Exercise No. 8.3

Find the eq. of the plane through the three given points (Problems 1–3):

Q1 $(2,1,1),(6,3,1),(-2,1,2)$.

Sol. Let the eq. of req. plane is

$$ax+by+cz+d=0 \quad \text{——— (1)}$$

Since it passes, through pts. $(2,1,1),(6,3,1)$ & $(-2,1,2)$

So.

$$2a+b+c+d=0 \quad \text{——— (2)}$$

$$6a+3b+c+d=0 \quad \text{——— (3)}$$

$$-2a+b+2c+d=0 \quad \text{——— (4)}$$

Subt. (2),(3) & (3),(4)

$$-4a-2b+0.c=0$$

or $$2a+b+0.c=0$$

& $$8a+2b-c=0$$

$$\frac{a}{-1-0}=\frac{-b}{-2-0}=\frac{c}{4-8}$$

$$\frac{a}{-1}=\frac{b}{2}=\frac{c}{-4}$$

Put these proportional values of a,b,c in (2)

$$2(-1)+2-4+d=0$$

$$\boxed{d=4}$$

Put all values in (1)

$$-x+2y-4z+4=0$$

or $\boxed{x-2y+4z-4=0}$ is req. eq.

Q2 (1,-1,2), (-3,-2,6) , (6,0,1)

Sol. Let the req. eq. of plane is

$$ax+by+cz+d=0 \quad \text{——— (1)}$$

Since it passes through (1,-1,2), (-3,-2,6) & (6,0,1)

So they will satisfy eq. (1)

Hence

$$a-b+2c+d = 0 \quad \text{——— (2)}$$
$$-3a-2b+6c+d = 0 \quad \text{——— (3)}$$
$$6a+0b+c+d = 0 \quad \text{——— (4)}$$

Subt. (2), (3) & (3), (4)

$$\left.\begin{aligned} 4a+b-4c &= 0 \\ -9a-2b+5c &= 0 \end{aligned}\right]$$

$$\frac{a}{5-8} = \frac{-b}{20-36} = \frac{c}{-8+9}$$

$$\frac{a}{-3} = \frac{b}{16} = \frac{c}{1}$$

Put these proportional values of a, b, c in (2)

$$-3-16+2+d=0$$

$$-17+d=0$$

$$\boxed{d=17}$$

Put all values in (1)

$$-3x+16y+z+17=0$$

$$\boxed{3x-16y-z-17=0}$$ is req. eq.

Q3 (-1,1,1), (5,-8,-2) & (4,1,0)

Sol. Let the eq. of plane is

$ax+by+cz+d=0$ —— ①

Since it passes through $(-1,1,1)$, $(5,-8,-2)$ & $(4,1,0)$

So these pts. will lie on ①

Hence
$$-a+b+c+d=0 \text{ —— ②}$$
$$5a-8b-2c+d=0 \text{ —— ③}$$
$$4a+b+0c+d=0 \text{ —— ④}$$

Subt. ②,③ & ③,④

$$-6a+9b+3c=0$$

or
$$\left.\begin{aligned} 2a-3b-c &= 0 \\ a-9b-2c &= 0 \end{aligned}\right]$$
&

$$\frac{a}{6-9}=\frac{-b}{-4+1}=\frac{c}{-18+3}$$

$$\frac{a}{-3}=\frac{b}{3}=\frac{c}{-15}$$

or $\dfrac{a}{1}=\dfrac{b}{-1}=\dfrac{c}{5}$

Put these proportional values of a,b,c in ②

$$-1-1+5+d=0$$
$$3+d=0$$
$$d=-3$$

Put all values in ①

$x-y+5z-3=0$ is req. eq.

