EXERCISE # 10.1

**Question # 1:** \((9D^2 - 12D + 4)y = 0\)

**Solution:**

Given equation is

\((9D^2 - 12D + 4)y = 0 \quad \text{(i)}\)

The characteristics equation of (i) will be

\(9D^2 - 12D + 4 = 0\)

\[\Rightarrow D = \frac{-(12) \pm \sqrt{(-12)^2 - 4(9)(4)}}{18}\]

\[\Rightarrow D = \frac{12 \pm \sqrt{144 - 144}}{18}\]

\[D = \frac{12}{18} + 0; \quad D = \frac{12}{18} - 0\]

\[D = \frac{2}{3}, \frac{2}{3}\]

Therefore, the complementary function is

\[y_c = c_1e^{\left(\frac{2}{3}x\right)} + c_2xe^{\left(\frac{2}{3}x\right)}\]

\[\Rightarrow y = (c_1 + c_2x)e^{\left(\frac{2}{3}x\right)}; \quad y_c = y\]

is required solution.

**Question # 2:** \((75D^2 + 50D + 12)y = 0\)

**Solution:**

Given equation is

\((75D^2 + 50D + 12)y = 0 \quad \text{(i)}\)

The characteristics equation of (i) will be

\(75D^2 + 50D + 12 = 0\)

\[\Rightarrow D = \frac{-50 \pm \sqrt{(50)^2 - 4(75)(12)}}{2(75)}\]

\[\Rightarrow D = \frac{-50 \pm \sqrt{2500 - 3600}}{150}\]

\[D = \frac{-50 \pm \sqrt{-1100}}{150}\]

\[\Rightarrow D = \frac{-50 \pm 10\sqrt{-11}}{150}\]

\[\Rightarrow D = \frac{-5 \pm i\sqrt{11}}{15}\]

\[\Rightarrow D = \frac{-1}{3} + \frac{i\sqrt{11}}{15}\]

Therefore, the complementary function is

\[y_c = \left( c_1\cos\left(\frac{\sqrt{11}}{15}x\right) + c_2\sin\left(\frac{\sqrt{11}}{15}x\right) \right)e^{-\frac{1}{3}x}\]

\[\Rightarrow y = \left( c_1\cos\left(\frac{\sqrt{11}}{15}x\right) + c_2\sin\left(\frac{\sqrt{11}}{15}x\right) \right)e^{-\frac{1}{3}x}\]

is required solution.

**Question # 3:** \((D^3 - 4D^2 + D + 6)y = 0\)

**Solution:**

Given equation is

\((D^3 - 4D^2 + D + 6)y = 0\)
The characteristics equation of (i) will be
\[ D^3 - 4D^2 + D + 6 = 0 \]
Since \( D = -1 \) is a root of characteristic equation. So we use synthetic division in order to find the other roots.

```
| 1 | -4 | 1 | 6 |
-1----------
| 0 | -1 | 5 | -6 |
```

Therefore, the complementary function is
\[ y_c = c_1e^{-x} + c_2 \cos x + c_3 \sin x \]
\[ y = c_1e^{-x} + c_2 \cos x + c_3 \sin x \implies y_c = y \]
is required solution.

```
Question #5: (D^3 - 6D^2 + 12D - 8)y = 0
```

Solution:

Given equation is
\[ (D^3 - 6D^2 + 12D - 8)y = 0 \]
The characteristics equation of (i) will be
\[ D^3 - 6D^2 + 12D - 8 = 0 \]
Since \( D = 2 \) is a root of characteristic equation. So we use synthetic division in order to find the other roots.

```
| 2 | 1 | -6 | 12 | -8 |
| 0 | 2 | -8 |  8 |
| 1 | -4 |  4 |  0 |
```

Therefore, the complementary function is
\[ y_c = e^{2x} + c_2x e^{2x} + c_3x^2 e^{3x} \]
\[ y = (c_1 + c_2x + c_3x^2)e^{2x} \implies y_c = y \]
is required solution.
**Question #6:** \((D^3 - 6D^2 + 3D + 10)y = 0\)

**Solution:**

Given equation is

\((D^3 - 6D^2 + 3D + 10)y = 0 - - - (i)\)

The characteristics equation of (i) will be

\[D^3 - 6D^2 + 3D + 10 = 0\]

Since \(D = 2\) is a root of characteristic equation. So we use synthetic division in order to find the other roots.

