EXERCISE # 10.1

Question # 1: $(9D^2 - 12D + 4)y = 0$

Solution:

Given equation is

$$(9D^2 - 12D + 4)y = 0 - - - (i)$$

The characteristics equation of (i) *will be*

$$9D^2 - 12D + 4 = 0$$

$$\Rightarrow D = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(9)(4)}}{18}$$

$$\Rightarrow D = \frac{12 \pm \sqrt{144 - 144}}{18}$$

$$D = \frac{12}{18} + 0 \; ; \; D = \frac{12}{18} - 0$$

$$D = \frac{2}{3}, \frac{2}{3}$$

Therefore, the complementary function is

$$y_c = c_1 e^{\left(\frac{2}{3}x\right)} + c_2 x e^{\left(\frac{2}{3}x\right)}$$

$$\Rightarrow \quad y = (c_1 + c_2 x) e^{\left(\frac{2}{3}x\right)} \quad \because y_c = y$$

is required solution.

Question # 2: $(75D^2 + 50D + 12)\boldsymbol{y} = \boldsymbol{0}$

Solution:

Given equation is

$$(75D^2 + 50D + 12)\boldsymbol{y} = \boldsymbol{0} - - - (i)$$

The characteristics equation of (i) *will be*

$$75D^2 + 50D + 12 = 0$$

$$\Rightarrow D = \frac{-50 \pm \sqrt{(50)^2 - 4(75)(12)}}{2(75)}$$

$$\Rightarrow D = \frac{-50 \pm \sqrt{2500 - 3600}}{150}$$

$$D = \frac{-50 \pm \sqrt{-1100}}{150}$$

$$\Rightarrow D = \frac{-50 \pm 10\sqrt{-11}}{150}$$

$$\Rightarrow D = 10\left(\frac{-5 \pm \sqrt{-11}}{150}\right)$$

$$D = \frac{-5 \pm i\sqrt{11}}{15}$$

$$\Rightarrow D = \frac{-5}{15} \pm \frac{i\sqrt{11}}{15}$$

$$D = \frac{-1}{3} + \frac{i\sqrt{11}}{15}$$

Therefore, the complementary function is

$$y_c = \left(c_1 cos\left(\frac{\sqrt{11}}{15}x\right) + c_2 sin\left(\frac{\sqrt{11}}{15}x\right)\right) e^{\frac{-1}{3}x}$$

$$\Rightarrow y = \left(c_1 cos\left(\frac{\sqrt{11}}{15}x\right) + c_2 sin\left(\frac{\sqrt{11}}{15}x\right)\right) e^{\frac{-1}{3}x}$$

is required solution.

Question # 3: $(\boldsymbol{D^3 - 4D^2 + D + 6})\boldsymbol{y} = \boldsymbol{0}$

Solution:

Given equation is

$$(\boldsymbol{D^3 - 4D^2 + D + 6})\boldsymbol{y} = \boldsymbol{0}$$

The characteristics equation of (i) *will be*

$D^3 - 4D^2 + D + 6 = 0$

Since $D = -1$ is a root of characteristic equation. So we use synthetic division in order to find the other roots.

	1	-4	1	6
-1	0	-1	5	-6
	1	-5	6	0

Now, the residual equation will be

$D^2 - 5D + 6 = 0$

$\Rightarrow D^2 - 2D - 3D + 6 = 0$

$\Rightarrow D(D-2) - 3(D-2)$

$\Rightarrow (D-2)(D-3) = 0$

$\Rightarrow \; D = 2 \;\; or \;\; D = 3$

Therefore, the complementary function is

$y_c = c_1e^{-x} + c_2e^{2x} + +c_3e^{3x}$

$\Rightarrow \;\; y = c_1e^{-x} + c_2e^{2x} + +c_3e^{3x} \because y_c = y$

is required solution.

