Exercise No. 8.7

Q1 Find the cylindrical co-ords. of the pt. whose rectangular co-ords. are

(a) $(2\sqrt{3}, 2, -2)$

Sol. Given pt. is $(2\sqrt{3}, 2, -2)$

we know that the relation b/w. rectangular & cylindrical polar co-ords. are

$$x = r\cos\theta \qquad r = \sqrt{x^2+y^2}$$
$$y = r\sin\theta \qquad \theta = \tan^{-1}(y/x)$$
$$z = z \qquad z = z$$

So $r = \sqrt{x^2+y^2}$

$= \sqrt{(2\sqrt{3})^2+(2)^2}$

$= \sqrt{12+4}$

$= \sqrt{16}$

$\boxed{r = 4}$

$\theta = \tan^{-1}(y/x)$

$= \tan^{-1}\left(\frac{2}{2\sqrt{3}}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\boxed{\theta = \frac{\pi}{6}}$

& $z = -2$

So corresponding cylindrical polar co-ords. is $(4, \frac{\pi}{6}, -2)$

(b) $\left(\frac{16}{5}, \frac{12}{5}, 3\right)$

Sol.- Given pt. is $\left(\frac{16}{5}, \frac{12}{5}, 3\right)$

We know the relation b/w rectangular & cylindrical polar Co-ords. are

$$x = r\cos\theta \qquad r = \sqrt{x^2+y^2}$$

$$y = r\sin\theta \qquad \theta = \tan^{-1}(y/x)$$

$$z = z \qquad z = z$$

Now $r = \sqrt{x^2+y^2}$

$$= \sqrt{\left(\frac{16}{5}\right)^2+\left(\frac{12}{5}\right)^2}$$

$$= \sqrt{\frac{256}{25}+\frac{144}{25}}$$

$$= \sqrt{\frac{400}{25}}$$

$$= \frac{20}{5}$$

$$\boxed{r = 4}$$

$$\theta = \tan^{-1}(y/x)$$

$$= \tan^{-1}\left(\frac{\frac{12}{5}}{\frac{16}{5}}\right)$$

$$= \tan^{-1}\left(\frac{12}{16}\right)$$

$$\boxed{\theta = \tan^{-1}(3/4)}$$

& $\boxed{z = 3}$

So corresponding cylindrical polar co-ords. are $\left(4, \tan^{-1}\left(\frac{3}{4}\right), 3\right)$

Q2 Changing the following from cylindrical Co-ords. 14o to rectangular Co-ords.

(a) $(5, \frac{\pi}{6}, 3)$

Sol. Given pt. is $(5, \frac{\pi}{6}, 3)$

We know the relation b/w rectangular & Cylindrical polar Co-ords. are

$$x = r\cos\theta \qquad r = \sqrt{x^2+y^2}$$
$$y = r\sin\theta \qquad \theta = \tan^{-1}(y/x)$$
$$z = z \qquad z = z$$

Now $x = 5\cos(\pi/6)$

$= 5 \cdot \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{5\sqrt{3}}{2}}$

$y = 5\sin\frac{\pi}{6}$

$= 5 \cdot \frac{1}{2}$

$\boxed{y = \frac{5}{2}}$

& $\boxed{z = 3}$

So Corresponding rectangular Co-ords. are $\left(\frac{5\sqrt{3}}{2}, \frac{5}{2}, 3\right)$.

(b) $(6, \frac{\pi}{3}, -5)$

Sol. Given pt. is $(6, \frac{\pi}{3}, -5)$

We know the relation b/w rect. & Cylindrical polar Co-ords. are

$$x = r\cos\theta \qquad r = \sqrt{x^2+y^2}$$
$$y = r\sin\theta \qquad \theta = \tan^{-1}(y/x)$$
$$z = z \qquad z = z$$