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<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>-8</td>
<td>-10</td>
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<td></td>
<td>1</td>
<td>-4</td>
<td>-5</td>
<td>0</td>
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</table>

Now, the residual equation will be

\[D^2 - 4D - 5 = 0\]

\[\Rightarrow D^2 - 5D + D - 5 = 0\]

\[\Rightarrow (D + 1)(D - 5) = 0\]

\[\Rightarrow D = -1 \text{ or } D = 5\]

Therefore, the complementary solution is

\[y_c = c_1e^{2x} + c_2e^{-x} + c_3e^{5x}\]

\[\Rightarrow y = c_1e^{2x} + c_2e^{-x} + c_3e^{5x} \quad \therefore y_c = y\]

is required solution.

**Question #7:** \((D^3 - 27)y = 0\)

**Solution:**

Given equation is

\((4D^4 - 4D^3 - 3D^2 + 4D - 1)y = 0 - - - (i)\)

The characteristics equation of (i) will be

\[4D^4 - 4D^3 - 3D^2 + 4D - 1 = 0\]

Since \(D = 1, -1\) are the roots of characteristic equation. So we use synthetic division in order
to find the other roots. The synthetic division is as follow:-

\[
\begin{array}{c|ccccc}
1 & 4 & -4 & -3 & 4 & -1 \\
0 & 4 & 0 & 0 & -3 & 1 \\
-1 & 0 & -4 & 4 & -1 \\
\hline
4 & -4 & 1 & 0 \\
\end{array}
\]

Now, the residual equation will be

\[4D^2 - 4D + 1 = 0\]

\[\Rightarrow (2D - 1)(2D - 1) = 0\]

\[\Rightarrow 2D = 1; 2D = 1\]

\[\Rightarrow D = \frac{1}{2}; D = \frac{1}{2}\]

Therefore, the complementary solution is

\[y_c = c_1 e^{x} + c_2 e^{-x} + c_3 e^{\frac{x}{2}} + c_4 e^{-\frac{x}{2}}\]

OR \ \[y = c_1 e^{x} + c_2 e^{-x} + (c_3 + c_4 x)e^{\frac{x}{2}}\]

is required solution.

**Question #9**: \((D^4 + 2D^3 - 2D^2 - 6D + 5)y = 0\)

**Solution:**

Given equation is

\((D^4 + 2D^3 - 2D^2 - 6D + 5)y = 0 \ \ \ \ \ (i)\)

The characteristics equation of \((i)\) will be

\[D^4 + 2D^3 - 2D^2 - 6D + 5 = 0\]

Since \(D = 1,1\) are the roots of characteristic equation. So we use synthetic division in order to find the other roots. The synthetic division is as follow:-

\[
\begin{array}{c|ccccc}
1 & 1 & 2 & -2 & -6 & 5 \\
0 & 1 & 3 & 1 & -5 \\
1 & 3 & 1 & -5 & 0 \\
0 & 1 & 4 & 5 \\
1 & 4 & 5 & 0 \\
\end{array}
\]

Now, the residual equation will be

\[D^2 + 4D + 5 = 0\]

\[\Rightarrow D = \frac{-4 \pm \sqrt{16 - 20}}{2}\]

\[\Rightarrow D = \frac{-4 \pm \sqrt{-4}}{2}\]

\[\Rightarrow D = \frac{-4 \pm 2t}{2}\]

\[\Rightarrow D = -2 \pm i\]

Therefore, the complementary solution is

\[y_c = c_1 e^{x} + c_2 xe^{x} + (c_3 \cos x + c_4 \sin x)e^{-2x}\]

\[\Rightarrow y = (c_1 + c_2 x)e^{x} + (c_3 \cos x + c_4 \sin x)e^{-2x}\]

is required solution.