<u>Question # 4</u>: $\boldsymbol{(D^3 + D^2 + D + 1)y = 0}$

Solution:

Given equation is

$\boldsymbol{(D^3 + D^2 + D + 1)y = 0 - - - (i)}$

The characteristics equation of (i) *will be*

$\boldsymbol{D^3 + D^2 + D + 1 = 0}$

$\Rightarrow \boldsymbol{D^2(D+1) + (D+1) = 0}$

$\Rightarrow \boldsymbol{(D+1)(D^2+1) = 0}$

$\Rightarrow \;\; D = -1 \;\; or \;\; D = \pm i$

Therefore, the complementary function is

$y_c = c_1e^{-x} + c_2\cos x + c_3\sin x$

$\Rightarrow \; y = c_1e^{-x} + c_2\cos x + c_3\sin x \;\; \because y_c = y$

is required solution.

<u>Question # 5</u>: $\boldsymbol{(D^3 - 6D^2 + 12D - 8)y = 0}$

Solution:

Given equation is

$\boldsymbol{(D^3 - 6D^2 + 12D - 8)y = 0 - - - (i)}$

The characteristics equation of (i) *will be*

$\boldsymbol{D^3 - 6D^2 + 12D - 8 = 0}$

Since $D = 2$ is a root of characteristic equation. So we use synthetic division in order to find the other roots.

	1	-6	12	-8
2	0	2	-8	8
	1	-4	4	0

Now the residual equation will be

$D^2 - 4D + 4 = 0$

$\Rightarrow (D-2)^2 = 0$

$\Rightarrow \mathrm{D} = 2,2$

Therefore, the complementary function is

$y_c = c_1e^{2x} + c_2xe^{2x} + +c_3x^2e^{3x}$

$\Rightarrow \;\; y = (c_1 + c_2x + +c_3x^2)e^{2x} \;\; \because y_c = y$

is required solution.

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Question # 6: $(D^3 - 6D^2 + 3D + 10)y = 0$

Solution:

Given equation is

$$(D^3 - 6D^2 + 3D + 10)y = 0 - - - (i)$$

The characteristics equation of (i) *will be*

$$D^3 - 6D^2 + 3D + 10 = 0$$

Since $D = 2$ is a root of characteristic equation. So we use synthetic division in order to find the other roots.

	1	-6	3	10
2	0	2	-8	-10
	1	-4	-5	0

Now, the residual equation will be

$$D^2 - 4D - 5 = 0$$

$$\Longrightarrow D^2 - 5D + D - 5 = 0$$

$$\Longrightarrow (D+1)(D-5) = 0$$

$$\Longrightarrow D = -1 \; or \; D = 5$$

Therefore, the complementary solution is

$$y_c = c_1e^{2x} + c_2e^{-x} + c_3e^{5x}$$

$$\Longrightarrow y = c_1e^{2x} + c_2e^{-x} + c_3e^{5x} \quad \because y_c = y$$

is required solution.

Question # 7: $(D^3 - 27)y = 0$

Solution:

Given equation is

$$(D^3 - 27)y = 0 - - - (i)$$

The characteristics equation of (i) *will be*

$$D^3 - 27 = 0$$

$$\Longrightarrow (D)^3 - (3)^3 = 0$$

$$\Longrightarrow (D-3)(D^2 + 3D + 9) = 0$$

$$\Longrightarrow (D-3) = 0 \; or \; (D^2 + 3D + 9) = 0$$

$$\Longrightarrow D = 3$$

$$or \; (D^2 + 3D + 9) = 0$$

$$\Longrightarrow D = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(9)}}{2}$$

$$\Longrightarrow D = \frac{-3 \pm \sqrt{9 - 36}}{2}$$

$$D = \frac{-3 \pm \sqrt{-27}}{2}$$

$$\Longrightarrow D = \frac{-3 \pm 3\sqrt{3}i}{2}$$

$$\Longrightarrow D = \frac{-3}{2} \pm i\frac{3\sqrt{3}}{2}$$

Therefore, the complementary solution is

$$y_c = c_1e^{3x} + \left[c_2cos\left(\frac{3\sqrt{3}}{2}x\right) + c_3sin\left(\frac{3\sqrt{3}}{2}x\right)\right]e^{-\frac{3x}{2}}$$

$$\Longrightarrow y = c_1e^{3x} + \left[c_2cos\left(\frac{3\sqrt{3}}{2}x\right) + c_3sin\left(\frac{3\sqrt{3}}{2}x\right)\right]e^{-\frac{3x}{2}}$$

is required solution.