$x = 6\cos \pi/3$

$= 6 \cdot \frac{1}{2}$

$\boxed{x = 3}$

$y = 6\sin \pi/3$

$= 6 \cdot \frac{\sqrt{3}}{2}$

$\boxed{y = 3\sqrt{3}}$

$\boxed{z = -5}$

So corresponding rectangular co-ords. are $(3, 3\sqrt{3}, -5)$

Q3 Find the spherical co-ords. of the pt. whose rectangular co-ords. are

(a): $(1, 1, \sqrt{6})$

Sol. Given pt. is

$(1, 1, \sqrt{6})$

We know the relation b/w rectangular & spherical polar co-ords. are

$x = \rho \sin\phi \cos\theta$ | $\rho = \sqrt{x^2+y^2+z^2}$

$y = \rho \sin\phi \sin\theta$ | $\theta = \tan^{-1}(y/x)$

$z = \rho \cos\phi$ | $\phi = \cos^{-1}\left(\frac{z}{\rho}\right)$

So

$\rho = \sqrt{1+1+6}$

$\boxed{\rho = \sqrt{8}}$

$\theta = \tan^{-1}(y/x)$

$= \tan^{-1}\left(\frac{1}{1}\right)$

$\theta = \tan^{-1}(1)$

$\boxed{\theta = \frac{\pi}{4}}$

$\phi = \cos^{-1}\left(\frac{\sqrt{6}}{\sqrt{8}}\right)$

$= \cos^{-1}\left(\frac{\sqrt{2}\cdot\sqrt{3}}{2\sqrt{2}}\right)$

$= \cos^{-1}\left(\frac{\sqrt{3}}{2}\right).$

$\boxed{\phi = \frac{\pi}{6}}$

So Corresponding spherical polar co-ords are $\left(\sqrt{8}, \frac{\pi}{4}, \frac{\pi}{6}\right)$

(b) $(1, 1, -\sqrt{6})$

Sol. Given pt. is

$(1, 1, -\sqrt{6})$

we know the relation b/w rectangular & spherical polar co-ords

$x = \rho \sin\phi \cos\theta$	$\rho = \sqrt{x^2+y^2+z^2}$
$y = \rho \sin\phi \sin\theta$	$\theta = \tan^{-1}(y/x)$
$z = \rho \cos\phi$	$\phi = \cos^{-1}\left(\frac{z}{\rho}\right)$

$\rho = \sqrt{x^2+y^2+z^2}$

$= \sqrt{1+1+6}$

$\boxed{\rho = \sqrt{8}}$

$\theta = \tan^{-1}\left(\frac{1}{1}\right)$

$= \tan^{-1}(1)$

$\boxed{\theta = \pi/4}$

$$\phi = \cos^{-1}\left(-\frac{\sqrt{6}}{\sqrt{8}}\right)$$

$$= \cos^{-1}\left(-\frac{\sqrt{2}.\sqrt{3}}{2\sqrt{2}}\right)$$

$$= \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\boxed{\phi = 5\frac{\pi}{6}}$$

So corresponding spherical polar Co-ords. are $(\sqrt{8}, \frac{\pi}{4}, 5\frac{\pi}{6})$.

(c) $P = (-\sqrt{3}, 1, -2)$

Sol. Given pt. is $P = (-\sqrt{3}, 1, -2)$.

We know the relation b/w rectangular & spherical polar Co-ords. are

$$x = \rho\sin\phi\cos\theta \qquad \rho = \sqrt{x^2+y^2+z^2}$$
$$y = \rho\sin\phi\sin\theta \qquad \theta = \tan^{-1}(y/x)$$
$$z = \rho\cos\phi \qquad \phi = \cos^{-1}\left(\frac{z}{\rho}\right)$$

Now $\rho = \sqrt{3+1+4}$

$$\rho = \sqrt{8}.$$

$$\boxed{\rho = 2\sqrt{2}}$$

$$\theta = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right)$$

$$= \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$\boxed{\theta = \frac{5\pi}{6}}$$

$$\phi = \cos^{-1}\left(-\frac{2}{2\sqrt{2}}\right)$$

$$= \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

$$\boxed{\phi = \frac{3\pi}{4}}$$

So Corresponding spherical polar Co-ords. are $\left(2\sqrt{2}, \frac{5\pi}{6}, \frac{3\pi}{4}\right)$

(d) $P = (4, -4\sqrt{3}, 6)$.