**Question #10**: \((D^4 - 5D^3 + 6D^2 + 4D - 8)y = 0\)

**Solution:**

Given equation is

\[(D^4 - 5D^3 + 6D^2 + 4D - 8)y = 0 \ \ \ \ \ (i)\]

The characteristics equation of \((i)\) will be

\[D^4 - 5D^3 + 6D^2 + 4D - 8 = 0\]

Since \(D = -1,2\) are the roots of characteristic equation. So we use synthetic division in order to find the other roots. The synthetic division is as follow:-
Now, the residual equation will be
\[ D^2 - 4D + 4 = 0 \]
\[ \Rightarrow D^2 - 2D - 2D + 4 = 0 \]
\[ \Rightarrow D(D - 2) - 2(D - 2) \]
\[ \Rightarrow (D - 2)(D - 2) = 0 \]
\[ \Rightarrow D = 2,2 \]
Therefore, the complementary solution is
\[ y_c = c_1 e^{-x} + c_2 e^{2x} + c_3 x e^{2x} + c_4 x^2 e^{2x} \]
\[ y_c = c_1 e^{-x} + (c_2 + c_3 x + c_4 x^2) e^{2x} \]
OR
\[ y = c_1 e^{-x} + (c_2 + c_3 x + c_4 x^2) e^{2x} \]
is required solution.

**Question # 11**

\[(D^4 - 4D^3 - 7D^2 + 22D + 24)y = 0\]

**Solution:**

Given equation is
\[(D^4 - 4D^3 - 7D^2 + 22D + 24)y = 0 - - - (i)\]

The characteristics equation of (i) will be
\[D^4 - 4D^3 - 7D^2 + 22D + 24 = 0\]
Since \(D = -1,-2\) are the roots of characteristic equation. So we use synthetic division in order to find the other roots. The synthetic division is as follow:-

\[\begin{array}{cccccc}
-1 & 1 & -5 & 6 & 4 & -8 \\
-1 & 1 & -1 & 6 & -12 & 8 \\
2 & 0 & 2 & -8 & 8 & 0 \\
1 & -4 & 4 & 0 & -1 & 6 & -12 & 8 \\
\end{array}\]

Now, the residual equation will be
\[ D^2 - 7D + 12 = 0 \]
\[ \Rightarrow D^2 - 3D - 4D + 12 = 0 \]
\[ \Rightarrow D(D - 3) - 4(D - 3) = 0 \]
\[ \Rightarrow (D - 3)(D - 4) = 0 \]
\[ \Rightarrow D = 3,4 \]
Therefore, the complementary solution is
\[ y_c = c_1 e^{1x} + c_2 e^{3x} + c_3 e^{-2x} + c_4 e^{-x} \]
OR
\[ y = c_1 e^{1x} + (c_2 + c_3 x + c_4 x^2) e^{2x} \]
is required solution.

**Question # 12**

\[(D^4 + 4)y = 0\]

**Solution:**

Given equation is
\[(D^4 + 4)y = 0 - - - (i)\]

The characteristics equation of (i) will be
\[D^4 + 4 = 0\]
\[ \Rightarrow (D^2 + 2)^2 = 0 \]
\[ \Rightarrow (D^2 + 2 + 2D)(D^2 + 2 - 2D) = 0 \]
\[ \Rightarrow D^2 + 2 + 2D = 0 \]
\[ : \quad D^2 - 2D + 2 = 0 \]
Chapter #10: Differential Equations of Higher Order

\( \Rightarrow D = \frac{-2 \pm \sqrt{4 - 8}}{2} \); \( D = \frac{-2 \pm \sqrt{4 - 8}}{2} \)

\( \Rightarrow D = \frac{-2 \pm 2i}{2} \); \( D = \frac{-2 \pm 2i}{2} \)

\( \Rightarrow D = \frac{2(-1 \pm i)}{2} \); \( D = \frac{2(-1 \pm i)}{2} \)

\( \Rightarrow D = -1 \pm i \); \( D = 1 \pm i \)

Therefore, the complementary solution is

\[ y_c = (c_1 \cos x + c_2 \sin x)e^{-x} + (c_3 \cos x + c_4 \sin x)e^x \]

is required solution.