Question # 8: $(4D^4 - 4D^3 - 3D^2 + 4D - 1)y = 0$

Solution:

Given equation is

$$(4D^4 - 4D^3 - 3D^2 + 4D - 1)y = 0 - - - (i)$$

The characteristics equation of (i) *will be*

$$4D^4 - 4D^3 - 3D^2 + 4D - 1 = 0$$

Since $D = 1, -1$ are the roots of characteristic equation. So we use synthetic division in order to find the other roots. The synthetic division is as follow:-

	4	-4	-3	4	-1
1	0	4	0	-3	1
	4	0	-3	1	0
-1	0	-4	4	-1	
	4	-4	1	0	

Now, the residual equation will be

$4D^2 - 4D + 1 = 0$

$\Rightarrow (2D - 1)(2D - 1) = 0$

$\Rightarrow 2D = 1; 2D = 1$

$\Rightarrow D = \frac{1}{2}; D = \frac{1}{2}$

Therefore, the complementary solution is

$y_c = c_1e^x + c_2e^{-x} + c_3e^{\frac{x}{2}} + c_4xe^{\frac{x}{2}}$

$OR \quad y = c_1e^x + c_2e^{-x} + (c_3 + c_4x)e^{\frac{x}{2}}$

is required solution.

Question # 9: $(D^4 + 2D^3 - 2D^2 - 6D + 5)y = 0$

Solution:

Given equation is

$(D^4 + 2D^3 - 2D^2 - 6D + 5)y = 0 - - - (i)$

The characteristics equation of (i) *will be*

$D^4 + 2D^3 - 2D^2 - 6D + 5 = 0$

Since $D = 1,1$ are the roots of characteristic equation. So we use synthetic division in order to find the other roots. The synthetic division is as follow:-

	1	2	-2	-6	5
1	0	1	3	1	-5
	1	3	1	-5	0
1	0	1	4	5	
	1	4	5	0	

Now, the residual equation will be

$D^2 + 4D + 5 = 0$

$\Longrightarrow D = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$\Longrightarrow D = \frac{-4 \pm \sqrt{-4}}{2}$

$\Longrightarrow D = \frac{-4 \pm 2i}{2}$

$\Longrightarrow D = -2 \pm i$

Therefore, the complementary solution is

$y_c = c_1e^x + c_2xe^x + (c_3cosx + c_4sinx)e^{-2x}$

$\Longrightarrow y = (c_1 + c_2x)e^x + (c_3cosx + c_4sinx)e^{-2x}$

is required solution.

Question # 10 $(D^4 - 5D^3 + 6D^2 + 4D - 8)y = 0$

Solution:

Given equation is

$$(D^4 - 5D^3 + 6D^2 + 4D - 8)y = 0 - - - (i)$$

The characteristics equation of (i) *will be*

$D^4 - 5D^3 + 6D^2 + 4D - 8 = 0$

Since $D = -1,2$ are the roots of characteristic equation. So we use synthetic division in order to find the other roots. The synthetic division is as follow:-

	1	-5	6	4	-8
-1	0	-1	6	-12	8
	1	-6	12	-8	0
2	0	2	-8	8	
	1	-4	4	0	

Now, the residual equation will be

$D^2 - 4D + 4 = 0$

$\Longrightarrow D^2 - 2D - 2D + 4 = 0$

$\Longrightarrow D(D-2) - 2(D-2)$

$\Longrightarrow (D-2)(D-2) = 0$

$\Longrightarrow D = 2,2$

Therefore, the complementary solution is

$y_c = c_1e^{-x} + c_2e^{2x} + c_3xe^{2x} + c_4x^2e^{2x}$

$y_c = c_1e^{-x} + (c_2 + c_3x + c_4x^2)e^{2x}$

OR

$y = c_1e^{-x} + (c_2 + c_3x + c_4x^2)e^{2x}$

is required solution.

