9.2-01

15

Differential Eqs of First order and First Degree...

An ordinary differential eq of first order and first degree can be expressed as $\frac{dy}{dx} = f(x,y)$

or $M(x,y)dx + N(x,y)dy = 0$

The General sol of such an eq will contain only one arbitrary const.

We discuss these types of diffl eqs of first degree & first order.

1) Separable Variables Eqs
2) Homogeneous Eqs
3) Non Homogenous Eqs.
4) Exact Eqs
5) Non Exact. Eqs
6) Linear Eqs
7) Bernoulli Eqs

Type 1 Separable Eq. is a diffl eq of the form $M(x)dx + N(y)dy = 0$

where $M(x)$ is fn of x alone and

$N(y)$ is fn of y alone.

To Solve we separate variables & integrate.

Note Constant of integration 'c' can be replaced by $\log c$, e^c, $\tan^{-1}c$ etc whichever is suitable for simplification.

Note Since '$\ln x$' when x is negative is not defined so it is better to write $\ln|x|$ i.e. modulus when there is possibility of a -ve number.

16

Ex 9.2

Solve

(1) $\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$

$y\,dy = \frac{x^2}{1+x^3}dx$

$\int y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^3}dx$

$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3) + c$

$\frac{3y^2}{2} = \ln(1+x^3) + 3c$

$3y^2 = 2\ln(1+x^3) + 6c$

$3y^2 = 2\ln(1+x^3) + c'$

(2) $\frac{dy}{dx} + y^2\sin x = 0$

$\frac{dy}{dx} = -y^2\sin x$

$\int\frac{dy}{y^2} = -\int\sin x\,dx$

$\frac{y^{-1}}{-1} = -(-\cos x) + c$

$-\frac{1}{y} = \cos x + c$

(3) $\frac{dy}{dx} = 1+x+y^2+xy^2$

$\frac{dy}{dx} = (1+x)+y^2(1+x)$

$\frac{dy}{dx} = (1+x)(1+y^2)$

$\int\frac{dy}{1+y^2} = \int(1+x)dx$

$\tan^{-1}y = x + \frac{x^2}{2} + c$

$2\tan^{-1}y = 2x+x^2+c'$

(5) $\frac{dy}{dx} = 2x^2+y-x^2y+xy-2x-2$

$= 2x^2-2x-2+y-x^2y+xy$

$= 2(x^2-x-1) - y(-1+x^2-x)$

$\frac{dy}{dx} = (x^2-x-1)(2-y)$

$\int\frac{dy}{2-y} = \int(x^2-x-1)dx$

$-\int\frac{-dy}{2-y} = \int(x^2-x-1)dx$

$-\ln|2-y| = \frac{x^3}{3} - \frac{x^2}{2} - x + c$

$-\ln|2-y| = \frac{2x^3-3x^2-6x+6c}{6}$

$-6\ln|2-y| = 2x^3-3x^2-6x+6c$

$\ln|2-y|^{-6} = (2x^3-3x^2-6x+6c)\ln e$

$\ln|2-y|^{-6} = \ln e^{2x^3-3x^2-6x+6c}$

$|2-y|^{-6} = e^{2x^3-3x^2-6x}\cdot e^{6c}$

$|2-y|^{-6} = c_1 e^{2x^3-3x^2-6x}$

(4) $(xy+2x+y+2)dx + (x^2+2x)dy = 0$

$[x(y+2)+(y+2)]dx + x(x+2)dy = 0$

$[(y+2)(x+1)]dx + x(x+2)dy = 0$

$\div$ by $x(x+2)(y+2)$

$\frac{x+1}{x(x+2)}dx + \frac{1}{y+2}dy = 0$

$\int\frac{x+1}{x^2+2x}dx + \int\frac{dy}{y+2} = 0$

$\frac{1}{2}\int\frac{2x+2}{x^2+2x}dx + \int\frac{dy}{y+2} = 0$

$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x)+\ln c$

$y+2 = \frac{c}{\sqrt{x^2+2x}}$

⑥ $\operatorname{Cosec} y\, dx + \sec x\, dy = 0$

$\div$ by $\operatorname{Cosec} y \sec x$

