Direction of Qibla

Example #41: Find the direction of Qibla of the Badshahi Mosque, Lahore, latitude $= 31^o35.5^{'}$ N and longitude $= 74^o18.7^{'}$ E

Solution: Given $\varphi = 31^o35.5^{'}N \quad , \quad \lambda = 74^o18.7^{'}E$

The Latitude φ_o and longitude λ_o of the Khana-e-Ka'ba are

$\varphi_o = 21^o25.2^{'}N \quad , \quad \lambda_o = 39^o49.2^{'}E$

Now classical length $l = (\lambda - \lambda_o)^o CE = (74^o18.7^{'} - 39^o49.2^{'})^o CE \implies l = 34^o29.5^{'}CE$

Suppose i be the inclination of direction of Qibla then

$$\tan i = \frac{sin\varphi \cos l - cos\varphi\, tan\varphi_o}{\sin l}$$

$$= \frac{\sin(31^o35.5^{'})\cos(34^o29.5^{'}) - \cos(31^o35.5^{'})\tan(21^o25.2^{'})}{\sin(34^o29.5^{'})}$$

$$= \frac{(0.5238)(0.8242) - (0.8518)(0.3923)}{0.5663}$$

$$\tan i = \frac{0.4317 - 0.3342}{0.5663} = \frac{0.0975}{0.5663}$$

$\tan i = 0.1722 \implies i = \tan^{-1}(0.1722) = 9^o46^{'}$

$i = 9^o46^{'}$ South of West

Example #42: Find the direction of Qibla of Bait-al-Mukarram Mosque, Dhaka, latitude $= 23^o42^{'}$ N and Longitude $= 90^o22^{'}$ E.

Solution: Given $\varphi = 23^o42^{'}N \quad , \quad \lambda = 90^o22^{'}E$

The Latitude φ_o and longitude λ_o of the Khana-e-Ka'ba are

$\varphi_o = 21^o25.2^{'}N \quad , \quad \lambda_o = 39^o49.2^{'}E$

Now classical length $l = (\lambda - \lambda_o)^o CE = (90^o22^{'} - 39^o49.2^{'})^o CE \implies l = 50^o32.8^{'}CE$

Suppose i be the inclination of direction of Qibla then

$$\tan i = \frac{sin\varphi \cos l - cos\varphi\, tan\varphi_o}{\sin l}$$

$$= \frac{\sin(23^o42^{'})\cos(50^o32.8^{'}) - \cos(23^o42^{'})\tan(21^o25.2^{'})}{\sin(50^o32.8^{'})}$$

$$= \frac{(0.4019)(0.6354) - (0.9157)(0.3923)}{0.7721}$$

$$\tan i = \frac{0.2554 - 0.3592}{0.7721} = \frac{-0.1038}{0.7721}$$

$\tan i = -0.1344 \implies i = \tan^{-1}(-0.1344) = -7^o39^{'}$

$i = -7^o39^{'}$ North of West

Exercise #8.13

Q#5: Find the direction of Qibla of each of the given places.

(a) Place : Islamabad , Latitude φ : $33^o40'$ N , Longitude λ :$73^o8'$ E

Solution: Given $\varphi = 33^o40'N \quad , \quad \lambda = 73^o8'E$

The Latitude φ_o and longitude λ_o of the Khana-e-Ka'ba are

$\varphi_o = 21^o25.2'N \quad , \quad \lambda_o = 39^o49.2'E$

Now classical length $l = (\lambda - \lambda_o)^o CE = (73^o8' - 39^o49.2')^o CE \quad \Rightarrow l = 33^o18.8'CE$

Suppose i be the inclination of direction of Qibla then

$$\tan i = \frac{sin\varphi \cos l - cos\varphi\, tan\varphi_o}{\sin l}$$

$$= \frac{\sin(33^o40')\cos(33^o18.8') - \cos(33^o40')\tan(21^o25.2')}{\sin(33^o18.8')}$$

$$= \frac{(0.5544)(0.8357) - (0.8323)(0.3923)}{0.5492}$$

$$= \frac{0.4633 - 0.3265}{0.5492} \quad = \frac{0.1368}{0.5492}$$

$\tan i = 0.2491 \quad \Rightarrow i = \tan^{-1}(0.2491) \ = 13.9877^o$

$i = 13^o59'$ South of West

(e) Place : New York , Latitude φ : $40^o49'$ N , Longitude λ :$74^o0'$ W

Solution: Given $\varphi = 40^o49'N \quad , \quad \lambda = 74^o0'W$

The Latitude φ_o and longitude λ_o of the Khana-e-Ka'ba are

$\varphi_o = 21^o25.2'N \quad , \quad \lambda_o = 39^o49.2'E$

Here $0^o < \lambda < 180^o - \lambda_o$,*so*

Now classical length $l = (\lambda + \lambda_o)^o CW = (74^o0' - 39^o49.2')^o CW \quad \Rightarrow l = 113^o49.2'CW$