Question # 11

$$(\boldsymbol{D^4 - 4D^3 - 7D^2 + 22D + 24)y = 0}$$

Solution:

Given equation is

$$(\boldsymbol{D^4 - 4D^3 - 7D^2 + 22D + 24)y = 0} - - - (\boldsymbol{i})$$

The characteristics equation of (i) will be

$$\boldsymbol{D^4 - 4D^3 - 7D^2 + 22D + 24 = 0}$$

Since $D = -1, -2$ are the roots of characteristic equation. So we use synthetic division in order to find the other roots. The synthetic division is as follow:-

	1	-4	-7	22	24
-1	0	-1	5	2	-24
	1	-5	-2	24	0
-2	0	-2	14	-24	
	1	-7	12	0	

Now, the residual equation will be

$D^2 - 7D + 12 = 0$

$\Longrightarrow D^2 - 3D - 4D + 12 = 0$

$\Longrightarrow D(D-3) - 4(D-3) = 0$

$\Longrightarrow (D-3)(D-4) = 0$

$\Longrightarrow D = 3,4$

Therefore, the complementary solution is

$y_c = c_1e^{4x} + c_2e^{3x} + c_3e^{-2x} + c_4e^{-x}$

OR

$y = c_1e^{-x} + (c_2 + c_3x + c_4x^2)e^{2x}$

is required solution.

Question # 12 $(\boldsymbol{D^4 + 4)y = 0}$

Solution:

Given equation is

$$(\boldsymbol{D^4 + 4)y = 0} - - - (\boldsymbol{i})$$

The characteristics equation of (i) will be

$$\boldsymbol{D^4 + 4 = 0}$$

$\Longrightarrow (D^2)^2 + (2)^2 + 2(D^2)(2) - 2(D^2)(2) = 0$

$\Longrightarrow (D^2 + 2)^2 - 4D^2 = 0$

$\Longrightarrow (D^2 + 2)^2 - (2D)^2 = 0$

$\Longrightarrow (D^2 + 2 + 2D)(D^2 + 2 - 2D) = 0$

$\Longrightarrow D^2 + 2D + 2 = 0 \quad : \quad D^2 - 2D + 2 = 0$

$\Longrightarrow D = \dfrac{-2 \pm \sqrt{4-8}}{2}$; $D = \dfrac{-2 \pm \sqrt{4-8}}{2}$

$\Longrightarrow D = \dfrac{-2 \pm 2i}{2}$; $D = \dfrac{-2 \pm 2i}{2}$

$\Longrightarrow D = \dfrac{2(-1 \pm i)}{2}$; $D = \dfrac{2(-1 \pm i)}{2}$

$\Longrightarrow D = -1 \pm i$; $D = 1 \pm i$

Therefore, the complementary solution is

$y_c = (c_1 cosx + c_2 sinx)e^{-x} + (c_3 cosx + c_4 sinx)e^{x}$

is required solution.

Question # 13 $(\boldsymbol{D^4 - D^3 - 3D^2 + D + 2})\boldsymbol{y = 0}$

Solution:

Given equation is

$(\boldsymbol{D^4 - D^3 - 3D^2 + D + 2})\boldsymbol{y = 0} - - - (\boldsymbol{i})$

The characteristics equation of (i) *will be*

$\boldsymbol{D^4 - D^3 - 3D^2 + D + 2 = 0}$

Since $D = 1,2$ are the roots of characteristic equation. So we use synthetic division in order to find the other roots. The synthetic division is as follow:-

	1	-1	-3	1	2
1	0	1	0	-3	-2
	1	0	-3	-2	0
2	0	2	4	2	
	1	2	1	0	

Now, the residual equation will be

$D^2 + 2D + 1 = 0$

$\Longrightarrow (D+1)^2 = 0$

$\Longrightarrow (D+1)(D+1) = 0$

$\Longrightarrow D = -1, -1$

Therefore, the complementary solution is

$y_c = c_1 e^{x} + c_2 e^{2x} + c_3 e^{-x} + c_4 x e^{-x}$

$\because y_p = 0.$ *Therefore,*

$y = c_1 e^{x} + c_2 e^{2x} + c_3 e^{-x} + c_4 x e^{-x}$

is required solution.

