Chapter 06: Vector Spaces Exercise 6.4 (Solutions) Mathematical Method by S.M. Yusuf, A. Majeed and M. Amin Published by Ilmi Kitab Khana, Lahore.

(1)

Exercise 6.4

 $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

 $T(x_{1}, x_{2}, X_{3}) = (x_{1}, x_{2}, 0)$ Standard basis for $\mathbb{R}^3 = \{e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1)\}$ T(1,0,0) = (1,0,0) = 1(1,0,0) + 0(0,1,0) + 0(0,0,1)T(0,1,0) = (0,1,0) = 0(1,0,0) + 1(0,1,0) + 0(0,0,1)

T(0,0,1) = (0,0,0) = 0(4,0,0) + 0(0,1,0) + 0(0,0,1)

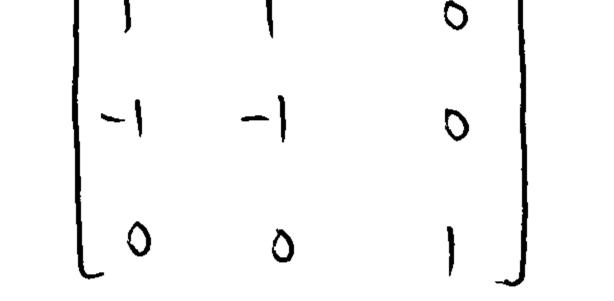
Hence the matrix of T write standard basis i

$$\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}$$
i)
$$T(x_1, X_2, X_3) = (x_1 + X_2, -X_1 - X_2, X_3)$$
Standard basis for $\mathbb{R}^3 = \begin{cases}
(1,0,0), (0,1,0), (0,0,1)\\
0, 0, 0
\end{cases}$

$$T(1,0,0) = (1,-1,0) = 1(1,0,0) - 1(0,1,0) + 0(0,0,1)$$

$$T(0,1,0) = (1,-1,0) = 1(1,0,0) - 1(0,1,0) + 0(0,0,1)$$

T(0,0,1) = (0,0,1) = 0(1,0,0) + 0(0,1,0) + 1(0,0,1)Hence the matrix of T wirt. standard basis is



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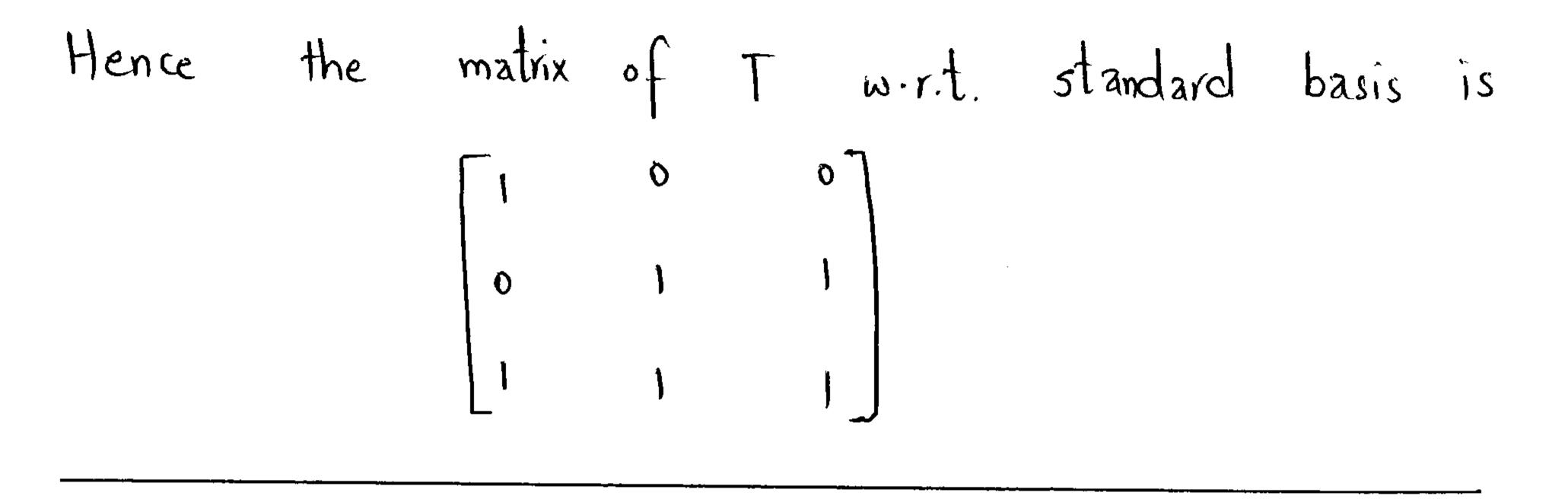
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(iii)
$$T(x_{1}, x_{2}, x_{3}) = (x_{2}, -x_{1}, -x_{3})$$

