Exercise No. 8.9

Write the eq. of the surface obtained by revolving the given curve about the specified Co-ord. axis. (Problems 1-5)

Q1 $x^2+2y^2 = 8$; $z = 0$ (a) x-axis (b) y-axis.

Sol. Given eq. of curve is

$$x^2+2y^2 = 8 \; ; \; z = 0 \text{ ———— } ①$$

a) Since this curve lies in xy-plane, so for eq. of surface of revolution about x-axis

Replace y^2 by y^2+z^2 in ①

$$\boxed{x^2+2(y^2+z^2) = 8} \text{ is req. eq.}$$

b) y-axis.

Since the curve lies in xy-plane, so for eq. of surface of revolution about y-axis

Replace x^2 by x^2+z^2 in ①

$$x^2+z^2+2y^2 = 8$$

$$\text{or } \boxed{x^2+2y^2+z^2 = 0} \text{ is req. eq.}$$

Q2 $4x^2-9z^2 = 5$; $y = 0$ (a) y-axis (b) z-axis

Sol. Given eq. of curve is

$$4x^2-9z^2 = 5, \text{ ———— } ①$$

(a) Since this curve lies in xz-plane so for eq.

of surface of revolution about y-axis

Replace z^2 by y^2+z^2 in ①

$\boxed{4x^2 - 9(y^2+z^2) = 5}$ is req. eq.

(b) z-axis

Since this curve lies in xz-plane, so for eq. of surface of revolution about z-axis

Replace x^2 by x^2+y^2 in ①

$\boxed{4(x^2+y^2) - 9z^2 = 5}$ is req. eq.

Q3 $6y^2+6z^2 = 7$; $x = 0$ (a) y-axis (b) z-axis

Sol. Given eq. of curve is

$6y^2 + 6z^2 = 7$ ——— ①

(a) y-axis

Since this curve lies in yz-plane, so for eq. of surface of revolution about y-axis

Replace z^2 by z^2+x^2 in ①

$6y^2 + 6(z^2+x^2) = 7$

$\boxed{6x^2+6y^2+6z^2 = 7}$ is req. eq.

(b) z-axis

Since this curve lies in yz-plane so for eq. of surface of revolution about z-axis

Replace y^2 by y^2+x^2 in ①

$$2x = 6-3y$$

$$4x^2 = (6-3y)^2$$

$$4(x^2+z^2) = 36-36y+9y^2$$

$$4x^2+4z^2-36+36y-9y^2 = 0$$

$4x^2-9y^2+4z^2+36y-36 = 0$ is req. eq.

Q5 $y=2$, $x=0$, (a) y-axis (b) z-axis

Sol:- Given eq. of curve is

$$y = 2 \quad \text{——— (1)}$$

(a) y-axis

Since this curve lies in yz-plane, so for eq. of surface of revolution about y-axis

Replace z^2 by z^2+x^2

$y = 2$ is req. eq.

(b) z-axis

Since this curve ~~lies in yz-plane~~ so for eq. of surface of revolution about z-axis

Replace y^2 by y^2+x^2 in (1)

as (1) is $y = 2$

$$y^2 = 4$$

$$y^2+x^2 = 4$$

or $x^2+y^2 = 4$ is req. eq.

$$6(y^2+x^2)+6z^2 = 7$$

$$\boxed{6x^2+6y^2+6z^2 = 7}$$ is req. eq.

Q4 $2x+3y = 6$; $z = 0$ (a) x-axis (b) y-axis.

Sol. Given eq. of curve is

$$2x+3y = 6 \quad \text{——— (1)}$$

(a) x-axis

Since this curve lies in xy-plane, so for eq. of surface of revolution about x-axis

Replace y^2 by y^2+z^2 in (1)

As (1) is $2x+3y = 6$

$$3y = 6-2x$$

$$9y^2 = (6-2x)^2$$

or $9(y^2+z^2) = 36-24x+4x^2$

$$4x^2-24x+36-9y^2-9z^2 = 0$$

$4x^2-9y^2-9z^2-24x+36 = 0$ is req. eq.

