If $f(x)=(\sin x)^{\cos x}+(\cos x)^{\sin x}$, find $f'(x)$ where $\sin x>0, \cos x>0$
**(ii)** Define error, relative error and percentage error.
The percentage error in measuring the edge of a cube is 2%. Find the percentage error in computing the volume of the cube.

**Q. 4(i)** Define pivot element, pivot column, and echelon form of a matrix. Find the rank of the matrix

$$\begin{pmatrix} 1 & 3 & 7 \\ 2 & 3 & 8 \\ 0 & 1 & 2 \\ 4 & 0 & 4 \end{pmatrix}$$

**(ii)** What is the difference between a diagonal and a scalar matrix?
Prove that the inverse of a diagonal matrix is a diagonal matrix.

**Q. 5(i)** what is the consistency criteria for a system of linear equations.
Is the following system consistent, if so find the solution?

$$x+y=0$$
$$2x-y+3z=3$$
$$x-2y-z=3$$

**(ii)** Define inverse of a matrix. Find the inverse of the following matrix using Gauss Jordan Elimination method

$$\begin{pmatrix} 2 & 2 & 3 \\ 1 & -2 & -3 \\ 4 & -2 & -3 \end{pmatrix}$$

**Q. 6(i)** Define a symmetric and a skew symmetric matrix. Prove that if a matrix is skew symmetric then all of its diagonal entries are zero.

**(ii)** When we can multiply two matrices?

Compute $(2-A)(3-A)(3-A)(-1-A)$, where $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 5 & 3 & 0 & 0 \\ 9 & 1 & 3 & 0 \\ 1 & 2 & 5 & -1 \end{pmatrix}$

………….



…………. .