Question # 14 $(\boldsymbol{16D^6 + 8D^4 + D^2})\boldsymbol{y = 0}$

Solution:

Given equation is

$(\boldsymbol{16D^6 + 8D^4 + D^2})\boldsymbol{y = 0} - - - (\boldsymbol{i})$

The characteristics equation of (i) *will be*

$\boldsymbol{16D^6 + 8D^4 + D^2 = 0}$

$\Longrightarrow D^2(16D^4 + 8D^2 + 1) = 0$

$\Longrightarrow D^2(4D^2 + 1)^2 = 0$

$\Longrightarrow D^2 = 0$; $(4D^2 + 1)^2 = 0$

$D = 0,0$

& $(4D^2 + 1)(4D^2 + 1) = 0$

$4D^2 = -1$ *or* $4D^2 = -1$

$\Longrightarrow D^2 = \dfrac{-1}{4}$ *or* $D^2 = \dfrac{-1}{4}$

$\Longrightarrow D = \sqrt{\dfrac{i^2}{4}}$ *or* $D = \sqrt{\dfrac{i^2}{4}}$

$\Longrightarrow D = \pm\dfrac{i}{2}; D = \pm\dfrac{i}{2}$

Therefore, the complementary solution is

$$y_c = c_1 e^{0x} + c_2 x e^{0x} + c_3 cos\frac{x}{2} + c_4 x cos\frac{x}{2} + c_5 sin\frac{x}{2} + c_6 x sin\frac{x}{2}$$

$$y_c = (c_1 + c_2x) + (c_3 + c_4x)cos\frac{x}{2} + (c_5 + c_6x)sin\frac{x}{2}$$

$\because y_p = 0.$ *Therefore,*

$$y = (c_1 + c_2x) + (c_3 + c_4x)cos\frac{x}{2} + (c_5 + c_6x)sin\frac{x}{2}$$

is required solution.

Question # 15

$$(\boldsymbol{D^4 + 6D^3 + 15D^2 + 20D + 12)y = 0}$$

Solution:

Given equation is

$(\boldsymbol{D^4 + 6D^3 + 15D^2 + 20D + 12)y = 0} \; --(\boldsymbol{i})$

The characteristics equation of (i) *will be*

$\boldsymbol{D^4 + 6D^3 + 15D^2 + 20D + 12 = 0}$

Since $D = -2, -2$ are the roots of characteristic equation. So we use synthetic division in order to find the other roots. The synthetic division is as follow:-

	1	6	15	20	12
-2	0	-2	-8	-14	-12
	1	4	7	6	0
-2	0	-2	-4	-6	
	1	2	3	0	

Now, the residual equation will be

$D^2 + 2D + 3 = 0$

$$\Rightarrow D = \frac{-2 \pm \sqrt{4 - 12}}{2}$$

$$\Rightarrow D = \frac{-2 \pm 2i\sqrt{2}}{2}$$

$\Rightarrow D = -1 \pm i\sqrt{2}$

Therefore, the complementary solution is

$$y_c = (c_1 + c_2x)e^{-2x} + \left(c_3cos\sqrt{2}x + c_4sin\sqrt{2}x\right)e^{-x}$$

$\because y_p = 0.$ *Therefore,*

$$y = (c_1 + c_2x)e^{-2x} + \left(c_3cos\sqrt{2}x + c_4sin\sqrt{2}x\right)e^{-x}$$

is required solution.