$$\Rightarrow \frac{1}{\sec x} dx + \frac{dy}{\operatorname{Cosec} y} = 0$$

$$\Rightarrow \int \cos x\, dx + \int \sin y\, dy = \int 0\, dx$$

$$\Rightarrow \sin x - \cos y = c$$ general Sol

⑦ $y(1+x)dx + x(1+y)dy = 0$

$\div$ by xy

$$\Rightarrow \frac{(1+x)}{x} dx + \frac{(1+y)}{y} dy = 0$$

$$\Rightarrow \int\left(\frac{1}{x}+1\right)dx + \int\left(\frac{1}{y}+1\right)dy = \int 0\, dx$$

$$\Rightarrow \ln x + x + \ln y + y = c$$

$$\Rightarrow x + y + \ln(xy) = c$$

⑧ $y\sqrt{1+x^2}\, dx + x\sqrt{1+y^2}\, dy = 0$

$\div$ by xy

$$\Rightarrow \int \frac{\sqrt{1+x^2}}{x} dx + \int \frac{\sqrt{1+y^2}}{y} dy = \int 0\, dx$$

Put $\sqrt{1+x^2} = t$

$1+x^2 = t^2$

$2x\,dx = 2t\,dt$

$x\,dx = t\,dt$

Put $\sqrt{1+y^2} = z$

$1+y^2 = z^2$

$2y\,dy = 2z\,dz$

$y\,dy = z\,dz$

Therefore

$$\Rightarrow \int \frac{\sqrt{1+x^2}}{x^2} x\,dx + \int \frac{\sqrt{1+y^2}}{y^2} y\,dy = \int 0\,dx$$

$$\Rightarrow \int \frac{t \cdot t\,dt}{t^2-1} + \int \frac{z \cdot z\,dz}{z^2-1} = c$$

$$\Rightarrow \int\left(\frac{t^2-1+1}{t^2-1}\right)dt + \int \frac{z^2-1+1}{z^2-1} dz = c$$

$$\Rightarrow \int\left(1 + \frac{1}{t^2-1}\right)dt + \int\left(1 + \frac{1}{z^2-1}\right)dz = c$$

$$\Rightarrow t + \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right) + z + \frac{1}{2}\ln\left(\frac{z-1}{z+1}\right) = c$$

$$\Rightarrow \sqrt{1+x^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + \sqrt{1+y^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+y^2}-1}{\sqrt{1+y^2}+1}\right) = c$$

⑨ $$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}} \quad \text{if } |x|<1, \; |y|<1$$

$$\text{or} \int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1} y = -\sin^{-1} x + c$$

$$y = \sin(c - \sin^{-1} x) \text{ is G.Sol.}$$

$$\frac{dy}{dx} + \sqrt{\frac{y^2-1}{x^2-1}} = 0 \quad \text{if } |x|>1, \; |y|>1$$

$$\frac{dy}{dx} = -\sqrt{\frac{y^2-1}{x^2-1}}$$

$$\int \frac{dy}{\sqrt{y^2-1}} = -\int \frac{dx}{\sqrt{x^2-1}}$$

$$\cosh^{-1} y = -\cosh^{-1} x + c$$

$$y = \cosh(c - \cosh^{-1} x)$$

⑩ $(e^x+1)y\,dy = (y+1)e^x\,dx$

$\div$ by $(e^x+1)(y+1)$

$$\Rightarrow \int \frac{y\,dy}{y+1} = \int \frac{e^x\,dx}{e^x+1}$$

$$\Rightarrow \int\left(\frac{y+1-1}{y+1}\right)dy = \int \frac{e^x}{e^x+1} dx$$

$$\Rightarrow \int\left(1 - \frac{1}{y+1}\right)dy = \int \frac{e^x\,dx}{e^x+1}$$

$$\Rightarrow y - \ln(y+1) = \ln(e^x+1) + \ln c$$

$$\Rightarrow y = \ln(y+1) + \ln(e^x+1) + \ln c$$

$$\Rightarrow y = \ln\{(y+1)(e^x+1)c\}$$

$$\Rightarrow e^y = c(y+1)(e^x+1)$$

(11) $\dfrac{dy}{dx} = \dfrac{y^3+2y}{x^2+3x}$

$\dfrac{dy}{y^3+2y} = \dfrac{dx}{x^2+3x}$

By Partial Fractions from (I) & (II)