Sol. Given pt. is $P = (4, -4\sqrt{3}, 6)$

We know the relation b/w rectangular & spherical polar Co-ords. are

$$x = \rho \sin\phi \cos\theta \qquad \rho = \sqrt{x^2+y^2+z^2}$$

$$y = \rho \sin\phi \sin\theta \qquad \theta = \tan^{-1}(y/x)$$

$$z = \rho \cos\phi \qquad \phi = \cos^{-1}\left(\frac{z}{\rho}\right)$$

Now

$$\rho = \sqrt{16+48+36}$$

$$= \sqrt{100}$$

$$\boxed{\rho = 10}$$

$$\theta = \tan^{-1}\left(\frac{-4\sqrt{3}}{4}\right)$$

$$= \tan^{-1}(-\sqrt{3})$$

$$\boxed{\theta = \frac{5\pi}{3}}$$

$$\phi = \cos^{-1}\left(\frac{6}{10}\right)$$

$$\boxed{\phi = \cos^{-1}\left(\frac{3}{5}\right)}$$

So Corresponding spherical polar Co-ords. are

$$\left(10, 5\frac{\pi}{3}, \cos^{-1}\left(\frac{3}{5}\right)\right)$$

Q4 Find the rectangular Co-ords. of the pt. whose, spherical Co-ords. are (a): $(5, \frac{\pi}{2}, \frac{\pi}{2})$ 145

Sol. Given pt. is

$(5, \frac{\pi}{2}, \frac{\pi}{2})$

we know the relation b/w rectangular & spherical polar Co-ords. are

$x = \rho \sin\phi \cos\theta$ | $\rho = \sqrt{x^2+y^2+z^2}$

$y = \rho \sin\phi \sin\theta$ | $\theta = \tan^{-1}(y/x)$

$z = \rho \cos\phi$ | $z = \cos^{-1}\left(\frac{z}{\rho}\right)$

$x = 5 \sin\frac{\pi}{2} \cos\frac{\pi}{2}$

$= 5(1).(0)$

$\boxed{x = 0}$

$y = 5 \sin\frac{\pi}{2} \sin\pi/2$

$= 5(1)(1)$

$\boxed{y = 5}$

$z = 5 \cos\pi/2$

$= 5(0)$

$\boxed{z = 0}$

So the Corresponding rectangular Co-ords. are $(0, 5, 0)$.

(b) $(4, \frac{\pi}{3}, 2\frac{\pi}{3})$

Sol. Given pt. is $(4, \frac{\pi}{3}, \frac{2\pi}{3})$

we know the relation b/w rectangular & spherical polar Co-ords. are

$x = \rho \sin\phi \cos\theta$ | $\rho = \sqrt{x^2+y^2+z^2}$

$y = \rho \sin\phi \sin\theta$ | $\theta = \tan^{-1}(y/x)$

$z = \rho \cos\phi$ | $\phi = \cos^{-1}\left(\frac{z}{\rho}\right)$

$$x = 4 \cdot \sin\left(\frac{2\pi}{3}\right) \cos\frac{\pi}{3}$$

$$= 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$$

$$\boxed{x = \sqrt{3}}$$

$$y = 4 \cdot \sin\left(\frac{2\pi}{3}\right) \sin\frac{\pi}{3}$$

$$= 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$\boxed{y = 3}$$

$$z = 4\cos\left(\frac{2\pi}{3}\right)$$

$$= 4 \cdot \left(-\frac{1}{2}\right)$$

$$\boxed{z = -2}$$

So corresponding rectangular Co-ords. are $(\sqrt{3}, 3, -2)$

(c) $\left(0, \frac{\pi}{15}, \frac{\pi}{5}\right)$

Sol. Given pt. is $\left(0, \frac{\pi}{15}, \frac{\pi}{5}\right)$

we know the relation b/w rectangular & spherical polar Co-ords. are

$x = \rho \sin\phi \cos\theta$ | $\rho = \sqrt{x^2+y^2+z^2}$

$y = \rho \sin\phi \sin\theta$ | $\theta = \tan^{-1}(y/x)$

$z = \rho \cos\phi$ | $\phi = \cos^{-1}\left(\frac{z}{\rho}\right)$

Now $x = 0 \cdot \sin\frac{\pi}{5} \cos\frac{\pi}{15} = 0$

$$y = 0 \cdot \sin\frac{\pi}{5} \cdot \sin\frac{\pi}{15}$$

$$\boxed{y = 0}$$

& $$z = 0 \cdot \cos\frac{\pi}{5}$$

$$\boxed{z = 0}$$

So Corresponding rectangular Co-ords are $(0,0,0)$.