Standard basis for $\mathbb{R}^{3} = \{(1, \circ, \circ), (\circ, 1, \circ), (\circ, \circ, 1)\}$
 $T(1, \circ, \circ) = (\circ, -1, \circ) = \circ(1, \circ, \circ) - 1(\circ, 1, \circ) + \circ(\circ, \circ, 0, 1)$
 $T(\circ, 1, \circ) = (1, \circ, \circ) = 1((1, \circ, \circ) + \circ(\circ, 1, \circ) + \circ(\circ, 0, 0, 1))$
 $T(\circ, 0, 1) = (\circ, \circ, -1) = \circ(1, \circ, \circ) + \circ(\circ, 1, \circ) - 1(\circ, 0, 0, 1)$
Hence the standard matrix of T wint standard basis
 $\begin{bmatrix} \circ & 1 & \circ \\ -1 & \circ & \circ \\ 0 & 0 & -1 \end{bmatrix}$
(iv) $T(x_{1}, x_{2}, x_{3}) = (x_{1}, x_{2} + x_{3}, x_{1} + x_{2} + x_{3})$
Standard basis for $\mathbb{R}^{3} = \{(1, \circ, \circ), (\circ, 1, \circ), (\circ, 0, 1)\} \}$
 $T(1, \circ, \circ) = (1, \circ, 1) = 1(1, \circ, 0) + \circ((\circ, 1, 0) + 1(\circ, 0, 1))$

$$T(0,1,0) = (0,1,1) = 0(1,0,0) + 1(0,1,0) + 1(0,0,1)$$

$$T(0,0,1) = (0,1,1) = 0(1,0,0) + 1(0,1,0) + 1(0,0,1)$$



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$$T(1, o) = (3, 5, 1, 4) = 3(1, 0, 0, 0) + 5(0, 1, 0, 0) + (0, 0, 1, 0) + 4(0, 0, 0, 1)$$

$$T(0, 1) = (4, -2, 7, 0) = 4(1, 0, 0, 0) - 2(0, 1, 0, 0) + 7(0, 0, 1, 0) + 0(0, 0, 0, 1)$$
So the matrix of T is
$$\begin{bmatrix} 3 & 4 \\ 5 & -2 \\ 1 & 7 \\ 4 & 0 \end{bmatrix}$$
(iv)
$$T: \mathbb{R}^{4} \to \mathbb{R} \quad defined \quad b\gamma$$

$$T(x_{1}, x_{2}, x_{3}, x_{4}) = 2x_{1} + 3x_{2} - 7x_{3} + x_{4}$$

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Standard basis for
$$\mathbb{R}^{4} = \{(1,0,0,0), (0,1,0,0), (0,0,0,1), (0,0,0,1)\}$$

Standard basis for $\mathbb{R} = \{1\}$
 $T((1,0,0,0) = 2 = 2(1)$
 $T(0,1,0,0) = 3 = 3(1)$
 $T(0,0,1,0) = -7 = -7(1)$
 $T(0,0,0,1) = 1 = 1(1)$

So the matrix of T is [2, 2, -7, 1].

$$3n \text{ (i)} \qquad 16 \text{ (i)} \qquad A = \begin{bmatrix} 6 & -1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$
Since matrix A is of order $3x^2$, so $m=3$, $n=2$.
i.e. $T:\mathbb{R}^2 \to \mathbb{R}^3$ is defined by $T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$
or $T(x_1, x_2) = (6x_1 - x_2, x_1 + 2x_2, x_1 + 3x_2).$

(ii) Let
$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 5 & 6 \\ -2 & 3 & -1 \end{bmatrix}$$