$\left[\dfrac{1}{2y} - \dfrac{y}{2(y^2+2)}\right]dy = \left[\dfrac{1}{3x} - \dfrac{1}{3(x+3)}\right]dx$

$\int\dfrac{dy}{2y} - \int\dfrac{(2y)\,dy}{4(y^2+2)} = \int\dfrac{dx}{3x} - \int\dfrac{dx}{3(x+3)}$

$\dfrac{1}{2}\ln y - \dfrac{1}{4}\ln(y^2+2) = \dfrac{1}{3}\ln x - \dfrac{1}{3}\ln(x+3) + \ln c$

$\ln y^{\frac{1}{2}} - \ln(y^2+2)^{\frac{1}{4}} = \ln x^{\frac{1}{3}} - \ln(x+3)^{\frac{1}{3}} + \ln c^{\frac{1}{3}}$

$\ln\left[\dfrac{y^{\frac{1}{2}}}{(y^2+2)^{\frac{1}{4}}}\right] = \ln\left[\dfrac{cx}{(x+3)}\right]^{\frac{1}{3}}$

$\dfrac{y^{\frac{1}{2}}}{(y^2+2)^{\frac{1}{4}}} = \left(\dfrac{cx}{x+3}\right)^{\frac{1}{3}}$ is G. Sol.

Partial Fractions

$\dfrac{1}{y(y^2+2)} = \dfrac{A}{y} + \dfrac{By+C}{y^2+2}$

$1 = A(y^2+2) + (By+C)y$

Put $y=0 \Rightarrow 1 = A(2) \Rightarrow \boxed{A = \frac{1}{2}}$

comparing coeff of $y^2 \rightarrow 0 = A+B$

$0 = \frac{1}{2} + B \quad \boxed{B = -\frac{1}{2}}$

comparing coeff of $y \Rightarrow \boxed{C=0}$

Thus $\dfrac{1}{y(y^2+2)} = \dfrac{1}{2y} - \dfrac{1(y)+0}{2(y^2+2)}$

$\dfrac{1}{y(y^2+2)} = \dfrac{1}{2y} - \dfrac{y}{2(y^2+2)}$ —— (I)

Now $\dfrac{1}{x(x+3)} = \dfrac{A}{x} + \dfrac{B}{x+3}$

$1 = A(x+3) + Bx$

Put $x=0 \Rightarrow \boxed{A = \frac{1}{3}}$

Put $x+3=0 \Rightarrow \boxed{B = -\frac{1}{3}}$

$\therefore \dfrac{1}{x(x+3)} = \dfrac{1}{3x} - \dfrac{1}{3(x+3)}$ —— (II)

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(12) $(\sin x + \cos x)dy + (\cos x - \sin x)dx = 0$

$\div$ by $(\sin x + \cos x)$

$\int dy + \int\dfrac{(\cos x - \sin x)}{(\sin x + \cos x)}dx = \int 0\,dx$

$y + \ln(\sin x + \cos x) = c$

$y\ln e + \ln(\sin x + \cos x) = c\ln e$

$\ln e^y + \ln(\sin x + \cos x) = \ln e^c$

$\ln\left[e^y(\sin x + \cos x)\right] = \ln e^c$

$e^y(\sin x + \cos x) = c'$

$e^y = \dfrac{c'}{\sin x + \cos x}$ G. Sol.