(b) y-axis

Since this curve lies in xy-plane ; so for eq. of surface of revolution about y-axis

Replace x^2 by x^2+z^2 in (1)

As (1) is $2x+3y = 6$

State which co-ord. axis is the axis of 165
revolution for the surface & write eq. for a
generatrix in the specified co-ord. plane (Prob. 6-9)

Q6 $x^2+y^2+z=2$; xz-plane

Sol. Given eq. of surface is

$$x^2+y^2+z=2 \quad \text{———} \quad (1)$$

This surface contains the term x^2+y^2 which has been replace for x^2 in the xz-plane which shows that the axis of revolution is z-axis.

For eq. of generatrix Put $y=0$ in (1)

$\boxed{x^2+z=2}$ is req. eq. of generatrix

Q7 $x^2-4y^2-4z^2=8$; xy-plane.

Sol. Given eq. of surface is

$$x^2-4(y^2+z^2)=8 \quad \text{———} \quad (1)$$

This surface contains the term y^2+z^2 which has been replaced for y^2 in the xy-plane which shows that the axis of revolution is x-axis.

For eq. of generatrix

Put $z=0$ in (1)

$x^2-4y^2=8$ is req. eq. of generatrix

Q9 $x^2-4y^2-4z^2=0$; xz-plane

Sol.- Given eq. of surface is

$$x^2-4y^2-4z^2=0$$

or $x^2-4(y^2+z^2)=0$ ——— (1)

This surface contains the term y^2+z^2 which has been replaced z^2 in xz-plane which shows that the axis of revolution is x-axis.

For eq. of generatrix

Put $y=0$ in (1)

$x^2-4z^2=0$ is req. eq.

Q9 $x^2y^2+y^2z^2=1$; yz-plane

Sol.- Given surface is

$$x^2y^2+y^2z^2=1$$

or $y^2(x^2+z^2)=1$ ——— (1)

This surface contains x^2+z^2 which has been replaced for z^2 in the yz-plane which shows that the axis of revolution is y-axis.

For eq. of generatrix

Put $x=0$ in (1)

$$y^2z^2=1$$

or $\boxed{yz=1}$ is req. eq. of generatrix

Q10 Find the eq. of the torus obtained by revolving about y-axis the circle in the xy-plane with centre at $(a,0,0)$ & radius b where $0<b<a$. 117

Sol. Eq. of the circle with centre at $(a,0)$ & radius b is

$$(x-a)^2+(y-0)^2=b^2$$

or $(x-a)^2+y^2=b^2$ ——— (1)

This curve lies in xy-plane so for eq. of surface of revolution about y-axis.

Replace x^2 by x^2+z^2

or x by $\sqrt{x^2+z^2}$ in (1)

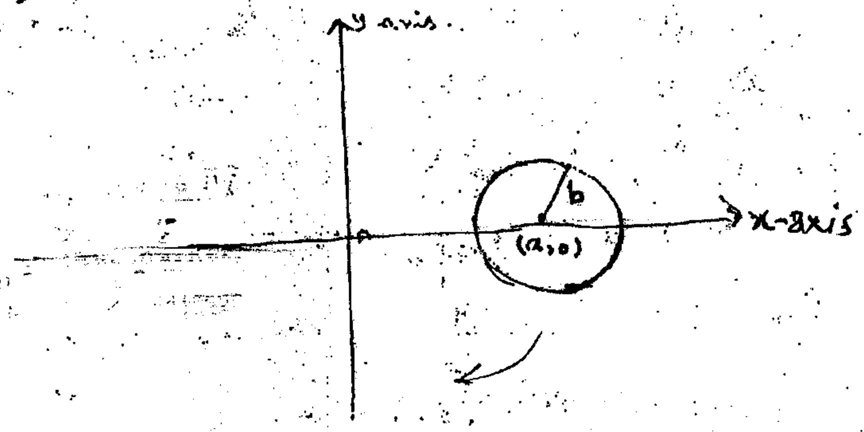
$$\left(\sqrt{x^2+z^2}-a\right)^2+y^2=b^2$$