Solve each of the following initial problem.

Question # 16

$(\boldsymbol{D^2 + 8D - 9)y = 0} \qquad \boldsymbol{y(1) = 1, y'(1) = 0}$

Solution:

Given equation is

$(\boldsymbol{D^2 + 8D - 9)y = 0} \; ---(\boldsymbol{i})$

The characteristics equation of (i) *will be*

$\boldsymbol{D^2 + 8D - 9 = 0}$

$\Rightarrow D^2 + 9D - D - 9 = 0$

$\Rightarrow D(D + 9) - 1(D + 9) = 0$

$\Rightarrow (D - 1)(D + 9) = 0$

$\Rightarrow D = 1 \;\; or \;\; D = -9$

Therefore, the complementary solution is

$y_c = c_1e^x + c_2e^{-9x}$

$\because y_p = 0.$ *Therefore*

$y = c_1e^x + c_2e^{-9x} \; ---(a)$

Differentiating w.r.t "x", we have

$y' = c_1e^x - 9c_2e^{-9x} \; ---(b)$

Applying $\mathbf{y(1) = 1}$ **on** $(\boldsymbol{a})$**, we have,**

$1 = c_1e^1 + c_2e^{-9}$

$$\Rightarrow c_1e + \frac{c_2}{e^9} = 1 \; ---(c)$$

Applying $\boldsymbol{y'(1) = 0}$ **on** $(\boldsymbol{b})$**, we have,**

$0 = c_1e - 9c_2e^{-9}$

$\Longrightarrow c_1e = 9c_2e^{-9} - - - (d)$

Using (d) in (c), we have

$9c_2e^{-9} + \frac{c_2}{e^9} = 1$

$\Longrightarrow \frac{9c_2}{e^9} + \frac{c_2}{e^9} = 1$

$\Longrightarrow \frac{10c_2}{e^9} = 1$

$\Longrightarrow \boxed{\boldsymbol{c_2 = \frac{e^9}{10}}}$

Now $(d) \Longrightarrow$

$c_1e = 9\frac{e^9}{10}e^{-9}$

$\Longrightarrow \boxed{\boldsymbol{c_1 = \frac{9}{10e}}}$

Thus equation (a) becomes

$y = \frac{9}{10e}.e^x + \frac{e^9}{10}.e^{-9x}$

$y = \frac{9}{10}e^{x-1} + \frac{1}{10}e^{-9(x-1)}$

is required solution.

Question # 17

$\boldsymbol{(D^2 + 6D + 9)y = 0 \quad y(0) = 2, y'(0) = -3}$

Solution:

Given equation is

$\boldsymbol{(D^2 + 6D + 9)y = 0 - - - (i)}$

The characteristics equation of (i) will be

$D^2 + 6D + 9 = 0$

$\Longrightarrow D^2 + 3D + 3D + 9 = 0$

$\Longrightarrow D(D + 3) + 3(D + 3)$

$\Longrightarrow (D + 3)(D + 3) = 0$

$\Longrightarrow D = -3 \quad D = -3$

Therefore, the complementary solution is

$y_c = c_1e^{-3x} + c_2xe^{-3x}$

$\because y_p = 0.$ *Therefore*

$y = c_1e^{-3x} + c_2xe^{-3x} - - - (a)$

Differentiating w.r.t "x", we have

$y' = -3c_1e^{-3x} + c_2e^{-3x} - 3xe^{-3x} - - - (b)$

Applying $\mathbf{y(0) = 2}$ **on** $\boldsymbol{(a)}$**, we have,**

$\boxed{2 = c_1}$

Applying $\mathbf{y'(0) = -3}$ **on** $\boldsymbol{(b)}$**, we have,**

$-3 = -3c_1 + c_2$

$\Longrightarrow c_2 = -3 + 6$

$\Longrightarrow \boxed{c_2 = 3}$

Thus equation (a) becomes

$y = 2e^{-3x} + 3xe^{-3x}$

$\Longrightarrow y = (2 + 3x)e^{-3x}$

is required solution.