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(13) $(2x\cos y)dx + x^2(\sec y - \sin y)dy = 0$

$\div$ by $x^2\cos y$

$\dfrac{2\cancel{x}\cancel{\cos y}}{x^{\cancel{2}}\cancel{\cos y}}dx + \dfrac{\cancel{x^2}(\sec y - \sin y)}{\cancel{x^2}\cos y}dy = 0$

$\int\dfrac{2dx}{x} + \int\left(\dfrac{\sec y}{\cos y} - \dfrac{\sin y}{\cos y}\right)dy = \int 0$

$2\int\dfrac{dx}{x} + \int(\sec^2 y - \tan y)\,dy = \int 0\,dx$

$2\ln x + \tan y - \ln\sec y = c$

$2\ln x = \ln\sec y - \tan c + c$
G. Sol

19

(13) $e^{x}\left(1+\frac{dy}{dx}\right) = xe^{-y}$

$$1+\frac{dy}{dx} = xe^{-y}e^{-x}$$

$$1+\frac{dy}{dx} = xe^{-(x+y)}$$

Not separable

So Put $z = x+y$

$$\frac{dz}{dx} = 1+\frac{dy}{dx}$$

$$\therefore \frac{dz}{dx} = xe^{-z}$$

$$\int e^{z}\,dz = \int x\,dx$$

$$e^{z} = \frac{x^2}{2}+c$$

$$e^{x+y} = \frac{x^2}{2}+c$$

$$\ln e^{x+y} = \ln\left(\frac{x^2}{2}+c\right)$$

$$(x+y)\ln e = \ln\left(\frac{x^2}{2}+c\right)$$

$$(x+y)\,1 = \ln\left(\frac{x^2}{2}+c\right)$$

$$y = \ln\left(\frac{x^2}{2}+c\right)-x \quad \text{G.Sol}$$

(14) $xe^{x^2+y}\,dx = y\,dy$

$$xe^{x^2}e^{y}\,dx = y\,dy$$

$$xe^{x^2}\,dx = ye^{-y}\,dy$$

$$\frac{1}{2}\int e^{x^2}(2x)\,dx = \int ye^{-y}\,dy \quad \text{IBP}$$

$$\because \begin{cases} e^{x^2} = t & e^{x^2}2x\,dx = dt \\ \int e^{x^2}2x\,dx = \int dt = t = e^{x^2} \end{cases}$$

$$\text{So } \frac{1}{2}e^{x^2} = y\frac{e^{-y}}{-1} - \int 1\cdot\frac{e^{-y}}{-1}\,dy$$

$$= -ye^{-y} + \int e^{-y}\,dy$$

$$= -ye^{-y} + \frac{e^{-y}}{-1} + c$$

$$\frac{1}{2}e^{x^2} = -ye^{-y} - e^{-y} + c$$

$$e^{x^2} = -2e^{-y}(y+1) + 2c$$

$$e^{x^2} = -2e^{-y}(y+1) + c' \quad \text{G.Sol}$$

(16) Solve the initial value problems.

$$2(y-1)\,dy = (3x^2+4x+2)\,dx \qquad y(0) = -1$$

$$2\int (y-1)\,dy = \int (3x^2+4x+2)\,dx$$

$$2\left(\frac{y^2}{2}-y\right) = 3\frac{x^3}{3}+4\frac{x^2}{2}+2x+c$$

$$\cancel{2}\left(\frac{y^2-2y}{\cancel{2}}\right) = x^3+2x^2+2x+c$$

$$\because y(0) = -1$$

$$\therefore (-1)^2-2(-1) = 0+0+0+c$$

$$\boxed{3 = c}$$

$$\text{Hence } y^2-2y = x^3+2x^2+2x+3$$

$$+1-1 \quad y^2-2y+1 = x^3+2x^2+2x+3+1$$

$$(y-1)^2 = x^3+2x^2+2x+4$$

$$y-1 = \pm\sqrt{x^3+2x^2+2x+4}$$

$$y = 1\pm\sqrt{x^3+2x^2+2x+4}$$

$y(0) = -1$ does not satisfy $y = 1+\sqrt{x^3+2x^2+2x+4}$

$$\because -1 = 1+\sqrt{0+0+0+4}$$

$$-1 = 1+2 \quad \text{Impossible}$$

$y(0) = -1$ satisfy $y = 1-\sqrt{x^3+2x^2+2x+4}$

$$-1 = 1-\sqrt{0+0+0+4}$$

$$-1 = 1-2 \quad \text{true}$$

So $y = 1-\sqrt{x^3+2x^2+2x+4}$ is Sol.