**Question #13** \((D^4 - D^3 - 3D^2 + D + 2)y = 0\)

**Solution:**

Given equation is

\((D^4 - D^3 - 3D^2 + D + 2)y = 0 \quad \ldots (i)\)

The characteristics equation of \((i)\) will be

\[ D^4 - D^3 - 3D^2 + D + 2 = 0 \]

Since \( D = 1,2 \) are the roots of characteristic equation. So we use synthetic division in order to find the other roots. The synthetic division is as follow:-

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</tbody>
</table>

Now, the residual equation will be

\[ D^2 + 2D + 1 = 0 \]

\( \Rightarrow (D + 1)^2 = 0 \)

\( \Rightarrow (D + 1)(D + 1) = 0 \)

\( \Rightarrow D = -1, -1 \)

Therefore, the complementary solution is

\[ y_c = c_1 e^x + c_2 e^{2x} + c_3 e^{-x} + c_4 xe^{-x} \]

\( \therefore y_p = 0. \) Therefore,

\[ y = c_1 e^x + c_2 e^{2x} + c_3 e^{-x} + c_4 xe^{-x} \]

is required solution.

**Question #14** \((16D^6 + 8D^4 + D^2)y = 0\)

**Solution:**

Given equation is

\((16D^6 + 8D^4 + D^2)y = 0 \quad \ldots (i)\)

The characteristics equation of \((i)\) will be

\[ 16D^6 + 8D^4 + D^2 = 0 \]

\( \Rightarrow D^2(16D^4 + 8D^2 + 1) = 0 \)

\( \Rightarrow D^2(4D^2 + 1)^2 = 0 \)

\( \Rightarrow D^2 = 0 \quad ; \quad (4D^2 + 1)^2 = 0 \)

\( D = 0,0 \)

& \( (4D^2 + 1)(4D^2 + 1) = 0 \)

\( 4D^2 = -1 \quad \text{or} \quad 4D^2 = -1 \)

\( \Rightarrow D^2 = \frac{-1}{4} \quad \text{or} \quad D^2 = \frac{-1}{4} \)

\( \Rightarrow D = \sqrt{\frac{i^2}{4}} \quad \text{or} \quad D = \sqrt{\frac{i^2}{4}} \)

\( \Rightarrow D = \pm \frac{i}{2}; D = \pm \frac{i}{2} \)

Therefore, the complementary solution is

\[ y_c = c_1 e^{0x} + c_2 xe^{0x} + c_3 \cos \frac{x}{2} + c_4 x \cos \frac{x}{2} + c_5 \sin \frac{x}{2} + c_6 x \sin \frac{x}{2} \]
\[ y_c = (c_1 + c_2x) + (c_3 + c_4x)\cos \frac{x}{2} + (c_5 + c_6x)\sin \frac{x}{2} \]
\[ \therefore y_p = 0. \text{ Therefore,} \]
\[ y = (c_1 + c_2x) + (c_3 + c_4x)\cos \frac{x}{2} + (c_5 + c_6x)\sin \frac{x}{2} \]

is required solution.

**Question #15**

\[ (D^4 + 6D^3 + 15D^2 + 20D + 12)y = 0 \]

**Solution:**

Given equation is
\[ (D^4 + 6D^3 + 15D^2 + 20D + 12)y = 0 \quad -(i) \]

The characteristics equation of \((i)\) will be
\[ D^4 + 6D^3 + 15D^2 + 20D + 12 = 0 \]

Since \(D = -2, -2\) are the roots of characteristic equation. So we use synthetic division in order to find the other roots. The synthetic division is as follow:

<table>
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<th>(-2)</th>
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<th>15</th>
<th>20</th>
<th>12</th>
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<td>0</td>
<td>2</td>
<td>-4</td>
<td>-6</td>
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<td>1</td>
<td>2</td>
<td>3</td>
<td>0</td>
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</table>