(d) $P = \left(2, 5\frac{\pi}{3}, \frac{3\pi}{4}\right)$

Sol:- Given pt. is $P = \left(2, \frac{5\pi}{3}, 3\frac{\pi}{4}\right)$

We know the relation b/w rectangular & spherical polar co-ords are

$$x = \rho\sin\phi\cos\theta \qquad \rho = \sqrt{x^2+y^2+z^2}$$

$$y = \rho\sin\phi\sin\theta \qquad \theta = \tan^{-1}(y/x)$$

$$z = \rho\cos\phi \qquad \phi = \cos^{-1}\left(\frac{z}{\rho}\right)$$

Now

$$x = 2\sin(3\pi/4)\cos(5\pi/3)$$

$$= 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$\boxed{x = \frac{1}{\sqrt{2}}}$$

$$y = 2 \cdot \sin(3\pi/4) \cdot \sin(5\pi/3)$$

$$= 2 \cdot \frac{1}{\sqrt{2}} \cdot -\frac{\sqrt{3}}{2}$$

$$\boxed{y = -\sqrt{\frac{3}{2}}}$$

$$z = 2 \cdot \cos\left(3\frac{\pi}{4}\right)$$

$$z = 2.\left(\frac{-1}{\sqrt{2}}\right)$$

$$\boxed{z = -\sqrt{2}}$$

So corresponding rectangular co-ords. are $\left(\frac{1}{\sqrt{2}}, -\sqrt{\frac{3}{2}}, -\sqrt{2}\right)$

In problems (5-10), express the given eq. in rectangular co-ords:

Q5 $\rho \cos\phi = 2$

Sol. Given eq. is $\rho \cos\phi = 2$

we know the relation b/w rectangular & spherical polar co-ords. are

$$x = \rho \sin\phi \cos\theta \qquad \rho = \sqrt{x^2+y^2+z^2}$$

$$y = \rho \sin\phi \sin\theta \qquad \theta = \tan^{-1}(y/x)$$

$$z = \rho \cos\phi \qquad \phi = \cos^{-1}\left(\frac{z}{\rho}\right)$$

Now as $\rho \cos\phi = 2$

$$\Rightarrow \boxed{z = 2}$$

Q6 $\rho = 2\cos\theta \sin\phi$

Sol. we know the relation b/w rectangular & spherical co-ords. of a pt. are

$$x = \rho \sin\phi \cos\theta \qquad \rho = \sqrt{x^2+y^2+z^2}$$

$$y = \rho \sin\phi \sin\theta \qquad \theta = \tan^{-1}(y/x)$$

$$z = \rho \cos\phi \qquad \phi = \cos^{-1}\left(\frac{z}{\rho}\right)$$

As $\rho = 2\sin\phi \cos\theta$

$$\rho^2 = 2\rho \sin\phi \cos\theta$$

$$x^2+y^2+z^2 = 2x$$

or $x^2+y^2+z^2-2x = 0$ is req. eq.

Q7 $\rho = 7\sin\theta \sin\phi$

Sol. we know the relation b/w rectangular & spherical polar Co-ords. are

$x = \rho \sin\phi \cos\theta$ | $\rho = \sqrt{x^2+y^2+z^2}$

$y = \rho \sin\phi \sin\theta$ | $\theta = \tan^{-1}(y/x)$

$z = \rho \cos\phi$ | $\phi = \cos^{-1}\left(\frac{z}{\rho}\right)$

as $\rho = 7\sin\phi \sin\theta$

$$\rho^2 = 7\rho \sin\phi \sin\theta$$

$$x^2+y^2+z^2 = 7y$$

or $x^2+y^2+z^2-7y = 0$ is req. eq.

Q8 $\rho^2 \cos 2\theta = a^2$

Sol. we know the relation b/w rectangular & spherical polar Co-ords. are

$x = \rho \sin\phi \cos\theta$ | $\rho = \sqrt{x^2+y^2+z^2}$

$y = \rho \sin\phi \sin\theta$ | $\theta = \tan^{-1}(y/x)$

$z = \rho \cos\phi$ | $\phi = \cos^{-1}\left(\frac{z}{\rho}\right)$

As $\rho^2 \cos 2\theta = a^2$

$$\rho^2(\cos^2\theta - \sin^2\theta) = a^2$$

$$\rho^2\cos^2\theta\,(1-\tan^2\theta) = a^2$$

$$\frac{\rho^2(1-\tan^2\theta)}{\sec^2\theta} = a^2$$

$$\frac{\rho^2(1-\tan^2\theta)}{1+\tan^2\theta} = a^2$$

$$(x^2+y^2+z^2)\left[\frac{1-y^2/x^2}{1+y^2/x^2}\right] = a^2$$

$$(x^2+y^2+z^2)\left(\frac{x^2-y^2}{x^2+y^2}\right) = a^2$$

$$(x^2+y^2+z^2)(x^2-y^2) = a^2(x^2+y^2) \text{ is req. eq.}$$

<u>Q9</u> $z = r^2\cos 2\theta$

<u>Sol.</u> We know the relation b/w rectangular & cylindrical polar Co-ords. are