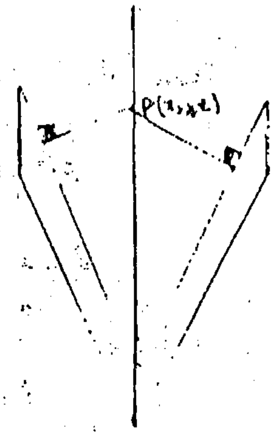
Q4 Find eqs. of the planes bisecting the angles b/w the planes $3x+2y-6z+1=0$ & $2x+y+2z-5=0$

Sol. Given eqs. of planes are

$$3x+2y-6z+1=0 \quad \& \quad 2x+y+2z-5=0$$

Let $P(x,y,z)$ be any pt. on the bisecting plane then the distance of pt. P from both planes should be equal. so

$$\frac{|3x+2y-6z+1|}{\sqrt{9+4+36}} = \frac{|2x+y+2z-5|}{\sqrt{4+1+4}}$$



$$\frac{|3x+2y-6z+1|}{7} = \frac{|2x+y+2z-5|}{3}$$

or $$\frac{3x+2y-6z+1}{7} = \pm\frac{2x+y+2z-5}{3}$$

$\frac{3x+2y-6z+1}{7} = \frac{2x+y+2z-5}{3}$	$\frac{3x+2y-6z+1}{7} = -\frac{2x+y+2z-5}{3}$
$14x+7y+14z-35 = 9x+6y-18z+3$	$9x+6y-18z+3 = -14x-7y-14z+35$
$14x+7y+14z-35-9x-6y+18z-3=0$	$9x+6y-18z+3+14x+7y+14z-35=0$
$5x+y+32z-38=0$	$23x+13y-4z-32=0$

are req. eqs of planes.

Q5 Transform the eqs. of the planes $3x-4y+5z=0$ & $2x-y-2z=5$ to normal forms & hence find measure of the angle b/w them.

Sol. Given eqs. of planes are

$$3x-4y+5z=0 \quad \text{——— (1)}$$
$$2x-y-2z=5 \quad \text{——— (2)}$$

Dividing both sides of plane ① by

$$\sqrt{9+16+25} = \sqrt{50} = 5\sqrt{2}$$

$$\frac{3}{5\sqrt{2}}x - \frac{4}{5\sqrt{2}}y + \frac{5}{5\sqrt{2}}z = 0 \quad \text{———} \quad (A)$$

which is req. normal form of plane ①

Now dividing both sides of plane ② by

$$\sqrt{4+1+4} = \sqrt{9} = 3$$

$$\frac{2}{3}x - \frac{1}{3}y - \frac{2}{3}z = \frac{5}{3} \quad \text{———} \quad (B)$$

which is req. normal form of plane ②

Now d.cs. of normal to plane (A) are $\frac{3}{5\sqrt{2}}, -\frac{4}{5\sqrt{2}}, \frac{5}{5\sqrt{2}}$

& d.cs. of normal to plane (B) are $\frac{2}{3}, \frac{-1}{3}, -\frac{2}{3}$

let θ be the angle b/w given planes then

$$\cos\theta = \left(\frac{3}{5\sqrt{2}}\right)\left(\frac{2}{3}\right) + \left(\frac{-4}{5\sqrt{2}}\right)\left(-\frac{1}{3}\right) + \left(\frac{5}{5\sqrt{2}}\right)\left(-\frac{2}{3}\right)$$

$$= \frac{6}{15\sqrt{2}} + \frac{4}{15\sqrt{2}} - \frac{10}{15\sqrt{2}}$$

$$= \frac{\cancel{10}}{15\sqrt{2}} - \frac{\cancel{10}}{\cancel{15\sqrt{2}}}$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\boxed{\theta = \frac{\pi}{2}}$$

Q6 Find the eqs. to the planes through the pts. 57
$(4,-5,3), (2,3,1)$ & parallel to the Co-ord. axes.

Sol.

Case(i) let the eq. of req. plane is

$$ax+by+cz+d=0 \quad \text{——— (1)}$$

Since it passes through $(4,-5,3)$ & $(2,3,1)$

so
$$4a-5b+3c+d=0 \quad \text{——— (2)}$$
$$2a+3b+c+d=0 \quad \text{——— (3)}$$

Since req. plane is $\parallel$ to x-axis whose d.rs. are $1,0,0$

Hence
$$a+0b+0c=0 \quad \text{——— (4)}$$
$$2a-8b+2c=0 \quad \text{——— (5)} \qquad \text{subt. (3) from (2)}$$

$$\frac{a}{0-0}=\frac{-b}{2-0}=\frac{c}{-8-0}$$

$$\frac{a}{0}=\frac{b}{-2}=\frac{c}{-8}$$

$$\frac{a}{0}=\frac{b}{1}=\frac{c}{4}$$

Put these values in (2)

$$0-5+3(4)+d=0$$
$$7+d=0$$
$$d=-7$$

Put all values in (1)

$$0x+y+4z-7=0$$

or $\boxed{y+4z-7=0}$ is req. eq.