Suppose i be the inclination of direction of Qibla then

$$\tan i = \frac{sin\varphi \cos l - cos\varphi\, tan\varphi_o}{\sin l}$$

$$= \frac{\sin(40^o49')\cos(113^o49.2') - \cos(40^o49')\tan(21^o25.2')}{\sin(113^o49.2')}$$

$$= \frac{(0.6536)(-0.4039) - (0.7568)(0.3923)}{0.9148}$$

$$= \frac{-0.2640 - 0.2969}{0.9148} \quad = \frac{-0.5609}{0.9148}$$

$\tan i = -0.6131 \quad \Rightarrow i = \tan^{-1}(-0.6131) \ = -31^o30'$

$i = 31^o30'$ North of East

(f) Place : Canberra , Latitude φ : $= 35^o17^{'}S$, Longitude λ $= 149^o8^{'}E$

Solution: Given $\varphi = 35^o17^{'}S \quad , \quad \lambda = 149^o8^{'}E$

The Latitude φ_o and longitude λ_o of the Khana-e-Ka'ba are

$\varphi_o = 21^o25.2^{'}N \quad , \quad \lambda_o = 39^o49.2^{'}E$

Here $\lambda_o < \lambda \leq 180^o$, so

Now classical length $l = (\lambda - \lambda_o)^o CE = (149^o8^{'} - 39^o49.2^{'})^o CE \quad \Longrightarrow l = 109^o18.8^{'}CE$

Suppose i be the inclination of direction of Qibla then

$$\tan i = \frac{sin\varphi \cos l - cos\varphi\, tan\varphi_o}{\sin l}$$

$$= \frac{\sin(35^o17^{'})\cos(109^o18.8^{'}) - \cos(35^o17^{'})\tan(21^o25.2^{'})}{\sin(109^o18.8^{'})}$$

$$= \frac{(-0.5776)(-0.3307) - (0.8163)(0.3923)}{0.9437}$$

$$\tan i = \frac{0.1910 - 0.3202}{0.9437} \quad = \frac{-0.1292}{0.9437}$$

$\tan i = -0.1369 \quad \Longrightarrow i = \tan^{-1}(-0.1369) \quad = -7^o47^{'}$

$or \;\; i = 7^o47^{'}$ North of West

Q#6: Prove that for a place on the equator the direction of Qibla is inclined at $\text{arc}\tan^{-1}(\tan \varphi_o \operatorname{cosec} l)$ north of west or north of east according as its classical longitude l is east or west.

Solution: Suppose i be the inclination of direction of Qibla then

$$\tan i = \frac{sin\varphi \cos l - cos\varphi\, tan\varphi_o}{\sin l} \quad ----(1)$$

For a place on the equator $latitude = \varphi = 0$

Put $\varphi = 0$ in eq. (1) $\tan i = \frac{\sin(0)\cos l - \cos(0)\tan\varphi_o}{\sin l}$

$$\tan i = \frac{0 - tan\varphi_o}{\sin l}$$

$$= \frac{-\tan\varphi_o}{\sin l}$$

$$= -\tan\varphi_o \cdot \frac{1}{\sin l}$$

$$\tan i = -\tan\varphi_o \,.\, cosec\, l$$

$$i = \tan^{-1}(-\tan\varphi_o \,.\, cosec\, l)$$

$$i = -\tan^{-1}(\tan\varphi_o \,.\, cosec\, l)$$

$$i = -arctan(\tan\varphi_o \,.\, cosec\, l)$$

Above equation shows that inclination is negative ,so direction is to north side. Above eq. shows that i is inclination to the north of west or north of east according as l to the east or west.

Q#7: Prove that for a place on the same parallel of latitude as the Khana-e-Ka'aba the direction of Qibla is inclined at $\arctan\left(\sin\varphi_o \tan\frac{l}{2}\right)$ north of west or north of east according as its classical longitude l is east or west.

Solution: Suppose i be the inclination of direction of Qibla then

$$\tan i = \frac{sin\varphi \cos l - cos\varphi\, tan\varphi_o}{\sin l} \quad ----(1)$$

For a place on the same parallel of latitude the Khana-e-Ka'aba we have $\varphi = \varphi_o$

Now put $\varphi = \varphi_o$ in equation (1)

$$\tan i = \frac{sin\varphi_o \cos l - cos\varphi_o\, tan\varphi_o}{\sin l}$$

$$\tan i = \frac{sin\varphi_o \cos l - sin\varphi_o}{\sin l}$$

$$\tan i = \frac{sin\varphi_o(\cos l - 1)}{\sin l}$$

$$\tan i = \frac{-sin\varphi_o(1 - \cos l)}{\sin l}$$

$$\tan i = -\frac{sin\varphi_o . 2sin^2\frac{l}{2}}{2\sin\frac{l}{2}\cos\frac{l}{2}}$$

$$\tan i = -sin\varphi_o . \frac{\sin\frac{l}{2}}{\cos\frac{l}{2}}$$

$$\tan i = -sin\varphi_o \tan\frac{l}{2}$$

$$i = \tan^{-1}\left(-sin\varphi_o \tan\frac{l}{2}\right)$$

$$i = -\tan^{-1}\left(sin\varphi_o \tan\frac{l}{2}\right)$$

$$i = -arctan\left(sin\varphi_o \tan\frac{l}{2}\right)$$

Above equation negative sign shows that the inclination is to the north side.

It is clear that the inclination is to the north of west or north of east according as l is to the east or west.

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