$$x^2+z^2+a^2-2a\sqrt{x^2+z^2}+y^2=b^2$$

$$x^2+y^2+z^2+a^2-b^2=2a\sqrt{x^2+z^2}$$

Sq. both sides

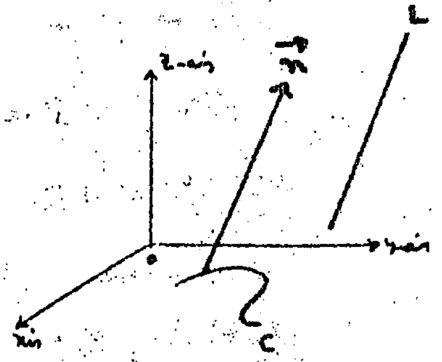
$(x^2+y^2+z^2+a^2-b^2)^2=4a^2(x^2+z^2)$ is req. eq.



<u>Cylinder</u>:- The set of all lines intersecting a Curve C & parallel to a line L define a cylinder.

Note it that the line L does not lie in the plane containing the curve C.

Each line intersecting C & parallel to L is called element or ruling of cylinder & curve C is called directrix of cylinder.



<u>Eq. of Cylinder</u>:

let the eq. of dTX be $f(x,y)=0$ lie in xy-plane & let $\vec{n}=[\lambda,\mu,\nu]$ be a vector not parallel to xy-plane.

The eq. of line L through a pt. $(x_1,y_1,0)$ & parallel to $\vec{n}$

is $\dfrac{x-x_1}{\lambda}=\dfrac{y-y_1}{\mu}=\dfrac{z}{\nu}=t$ (say)

$\Rightarrow x = x_1+\lambda t$ ——— (i)

$y = y_1 + \mu t$ ——— (ii)

$z = \nu t$ ——— (iii)

From (iii) $t=\dfrac{z}{\nu}$

Put in (i) & (ii)

$$\left.\begin{aligned} x &= x_1 + \frac{\lambda}{\nu} z \\ y &= y_1 + \frac{\mu}{\nu} z \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} x_1 &= x - \frac{\lambda}{\nu} z \\ y_1 &= y - \frac{\mu}{\nu} z \end{aligned}\right]$$

Since $(x_1, y_1, 0)$ lie on Curve C

So $f(x_1, y_1) = 0$

Using Values of x_1 & y_1

$$f\left(x - \frac{\lambda}{\nu} z,\ y - \frac{\mu}{\nu} z\right) = 0$$

is eq. of Cylinder.

Right Cylinder:

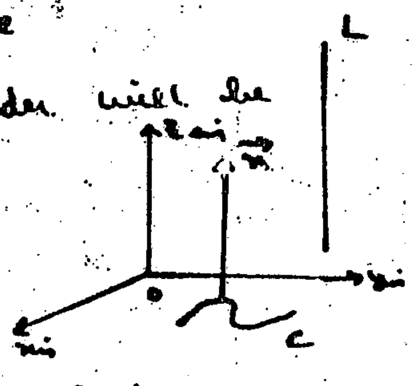
If the ruling of a cylinder are perp. to the plane containing directrix is called right Cylinder.

Eq of right Cylinder:

Let the eq. of directrix be

$C : f(x,y) = 0$ lie in x,y-plane

then the ruling of right cylinder will be parallel to z-axis



i.e., $\vec{n} = [0, 0, 1]$

$\Rightarrow \lambda = 0,\ \mu = 0,\ \nu = 1$

Using these Values in eq. of Cylinder

$$f\left(x - \frac{\lambda}{\nu} z,\ y - \frac{\mu}{\nu} z\right) = 0$$

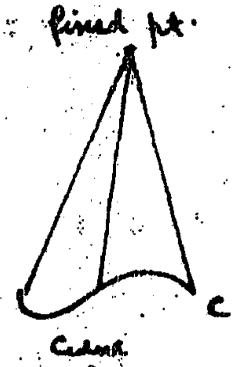
$$f(x - 0,\ y - 0) = 0$$

$\Rightarrow f(x, y) = 0$ is eq. of right cylinder.

Hence eq. of right cylinder is same as the eq. of its directrix.

<u>Cone</u> :- The set of all lines passing through a fixed pt. & intersecting a curve C defines a cone.

The fixed pt. is called vertex & the curve C is called directrix of cone.



Each st. line through vertex & intersecting curve C is called element or ruling of cone.