Now, the residual equation will be
\[ D^2 + 2D + 3 = 0 \]
\[ \Rightarrow D = \frac{-2 \pm \sqrt{4 - 12}}{2} \]
\[ \Rightarrow D = -1 \pm i\sqrt{2} \]

Therefore, the complementary solution is
\[ y_c = (c_1 + c_2x)e^{-2x} + (c_3\cos\sqrt{2}x + c_4\sin\sqrt{2}x)e^{-x} \]
\[ \therefore y_p = 0. \text{ Therefore,} \]
\[ y = (c_1 + c_2x)e^{-2x} + (c_3\cos\sqrt{2}x + c_4\sin\sqrt{2}x)e^{-x} \]
is required solution.

**Solve each of the following initial problem.**

**Question #16**

\[ (D^2 + 8D - 9)y = 0 \quad y(1) = 1, y'(1) = 0 \]

**Solution:**

Given equation is
\[ (D^2 + 8D - 9)y = 0 \quad -(i) \]

The characteristics equation of \((i)\) will be
\[ D^2 + 8D - 9 = 0 \]
\[ \Rightarrow D^2 + 9D - D - 9 = 0 \]
\[ \Rightarrow D(D + 9) - 1(D + 9) = 0 \]
\[ \Rightarrow (D - 1)(D + 9) = 0 \]
\[ \Rightarrow D = 1 \quad \text{or} \quad D = -9 \]

Therefore, the complementary solution is
\[ y_c = c_1e^x + c_2e^{-9x} \]
\[ \therefore y_p = 0. \text{ Therefore} \]
\[ y = c_1e^x + c_2e^{-9x} \quad -(a) \]

Differentiating w.r.t "x", we have
\[ y' = c_1e^x - 9c_2e^{-9x} \quad -(b) \]

Applying \(y(1) = 1\) on \((a)\), we have,
\[ 1 = c_1e^1 + c_2e^{-9} \]
\[ \Rightarrow c_1e + \frac{c_2}{e^9} = 1 \quad -(c) \]

Applying \(y'(1) = 0\) on \((b)\), we have,
\[0 = c_1 e^{-9c_2 e^{-9}}\]
\[\Rightarrow c_1 e = 9c_2 e^{-9} \quad \text{--- (d)}\]

Using (d) in (c), we have
\[9c_2 e^{-9} + \frac{c_2}{e^9} = 1\]
\[\Rightarrow \frac{9c_2}{e^9} + \frac{c_2}{e^9} = 1\]
\[\Rightarrow \frac{10c_2}{e^9} = 1\]
\[\Rightarrow c_2 = \frac{e^9}{10}\]

Now (d) \[\Rightarrow \]
\[c_1 e = \frac{9e^9}{10} e^{-9}\]
\[\Rightarrow c_1 = \frac{9}{10e}\]

Thus equation (a) becomes
\[y = \frac{9}{10e} e^x + \frac{e^9}{10} e^{-9x}\]
\[y = \frac{9}{10} e^{x-1} + \frac{1}{10} e^{-9(x-1)}\]
is required solution.

**Question #17**

\[(D^2 + 6D + 9)y = 0 \quad y(0) = 2, \ y'(0) = -3\]

**Solution:**

**Given equation is**
\[(D^2 + 6D + 9)y = 0 \quad \text{--- (i)}\]

**The characteristics equation of (i) will be**
\[D^2 + 6D + 9 = 0\]
\[\Rightarrow D^2 + 3D + 3D + 9 = 0\]
\[\Rightarrow D(D + 3) + 3(D + 3)\]
\[\Rightarrow (D + 3)(D + 3) = 0\]
\[\Rightarrow D = -3 \quad D = -3\]

Therefore, the complementary solution is
\[y_c = c_1 e^{-3x} + c_2 xe^{-3x}\]
\[\therefore y_p = 0. \text{ Therefore}\]
\[y = c_1 e^{-3x} + c_2 xe^{-3x} \quad \text{--- (a)}\]