Question # 18

$\boldsymbol{(D^2 + 6D + 13)y = 0 \quad y(0) = 3, y'(0) = -1}$

Solution:

Given equation is

$\boldsymbol{(D^2 + 6D + 13)y = 0 - - - (i)}$

The characteristics equation of (i) will be

$\boldsymbol{D^2 + 6D + 13 = 0}$

$\Longrightarrow D = \frac{-6 \pm \sqrt{36 - 52}}{2}$

$\Longrightarrow D = \dfrac{-6 \pm \sqrt{16i^2}}{2}$

$\Longrightarrow D = \dfrac{-6 \pm 4i}{2}$

$\Longrightarrow D = \dfrac{2(-3 \pm 2i)}{2}$

$\Longrightarrow D = -3 \pm 2i$

Therefore, the complementary solution is

$y_c = (c_1 cos2x + c_2 sin2x)e^{-3x}$

$\because y_p = 0.$ *Therefore*

$y = (c_1 cos2x + c_2 sin2x)e^{-3x} - - - (a)$

Differentiating w.r.t "x", we have

$y' = (-2c_1 sin2x + 2c_2 cos2x)e^{-3x} - 3(c_1 cos2x + c_2 sin2x)e^{-3x} - - - (b)$

Applying $\mathbf{y(0) = 3}$ **on** $(\boldsymbol{a})$**, we have,**

$3 = (c_1 cos0 + c_2 sin0)e^0$

$\Longrightarrow \boxed{c_1 = 3}$

Applying $\mathbf{y'(0) = -1}$ **on** $(\boldsymbol{b})$**, we have,**

$-1 = 2c_2 - 3c_1$

$\Longrightarrow -1 = 2c_2 - 3(3)$

$\Longrightarrow -1 = 2c_2 - 9$

$\Longrightarrow 2c_2 = 8$

$\Longrightarrow \boxed{c_2 = 4}$

Thus equation (a) becomes

$y = (3cos2x + 4sin2x)e^{-3x}$

`is required solution.

Question # 19

$\boldsymbol{(D^3 - 6D^2 + 11D - 6)y = 0}$

$\boldsymbol{y(0) = 0, y'(0) = 0, y''(0) = 2}$

Solution:

Given equation is

$\boldsymbol{(D^3 - 6D^2 + 11D - 6)y = 0} - - - (\boldsymbol{i})$

The characteristics equation of (i) *will be*

$\boldsymbol{D^3 - 6D^2 + 11D - 6 = 0}$

Since $D = 1$ is a root of characteristic equation. So we use synthetic division in order to find the other roots.

	1	-6	11	-6
1	0	1	-5	6
	1	-5	6	0

Now, the residual equation will be

$D^2 - 5D + 6 = 0$

$\Longrightarrow D^2 - 2D - 3D + 6 = 0$

$\Longrightarrow D(D-2) - 3(D-2) = 0$

$\Longrightarrow (D-2)(D-3) = 0$

$\Longrightarrow D = 2 \ or \ D = 3$

Therefore, the complementary solution is

$y_c = c_1 e^x + c_2 e^{2x} + c_3 e^{3x}$

$\because y_p = 0.$ *Therefore*

$y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x} - - - (a)$

Differentiating w.r.t "x", we have

$y' = c_1 e^x + 2c_2 e^{2x} + 3c_3 e^{3x} - - - (b)$

Umer Asghar (umermth2016@gmail.com) **For Online Skype Tuition (Skype ID): sp15mmth06678**