20

⑰ $(3x+8)(y^2+4)dx - 4y(x^2+5x+6)dy = 0$, $y(1) = 2$

$\div$ by $(y^2+4)(x^2+5x+6)$

$$\Rightarrow \frac{3x+8}{x^2+5x+6}dx - \frac{4y}{y^2+4}dy = 0$$

By Partial Fractions

$$\frac{3x+8}{x^2+5x+6}$$

$$\therefore \frac{3x+8}{(x+2)(x+3)} = \frac{A}{(x+2)} + \frac{B}{x+3} \quad \text{—(I)}$$

$$3x+8 = A(x+3) + B(x+2) \quad \text{—(II)}$$

Put $x+3=0 \Rightarrow x=-3$
$\Rightarrow \boxed{B=1}$

Put $x+2=0 \Rightarrow x=-2$
$\Rightarrow \boxed{A=2}$

$$\therefore \frac{3x+8}{(x+2)(x+3)} = \frac{2}{(x+2)} + \frac{1}{x+3}$$

$$\Rightarrow \left(\frac{2}{x+2} + \frac{1}{x+3}\right)dx - \frac{4y}{y^2+4}dy = 0$$

$$\Rightarrow \int\frac{2}{x+2}dx + \int\frac{dx}{x+3} - 2\int\frac{2y}{y^2+4}dy = \int 0\,dx$$

$$\Rightarrow 2\ln(x+2) + \ln(x+3) - 2\ln(y^2+4) = \ln c$$

$$\ln\left[\frac{(x+2)^2(x+3)}{(y^2+4)^2}\right] = \ln c$$

antilog $\dfrac{(x+2)^2(x+3)}{(y^2+4)^2} = c$

$\because y(1)=2 \Rightarrow \left.\begin{matrix} y=2 \\ x=1\end{matrix}\right\} \Rightarrow c = \dfrac{(1+2)^2(1+3)}{(2^2+4)^2}$

$$c = \frac{36}{64} = \frac{9}{16}$$

$$\therefore \frac{(x+2)^2(x+3)}{(y^2+4)^2} = \frac{9}{16}$$

$16(x+2)^2(x+3) = 9(y^2+4)^2$ Ans.

(18) $(1+2y^2)\,dy = y\cos x\,dx$, $y(0)=1$

$\div$ by y.

$\Rightarrow \left(\dfrac{1+2y^2}{y}\right)dy = \cos x\,dx$

$\Rightarrow \int\left(\dfrac{1}{y}+2y\right)dy = \int \cos x\,dx$

$\Rightarrow \ln y + \cancel{2}\dfrac{y^2}{\cancel{2}} = \sin x + c$

$\because y(0)=1$

$\therefore \ln 1 + 1 = 0 + c$

$\boxed{1 = c}$

$\Rightarrow \therefore \ln y + y^2 = \sin x + 1$

(19) $8\cos^2 y\,dx + \operatorname{cosec}^2 x\,dy = 0$, $y\left(\frac{\pi}{12}\right)=\frac{\pi}{4}$

$\div$ by $\cos^2 y \operatorname{cosec}^2 x$

$\Rightarrow \dfrac{8}{\operatorname{cosec}^2 x}dx + \dfrac{1}{\cos^2 y}dy = 0$

$\Rightarrow \int 8\sin^2 x\,dx + \int \sec^2 y\,dy = \int 0\,dx$

$\Rightarrow 4\int 2\sin^2 x\,dx + \tan y = c$

$\Rightarrow 4\int(1-\cos 2x)\,dx + \tan y = c$

$\Rightarrow 4\left(x - \dfrac{\sin 2x}{2}\right) + \tan y = c$

$\Rightarrow 4x - 2\sin 2x + \tan y = c$

$\Rightarrow \tan y = -4x + 2\sin 2x + c$

$\because y\left(\frac{\pi}{12}\right) = \frac{\pi}{4}$

$\therefore \tan\left(\frac{\pi}{4}\right) = -4\left(\frac{\pi}{12}\right) + 2\sin 2\left(\frac{\pi}{12}\right) + c$

$1 = -\frac{\pi}{3} + 2\sin\frac{\pi}{6} + c$

$1 = -\frac{\pi}{3} + 2\left(\frac{1}{2}\right) + c$

$\cancel{1} - \cancel{1} + \frac{\pi}{3} = c \quad \Rightarrow \boxed{c = \frac{\pi}{3}}$