Differentiating w.r.t "x", we have
\[y' = -3c_1 e^{-3x} + c_2 e^{-3x} - 3xe^{-3x} \quad \text{--- (b)}\]

Applying \(y(0) = 2\) on (a), we have,
\[2 = c_1\]

Applying \(y'(0) = -3\) on (b), we have,
\[-3 = -3c_1 + c_2\]
\[\Rightarrow c_2 = 3\]

Thus equation (a) becomes
\[y = 2e^{-3x} + 3xe^{-3x}\]
\[\Rightarrow y = (2 + 3x)e^{-3x}\]
is required solution.

**Question #18**

\[(D^2 + 6D + 13)y = 0 \quad y(0) = 3, \ y'(0) = -1\]

**Solution:**

**Given equation is**
\[(D^2 + 6D + 13)y = 0 \quad \text{--- (i)}\]

**The characteristics equation of (i) will be**
\[D^2 + 6D + 13 = 0\]
\[\Rightarrow D = \frac{-6 \pm \sqrt{36 - 52}}{2}\]

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Therefore, the complementary solution is

\[ y_c = (c_1 \cos 2x + c_2 \sin 2x)e^{-3x} \]

\[ \therefore y_p = 0. \text{ Therefore} \]

\[ y = (c_1 \cos 2x + c_2 \sin 2x)e^{-3x} \]  \hspace{1cm} (a)

Differentiating w.r.t \( x \), we have

\[ y' = (-2c_1 \sin 2x + 2c_2 \cos 2x)e^{-3x} - 3(c_1 \cos 2x + c_2 \sin 2x)e^{-3x} \]  \hspace{1cm} (b)

Applying \( y(0) = 3 \) on (a), we have,

\[ 3 = (c_1 \cos 0 + c_2 \sin 0)e^0 \]

\[ \Rightarrow c_1 = 3 \]

Applying \( y'(0) = -1 \) on (b), we have,

\[ -1 = 2c_2 - 3c_1 \]

\[ \Rightarrow -1 = 2c_2 - 3(3) \]

\[ -1 = 2c_2 - 9 \]

\[ 2c_2 = 8 \]

\[ \Rightarrow c_2 = 4 \]

Thus equation (a) becomes

\[ y = (3 \cos 2x + 4 \sin 2x)e^{-3x} \]

'is required solution.

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**Question # 19**

\[ (D^3 - 6D^2 + 11D - 6)y = 0 \]

\[ y(0) = 0, y'(0) = 0, y''(0) = 2 \]

**Solution:**

Given equation is

\[ (D^3 - 6D^2 + 11D - 6)y = 0 \]  \hspace{1cm} (i)

The characteristics equation of \( (i) \) will be

\[ D^3 - 6D^2 + 11D - 6 = 0 \]

Since \( D = 1 \) is a root of characteristic equation. So we use synthetic division in order to find the other roots.

\[ \begin{array}{c|cccc}
1 & -6 & 11 & -6 \\
\hline
1 & 0 & 1 & -5 & 6 \\
1 & 0 & -5 & 6 & 0
\end{array} \]

Now, the residual equation will be

\[ D^2 - 5D + 6 = 0 \]

\[ \Rightarrow D^2 - 2D - 3D + 6 = 0 \]

\[ \Rightarrow D(D - 2) - 3(D - 2) = 0 \]

\[ \Rightarrow (D - 2)(D - 3) = 0 \]

\[ \Rightarrow D = 2 \text{ or } D = 3 \]

Therefore, the complementary solution is

\[ y_c = c_1 e^x + c_2 e^{2x} + c_3 e^{3x} \]

\[ \therefore y_p = 0. \text{ Therefore} \]

\[ y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x} \]  \hspace{1cm} (a)