Since matrix A is of order 3x3, so $m = 3$, $n = 3$.
i.e., $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ is defined by
 $T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 5 & 6 \\ -2 & 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 + 2x_3 \\ 2x_1 + 5x_2 + 6x_3 \\ -2x_1 + 3x_2 - x_3 \end{bmatrix}$
or $T (x_1, x_2, x_3) = (x_1 + x_2 + 2x_3, 2x_1 + 5x_2 + 6x_3, -2x_1 + 3x_2 - x_3)$

 $(iii) \quad Let \quad A = \begin{bmatrix} 3 & 1 & 0 & 2 & 0 \\ A = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \end{bmatrix}$

Since matrix A is of order
$$3x 5$$
, so $m=3$, $n=5$.
i.e., $T: \mathbb{R} \to \mathbb{R}^3$ is defined by
 $T \begin{bmatrix} x_1 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & -1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & -1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3x_1 + x_2 + 2x_4 \\ x_4 + x_5 \\ -x_2 + x_3 + x_4 + x_5 \end{bmatrix}$
or
 $T (x_1, x_2, x_3, x_4, x_5) = (3x_1 + x_2 + 2x_4, x_1 + x_4 + x_5, -x_2 + x_3 + x_4 + x_5).$
The matrix of $T = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$
T in terms of coordinates is Available at www.mathety.org
 $T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_2 + x_3 \\ -x_1 - x_2 \end{bmatrix}$

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$$T(x_1, x_2, x_3) = (x_2 + x_3, x_1 - x_3, - x_1 - x_2)$$

Given basis for
$$\mathbb{R}^3 = \{v_1 = (0, 1, 2), v_2 = (1, 1, 1), v_3 = (1, 0, -2)\}$$

$$T(v_{1}) = T(v_{1}, 2) = (3, -2, -1)$$

$$(3, -2, -1) = \alpha_{1}v_{1} + \alpha_{2}v_{2} + \alpha_{3}v_{3}$$

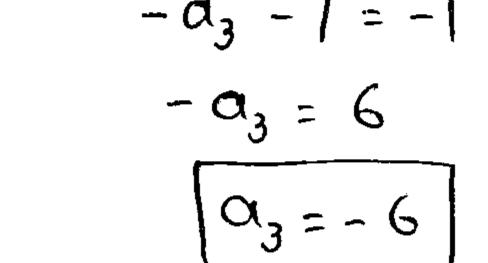
$$(3, -2, -1) = \alpha_{1}(v_{1}, 2) + \alpha_{2}(1, 1, 1) + \alpha_{3}(3, -2, -1)$$

$$(3, -2, -1) = (\alpha_{2} + \alpha_{3}, \alpha_{1} + \alpha_{2}, 2\alpha_{1} + \alpha_{2} - 2\alpha_{3})$$

$$\alpha_{1} + \alpha_{3} = 3, \qquad \alpha_{1} + \alpha_{2} = -2, \qquad 2\alpha_{1} + \alpha_{2} - 2\alpha_{3} = -1$$

$$\alpha_{2} = 3 - \alpha_{3}, \qquad \alpha_{1} + (3 - \alpha_{3}) = -2, \qquad 2(-5 + \alpha_{3}) + (3 - \alpha_{3}) - 2\alpha_{3} = -1$$

$$\alpha_{1} = -5 + \alpha_{3}, \qquad -\alpha_{3} - 7 = -1$$



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So
$$T(v_1) = -11v_1 + 9v_2 - 6v_3 - 0$$

Similarly $T(v_2) = -2v_1 + 2v_2 + 0v_3 - 2$
and $T(v_3) = 14v_1 - 11v_2 + 9v_3 - 3$

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Hence the matrix of 1 wird. new basis is
$$\begin{bmatrix} -11 & -2 & 14 \\ 9 & 2 & -11 \\ -6 & 0 & 9 \end{bmatrix}$$

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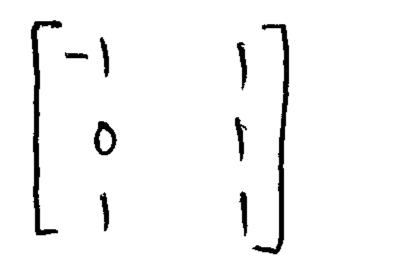
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standard basis is

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$$T(v_3) = W_6 = 0W_1 + 0W_2 + 0W_3 + 0W_4 + 0W_5 + W_6$$

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