$\Rightarrow \therefore \tan y = -4x + 2\sin 2x + \frac{\pi}{3}$ Ans

(20) $\dfrac{dy}{dx} = \dfrac{x(x^2+1)}{4y^3}$, $y(0) = -\dfrac{1}{\sqrt{2}}$

$\Rightarrow 4y^3\,dy = x(x^2+1)\,dx$

$\Rightarrow 4\int y^3\,dy = \int (x^3+x)\,dx$

$\Rightarrow \cancel{4}\dfrac{y^4}{\cancel{4}} = \dfrac{x^4}{4} + \dfrac{x^2}{2} + c$

$\because y(0) = -\frac{1}{\sqrt{2}}$

$\therefore \left(-\frac{1}{\sqrt{2}}\right)^4 = 0 + 0 + c$

$\boxed{\frac{1}{4} = c}$

$\Rightarrow \therefore y^4 = \dfrac{x^4}{4} + \dfrac{x^2}{2} + \dfrac{1}{4}$

$\Rightarrow 4y^4 = x^4 + 2x^2 + 1$

$\Rightarrow 4y^4 = (x^2+1)^2$

$\Rightarrow 2y^2 = x^2+1$

$\Rightarrow y^2 = \dfrac{x^2+1}{2}$

$\Rightarrow y = \pm\sqrt{\dfrac{x^2+1}{2}}$

$\Rightarrow y = -\sqrt{\dfrac{x^2+1}{2}}$ Ans.

Note $y = +\sqrt{\dfrac{x^2+1}{2}}$ is not satisfied by $y(0) = -\frac{1}{\sqrt{2}}$ So leave it.

9.3—1

Homogeneous Diff Eq. (H.DE)

A differential eq. of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ is said to be homogeneous diff eq. if both fns $f(x,y)$ & $g(x,y)$ are homogeneous of same degree.

Homogeneous Fn:-

A function $f(x,y)$ is said to be homogeneous of degree 'n', if it can be written as $f(tx, ty) = t^n f(x,y)$

e.g $f(x,y) = \sqrt{xy}$

$f(tx,ty) = \sqrt{tx\,ty} = t\sqrt{xy}$

$f(x,y)$ is homogeneous fn of degree '1'

To Solve Put $y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

then by method of separable variable we solve.

Ex 9.3

① $(x-y)dx + (x+y)dy = 0$

$x\,dx - y\,dx + x\,dy + y\,dy = 0$
Not separable.

$(x+y)\,dy = -(x-y)\,dx$

$\frac{dy}{dx} = \frac{y-x}{x+y}$ H.DE —— (i)

Put $y = vx$ —— (ii)

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ —— (iii)

using (ii) (iii) in (i)

$v + x\frac{dv}{dx} = \frac{vx - x}{x + vx}$

$x\frac{dv}{dx} = \frac{\cancel{x}(v-1)}{\cancel{x}(1+v)} - v$

$= \frac{\cancel{v} - 1 - \cancel{v} - v^2}{1+v}$

$x\frac{dv}{dx} = -\frac{(v^2+1)}{1+v}$

$\int \frac{v+1}{v^2+1}\,dv = -\int \frac{dx}{x}$

$\frac{1}{2}\int \frac{2v\,dv}{v^2+1} + \int \frac{dv}{v^2+1} = -\int \frac{dx}{x}$

$\frac{1}{2}\ln(v^2+1) + \tan^{-1} v = -\ln x + c$

$\rightarrow \ln(v^2+1)^{\frac{1}{2}} + \tan^{-1} v + \ln x = c$

$\ln\sqrt{\frac{y^2}{x^2} + 1} + \tan^{-1}\left(\frac{y}{x}\right) + \ln x = c$

$\ln\sqrt{y^2+x^2} - \cancel{\ln\sqrt{x^2}} + \tan^{-1}\left(\frac{y}{x}\right) + \cancel{\ln x} = c$

$\ln\sqrt{y^2+x^2} + \tan^{-1}\left(\frac{y}{x}\right) = c$