Case(ii) Let the eq. of req. plane is

$$ax+by+cz+d=0 \quad \text{———} \ (1)$$

Since it passes through $(4,-5,3)$ & $(2,3,1)$

So $$4a-5b+3c+d=0 \quad \text{———} \ (2)$$

$$2a+3b+c+d=0 \quad \text{———} \ (3)$$

Since plane (1) is $\parallel$ to y-axis whose d.rs. are 0,1,0

So $$0.a+b+0c=0 \quad \text{———} \ (4)$$

$$2a-8b+2c=0 \quad \text{———} \ (5)$$ subt. (3) from (2)

$$\frac{a}{2+0} = \frac{-b}{0-0} = \frac{c}{0-2}$$

$$\frac{a}{2} = \frac{b}{0} = \frac{c}{-2}$$

$$\frac{a}{1} = \frac{b}{0} = \frac{c}{-1}$$

Put these values in (2)

$$4(1)-5(0)+3(-1)+d=0$$

$$4-3+d=0$$

$$1+d=0$$

$$d=-1$$

Put all values in (1)

$$x+0y-z-1=0$$

or $x-z-1=0$ is req. eq.

Case(iii) Let the eq. of req. plane is

$$ax+by+cz+d=0 \quad \text{———} \ (1)$$

Since it passes through $(4,-5,3)$ & $(2,3,1)$ 59

So $\quad 4a-5b+3c+d=0$ —— ②

$\quad 2a+3b+c+d=0$ —— ③

Since plane ① is ∥ to z-axis whose d.r.s.

are $0,0,1$

So $\quad 0a+0b+c=0$ —— ④

$\quad 2a-8b+2c=0$ —— ⑤ Subt. ③ from ②

$$\frac{a}{0+8}=\frac{-b}{0-2}=\frac{c}{0-0}$$

$$\frac{a}{8}=\frac{b}{2}=\frac{c}{0}$$

$$\frac{a}{4}=\frac{b}{1}=\frac{c}{0}$$

Put these values in ②

$$4(4)-5(1)+3(0)+d=0$$

$$16-5+d=0$$

$$11+d=0$$

$$d=-11$$

Put all values in ①

$$4x+y+0z-11=0$$

$4x+y-11=0$ is req. eq.

Q7 Find the eq. of the plane through the pts. (1,0,1) & (2,2,1) & perpendicular to the plane $x-y-z+4=0$

Sol:- Let the eq. of req. plane is

$$ax+by+cz+d=0 \quad \text{—— (1)}$$

Since it passes through (1,0,1) & (2,2,1)

So $$a+0b+c+d=0 \quad \text{—— (2)}$$

$$2a+2b+c+d=0 \quad \text{—— (3)}$$

Since plane (1) is $\perp$ to plane $x-y-z+4=0$

So by condition of perpendicularity

$$a-b-c=0 \quad \text{—— (4)}$$

$$-a-2b+0c=0 \quad \text{—— (5)} \qquad \text{Subt. (3) f- (2)}$$

$$\frac{a}{0-2}=\frac{-b}{0-1}=\frac{c}{-2-1}$$

$$\frac{a}{-2}=\frac{b}{1}=\frac{c}{-3}$$

Put these values in (2)

$$-2+0-3+d=0$$

$$d=5$$

Put all values in (1)

$$-2x+y-3z+5=0$$

$$2x-y+3z-5=0 \text{ is req. eq.}$$

Q8 Find the eq. of plane which is perp. bisector of the segment joining the pts. $(3,4,-1)$ & $(5,2,7)$.

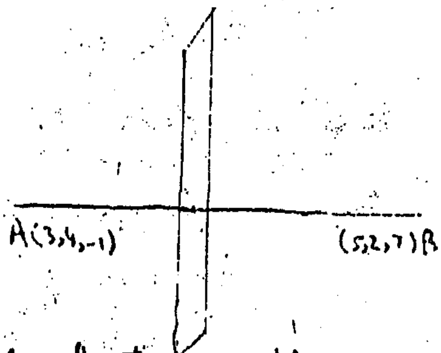
Sol.