$$x = r\cos\theta \qquad r = \sqrt{x^2+y^2}$$
$$y = r\sin\theta \qquad \theta = \tan^{-1}(y/x)$$
$$z = z \qquad z = z$$

Now $z = r^2\cos 2\theta$

$$z = r^2(\cos^2\theta - \sin^2\theta)$$

$$= r^2\cos^2\theta - r^2\sin^2\theta$$

$$z = x^2 - y^2 \text{ is req. eq.}$$

Q10 $z = 1+\sin\theta$

Sol. we know the relation b/w rectangular & cylindrical polar co-ords. are

$$x = r\cos\theta \qquad r = \sqrt{x^2+y^2}$$
$$y = r\sin\theta \qquad \theta = \tan^{-1}(y/x)$$
$$z = z \qquad z = z$$

As $z = 1+\sin\theta$

$$z-1 = \sin\theta$$
$$r(z-1) = r\sin\theta$$
$$r^2(z-1)^2 = r^2\sin^2\theta$$
$$(x^2+y^2)(z-1)^2 = y^2 \quad \text{is req. eq.}$$

In problems (11-14) express the given eq. in cylindrical & spherical co-ords

Q11 $(x+y)^2 - z^2 + 4 = 0$

Sol. Given eq. is

$$(x+y)^2 - z^2 + 4 = 0 \quad \text{—— (1)}$$

For spherical co-ords

Put $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$ in eq. (1)

$$(\rho\sin\phi\cos\theta + \rho\sin\phi\sin\theta)^2 - \rho^2\cos^2\phi + 4 = 0$$

$\rho^2\sin^2\phi\,(\cos\theta+\sin\theta)^2 - \rho^2\cos^2\phi + 4 = 0$

$\rho^2\sin^2\phi\,(\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta) - \rho^2\cos^2\phi + 4 = 0$

$\rho^2\sin^2\phi\,(1+\sin 2\theta) - \rho^2\cos^2\phi + 4 = 0$

$\rho^2\sin^2\phi + \rho^2\sin^2\phi\sin 2\theta - \rho^2\cos^2\phi + 4 = 0$

$\rho^2(\sin^2\phi - \cos^2\phi) + \rho^2\sin^2\phi\sin 2\theta + 4 = 0$

$-\rho^2(\cos^2\phi - \sin^2\phi) + \rho^2\sin^2\phi\sin 2\theta + 4 = 0$

$-\rho^2\cos 2\phi + \rho^2\sin^2\phi\sin 2\theta + 4 = 0$

$\rho^2(\sin^2\phi\sin 2\theta - \cos 2\phi) + 4 = 0$ is req. eq.

Now for Cylindrical Co-ords.

Put $x = r\cos\theta$

$y = r\sin\theta$

$z = z$ in (1)

$(r\cos\theta + r\sin\theta)^2 - z^2 + 4 = 0$

$\cancel{r^2(\cos\theta+\sin\theta)^2} - z^2 + 4 = 0$

$r^2(\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta) - z^2 + 4 = 0$

$r^2(1+\sin 2\theta) - z^2 + 4 = 0$ is req. eq.

Q12 $x^2+y^2+2z=6$

Sol. Given eq. is

$$x^2+y^2+2z=6 \quad \text{———} \quad (1)$$

For spherical Co-ords.

Put $x=\rho\sin\phi\cos\theta$

$y=\rho\sin\phi\sin\theta$

$z=\rho\cos\phi$ in (1)

$$\rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta+2\rho\cos\phi=6$$

$$\rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta)+2\rho\cos\phi=6$$

$\boxed{\rho^2\sin^2\phi+2\rho\cos\phi=6}$ is req. eq.

For Cylindrical Co-ords.