Differentiating w.r.t "x", we have

\[ y' = c_1 e^x + 2c_2 e^{2x} + 3c_3 e^{3x} \]  \hspace{1cm} (b)
Applying \( y(0) = 0 \) on \( (b) \), we have,
\[
0 = c_1 + c_2 + c_3
\]
\[
\Rightarrow c_1 + c_2 + c_3 = 0 \quad -(c)
\]
Applying \( y'(0) = 0 \) on \( (b) \), we have,
\[
0 = c_1 + 2c_2 + 3c_3
\]
\[
\Rightarrow c_1 + 2c_2 + 3c_3 = 0 \quad -(d)
\]
Now, differentiating \( (b) \) w.r.t "\( x \)", we have
\[
y'' = c_1 e^x + 4c_2 e^{2x} + 9c_3 e^{3x} \quad -(e)
\]
Applying \( y''(0) = 2 \) on \( (b) \), we have,
\[
2 = c_1 + 4c_2 + 9c_3
\]
\[
\Rightarrow c_1 + 4c_2 + 9c_3 = 2 \quad -(f)
\]
By \( (c) - (d) \), we have
\[
-c_2 - 2c_3 = 0
\]
\[
\Rightarrow c_2 = -2c_3 \quad -(g)
\]
By \( (c) - (f) \), we have
\[
-3c_2 - 8c_3 = -2
\]
\[
\Rightarrow -(3c_2 + 8c_3) = -2
\]
\[
\Rightarrow 3c_2 + 8c_3 = 2
\]
\[
\Rightarrow 3(-2c_3) + 8c_3 = 2 \quad \text{∴ } c_2 = -2c_3
\]
\[
\Rightarrow -6c_3 + 8c_3 = 2
\]
\[
\Rightarrow c_3 = 1
\]
\[
\Rightarrow c_2 = -2
\]
\[
\Rightarrow c_1 - 2 + 1 = 0
\]
\[
\Rightarrow c_1 = 1
\]

Thus equation \((a)\) becomes
\[
y = e^x - 2e^{2x} + e^{3x}
\]
is required solution.

**Question #20**

\[
(D^4 - D^3)y = 0
\]
\[
y(0) = 0, y'(0) = 1, y''(1) = 3e, y'''(1) = e
\]

**Solution:**

Given equation is
\[
(D^4 - D^3)y = 0 \quad -(i)
\]
The characteristics equation of \((i)\) will be
\[
D^4 - D^3 = 0
\]
\[
\Rightarrow D^3(D - 1) = 0
\]
\[
\Rightarrow D^3 = 0; D - 1 = 0
\]
\[
\Rightarrow D = \{0,0,0,1\}
\]
Therefore, the complementary solution is
\[
y_c = c_1 e^0 + c_2 xe^0 + c_3 x^2 e^0 + c_4 e^x
\]
\[
\therefore y_p = 0. \text{ Therefore}
\]
\[
y = c_1 + c_2 x + c_3 x^2 + c_4 e^x \quad -(a)
\]
Differentiating w.r.t "\( x \)", we have
\[
y' = c_2 + 2c_3 x + c_4 e^x \quad -(b)
\]
Again differentiating w.r.t "\( x \)", we have
\[
y'' = 2c_3 + c_4 e^x \quad -(c)
\]
Again differentiating w.r.t "\( x \)", we have
\[
y''' = c_4 e^x \quad -(d)
\]
Applying \( y(0) = 1 \) on \( (a) \), we have,
\[
1 = c_1 + c_4
\]
\[ c_1 + c_4 = 1 \quad \text{ (1)} \]

Applying \( y'(0) = 1 \) on (b), we have,
\[ 1 = c_2 + c_4 \]

Applying \( y''(1) = 3e \) on (c), we have,
\[ 3e = 2c_3 + c_4 e \]
\[ \Rightarrow 2c_3 + c_4 e = 3e - - - \quad (3) \]

Applying \( y'''(1) = e \) on (d), we have,
\[ e = c_4 e \]
\[ \Rightarrow c_4 = 1 \]

(3) \[ \Rightarrow c_3 = 1 \]

(2) \[ \Rightarrow c_2 = 0 \]

(1) \[ \Rightarrow c_1 = 0 \]

Thus equation (a) becomes
\[ y = x^2 + e^x \]

is required solution.

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