Applying $\mathbf{y(0) = 0}$ **on** $(\boldsymbol{b})$**, we have,**

$0 = c_1 + c_2 + c_3$

$\Longrightarrow c_1 + c_2 + c_3 = 0 - - - (c)$

Applying $\mathbf{y'(0) = 0}$ **on** $(\boldsymbol{b})$**, we have,**

$0 = c_1 + 2c_2 + 3c_3$

$\Longrightarrow c_1 + 2c_2 + 3c_3 = 0 - - - (d)$

Now, differentiating (b) $w.r.t$ "x", we have

$y'' = c_1e^x + 4c_2e^{2x} + 9c_3e^{3x} - - - (e)$

Applying $\mathbf{y''(0) = 2}$ **on** $(\boldsymbol{b})$**, we have,**

$2 = c_1 + 4c_2 + 9c_3$

$\Longrightarrow c_1 + 4c_2 + 9c_3 = 2 - - - (f)$

BY $(c) - (d)$, we have

$-c_2 - 2c_3 = 0$

$\Longrightarrow c_2 = -2c_3 - - - (g)$

BY $(c) - (f)$, we have

$-3c_2 - 8c_3 = -2$

$\Longrightarrow -(3c_2 + 8c_3) = -2$

$\Longrightarrow 3c_2 + 8c_3 = 2$

$\Longrightarrow 3(-2c_3) + 8c_3 = 2 \quad \because c_2 = -2c_3$

$\Longrightarrow -6c_3 + 8c_3 = 2$

$\Longrightarrow \boxed{\boldsymbol{c_3 = 1}}$

$\Longrightarrow \boxed{\boldsymbol{c_2 = -2}}$

$\Longrightarrow c_1 - 2 + 1 = 0$

$\Longrightarrow \boxed{c_1 = 1}$

Thus equation (a) becomes

$y = e^x - 2e^{2x} + e^{3x}$

is required solution.

Question # 20

$(\boldsymbol{D^4 - D^3})\boldsymbol{y = 0}$

$\boldsymbol{y(0) = 0, y'(0) = 1, y''(1) = 3e, y'''(1) = e}$

Solution:

Given equation is

$(\boldsymbol{D^4 - D^3})\boldsymbol{y = 0} - - - (\boldsymbol{i})$

The characteristics equation of (i) *will be*

$\boldsymbol{D^4 - D^3 = 0}$

$\Longrightarrow D^3(D - 1) = 0$

$\Longrightarrow D^3 = 0; D - 1 = 0$

$\Longrightarrow D = \{0,0,0,1\}$

Therefore, the complementary solution is

$y_c = c_1e^0 + c_2xe^0 + c_3x^2e^0 + c_4e^x$

$\because y_p = 0.$ *Therefore*

$y = c_1 + c_2x + c_3x^2 + c_4e^x - - - (a)$

Differentiating w.r.t "x", we have

$y' = c_2 + 2c_3x + c_4e^x - - - (b)$

Again differentiating w.r.t "x", we have

$y'' = 2c_3 + c_4e^x - - - -(c)$

Again differentiating w.r.t "x", we have

$y''' = c_4e^x - - - -(d)$

Applying $\mathbf{y(0) = 1}$ **on** $(\boldsymbol{a})$**, we have,**

$1 = c_1 + c_4$

$\Longrightarrow c_1 + c_4 = 1 - - - (1)$

Applying $\mathbf{y'(0) = 1}$ **on** $(\boldsymbol{b})$**, we have,**

$1 = c_2 + c_4$

Applying $\mathbf{y''(1) = 3e}$ **on** $(\boldsymbol{c})$**, we have,**

$3e = 2c_3 + c_4 e$

$\Longrightarrow 2c_3 + c_4 e = 3e - - - (3)$

Applying $\mathbf{y'''(1) = e}$ **on** $(\boldsymbol{d})$**, we have,**

$e = c_4 e$

$\Longrightarrow \boxed{c_4 = 1}$

$(3) \Longrightarrow \boxed{\boldsymbol{c_3 = 1}}$

$(2) \Longrightarrow \boxed{\boldsymbol{c_2 = 0}}$

$(1) \Longrightarrow \boxed{\boldsymbol{c_1 = 0}}$

Thus equation (a) becomes

$y = x^2 + e^x$

is required solution.

THE END

Written by:

Muhammad umer asghar

Sese (math)

Govt. elementary school malwal

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Contact #:

03074896454