Exercise No. 8.8

157

Find the eq. of the trace of the given surface in the specified Co-ord. plane. Identify the trace (Problems 1–4).

Q1 $x^2+y^2+z^2-2xz+5z-4=0$; xy-plane.

Sol: Given eq. of surface is

$$x^2+y^2+z^2-2xz+5z-4=0 \quad \text{——— (1)}$$

For the trace in xy-plane

Put $z=0$ in (1)

$$x^2+y^2-4=0$$

or $\boxed{x^2+y^2=4}$ is req. trace.

which is a circle with centre at origin & radius 2.

Q2 $xy+yz+zx=1$; xz-plane

Sol: Given eq. of surface is

$$xy+yz+zx=1 \quad \text{——— (1)}$$

For the trace in xz-plane

Put $y=0$ in (1)

$\boxed{zx=1}$ is req. trace

which is the eq. of a hyperbola.

Q3 $x^2+4y^2+z^2+4xy-2xz-2x-4y+z+1=0$; xy-plane 158

Sol: Given eq. of surface is

$$x^2+4y^2+z^2+4xy-2xz-2x-4y+z+1=0 \quad \text{——} (1)$$

For the trace in xy-plane

Put $z=0$ in (1)

$$x^2+4y^2+4xy-2x-4y+1=0$$

$$(x)^2+(2y)^2+(-1)^2+2(x)(2y)+2(x)(-1)+2(2y)(-1)=0$$

$$(x+2y-1)^2=0 \text{ is req. trace}$$

which is a pair of coincident lines.

Q4 $x^2+xy-3xz-2=0$; yz-plane.

Sol:- Given eq. of surface is

$$x^2+xy-3xz-2=0 \quad \text{——} (1)$$

For the trace in yz-plane

Put $x=0$ in (1)

$$-2=0$$

which is impossible

Hence there is no trace in yz-plane.

Find the intercepts of the given surface on the Co-ord. axes: (Problems 5-6):

Q5 $x^2+4y^2+5xz-2x+4y-3=0$

Sol. Given eq. of surface is 159

$$x^2 + 4y^2 + 5xz - 2x + y - 3 = 0 \quad \text{——— (1)}$$

For x-int. Put $y = z = 0$ in (1)

$$x^2 - 2x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$(x-3)(x+1) = 0$$

$$\Rightarrow \boxed{x = 3, -1} \text{ are x-int.}$$

For y-int.

Put $x = z = 0$ in (1)

$$4y^2 + y - 3 = 0$$

$$4y^2 + 4y - 3y - 3 = 0$$

$$4y(y+1) - 3(y+1) = 0$$

$$(y+1)(4y-3) = 0$$

$$\Rightarrow \boxed{y = -1, \frac{3}{4}} \text{ are y-int.}$$

For z-int.

Put $x = y = 0$ in (1)

$$-3 = 0$$

which is impossible.

Hence there is no z-int.

Q6 $2x^2 - z^2 - xy - 8yz + y - z - 2 = 0$ 160

Sol: Given eq. of surface is

$$2x^2 - z^2 - xy - 8yz + y - z - 2 = 0 \quad \text{---- (1)}$$

For x-int: Put $y = z = 0$ in (1)

$$2x^2 - 2 = 0$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$\boxed{x = \pm 1}$ are x-ints.

For y-int: Put $x = z = 0$ in (1)

$$y - 2 = 0$$

$\boxed{y = 2}$ is y-int.

For z-int Put $x = y = 0$ in (1)

$$-z^2 - z - 2 = 0$$

or $z^2 + z + 2 = 0$

$$z = \frac{-1 \pm \sqrt{1 - 4.1.2}}{2.1}$$

$$z = \frac{-1 \pm \sqrt{-7}}{2}$$

which are imaginary

Hence there is no z-intercept.