Put $x=r\cos\theta$

$y=r\sin\theta$

$z=z$ in (1)

$$r^2\cos^2\theta+r^2\sin^2\theta+2z=6$$

$$r^2(\cos^2\theta+\sin^2\theta)+2z=6$$

$\boxed{r^2+2z=6}$ is req. eq.

Q13 $x^2-y^2-z^2=1$

Sol. Given eq. is

$$x^2-y^2-z^2=1 \quad \text{———} \quad (1)$$

For spherical Co-ords.

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$ in (1)

$$\rho^2 \sin^2\phi \cos^2\theta - \rho^2 \sin^2\phi \sin^2\theta - \rho^2 \cos^2\phi = 1$$

$$\rho^2 \sin^2\phi (\cos^2\theta - \sin^2\theta) - \rho^2 \cos^2\phi = 1$$

$$\rho^2 \sin^2\phi \cos 2\theta - \rho^2 \cos^2\phi = 1$$

$$\boxed{\rho^2 (\sin^2\phi \cos 2\theta - \cos^2\phi) = 1}$$ is req. eq.

For cylindrical Co-ords.

Put $x = r\cos\theta$

$y = r\sin\theta$

$z = z$ in (1)

$$r^2\cos^2\theta - r^2\sin^2\theta - z^2 = 1$$

$$r^2(\cos^2\theta - \sin^2\theta) - z^2 = 1$$

$$\boxed{r^2\cos 2\theta - z^2 = 1}$$ is req.

Q14. $3x + y - 4z = 12$

Sol. Given eq. is

$$3x + y - 4z = 12 \quad \text{——— (1)}$$

For spherical Co-ords.

Put $x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$ in (1)

$$3(\rho \sin\phi \cos\theta) + \rho \sin\phi \sin\theta - 4\rho\cos\phi = 12$$

$$\boxed{\rho\left(3\sin\phi\cos\theta + \sin\phi\sin\theta - 4\cos\phi\right) = 12}$$ is req. eq.

For Cylindrical Co-ords.

Put $x = r\cos\theta$

$y = r\sin\theta$

$z = z$ in ①

$$\boxed{3r\cos\theta + r\sin\theta - 4z = 12}$$ is req. eq.

Q15 Write the eq. of surface defined by

$$\frac{(z-1)^2}{4} - \frac{(y+2)^2}{1} = 4(x-y)$$ relative to a new set of parallel axes with origin at $(4,-2,1)$.

Sol:- Given eq. of surface is

$$\frac{(z-1)^2}{4} - \frac{(y+2)^2}{1} = 4(x-4) \quad \text{———} ①$$

Let the new origin be at $O'(4,-2,1)$. If a pt. P has Co-ords. (x,y,z) relative to original axes & ~~P has Co-ords~~ (x',y',z') relative to new set of parallel axes then

$x = x'+4$

$y = y'-2$

$z = z'+1$

Put values in ①

$$\frac{(z'+1-1)^2}{4} - \frac{(y'-2+2)^2}{1} = 4(x'+4-4)$$

or $$\boxed{\frac{z'^2}{4} - \frac{y'^2}{1} = 4x'}$$ is req. eq.

Q16 Write the eq. $x^2-9y^2-4z^2-6x+18y+16z+20=0$ referred to new set of parallel axes with the origin at $(3,1,2)$.

Sol:- Given eq. of surface is

$$x^2-9y^2-4z^2-6x+18y+16z+20=0$$

$$(x^2-6x)-9y^2+18y-4z^2+16z=-20$$

$$x^2-6x-9(y^2-2y)-4(z^2-4z)=-20$$

$$x^2-6x+9-9(y^2-2y+1)-4(z^2-4z+4)=-20+9-9-16$$

$$(x-3)^2-9(y-1)^2-4(z-2)^2=-36$$

$$\frac{(y-1)^2}{4}+\frac{(z-2)^2}{9}-\frac{(x-3)^2}{36}=1$$

Let the new origin be at $(3,1,2)$.

If a pt. P has co-ord (x,y,z) relative to original axes & P has co-ords (x',y',z') relative to new set of parallel axes then

$$x=x'+3$$

$$y=y'+1$$

$$z=z'+2$$

Put in ①

$$\frac{(y'+1-1)^2}{4}+\frac{(z'+2-2)^2}{9}-\frac{(x'+3-3)^2}{36}=1$$

$$\frac{y'^2}{4}+\frac{z'^2}{9}-\frac{x'^2}{36}=1 \quad \text{is the req. eq.}$$