Let given pts. are $A(3,4,-1)$ & $B(5,2,7)$.

Now d.rs. of the line AB

are $5-3, 2-4, 7+1$

or $2, -2, 8$



Since AB is perp. to req. plane

So $2, -2, 8$ are the d.rs. of normal to req. plane

Now mid pt. of AB is $\left(\frac{3+5}{2}, \frac{4+2}{2}, \frac{-1+7}{2}\right)$

$$= (4,3,3)$$

Since req. plane is perp. bisector of line AB

So $(4,3,3)$ lies on req. plane.

Hence req. eq. of plane is

$$2(x-4)-2(y-3)+8(z-3)=0$$

$$2x-2y+8z-8+6-24=0$$

$$2x-2y+8z-26=0$$

or $x-y+4z-13=0$ is req. eq.

Q9 Show that the join of $(0,-1,0)$ & $(2,4,-1)$ intersect the join of $(1,1,1)$ & $(3,3,9)$.

Sol.

First we will show that the four given pts. are Coplanar.

Now we find eq. of plane through three pts.

Let the req. eq. of plane is

$$ax+by+cz+d=0 \quad \text{——} \ ①$$

Since it passes through $(0,-1,0)$, $(2,4,-1)$ & $(1,1,1)$

So
$$0a-b+0c+d=0 \quad \text{——} \ ②$$
$$2a+4b-c+d=0 \quad \text{——} \ ③$$
$$a+b+c+d=0 \quad \text{——} \ ④$$

Subt. ②, ③ & ③, ④

$$\left.\begin{array}{l} -2a-5b+c=0 \\ a+3b-2c=0 \end{array}\right]$$

$$\frac{a}{10-3} = \frac{-b}{4-1} = \frac{c}{-6+5}$$

$$\frac{a}{7} = \frac{b}{-3} = \frac{c}{-1}$$

Put values in ②

$$0+3+0+d=0$$
$$d=-3$$

Put all values in ①

$$7x-3y-z-3=0$$

Put the fourth pt. $(3,3,9)$ in above eq.

$$7(3)-3(3)-9-3=0$$
$$21-9-9-3=0$$
$$21-21=0$$
$$0=0$$

Since the eq. of plane is satisfied

Hence the four given pts. are coplanar.

So the two joins are coplanar.

Now d.rs. of join of $(0,-1,0)$ & $(2,4,-1)$ are

$2-0, 4+1, -1-0$

or $2, 5, -1$

& d.rs. of join of $(1,1,1)$ & $(3,3,9)$ are $3-1, 3-1, 9-1$

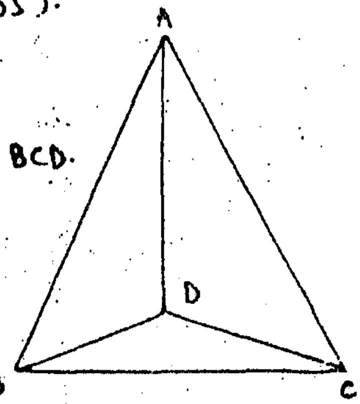
or $2, 2, 8$

Since d.rs. of both joins are not proportional so the two joins are not parallel & so being coplanar, they intersect each other.

Q10 The vertices of a tetrahedron are $(0,0,0)$, $(3,0,0)$, $(0,-4,0)$ & $(0,0,5)$. Find the eqs. of the planes of its faces.

Soln Let the vertices of given tetrahedron are $A(0,0,0)$, $B(3,0,0)$, $C(0,-4,0)$ & $D(0,0,5)$.

Then we want to find the eqs. of the plane faces ABC, ABD, ACD & BCD.



(i) Eq. of plane face ABC

Let the req. eq. of plane is

$$ax+by+cz+d=0 \quad \text{---} (1)$$

Since it passes through $(0,0,0)$, $(3,0,0)$ & $(0,-4,0)$

So

$0a + 0b + 0c + d = 0$ —— (2)

$3a + 0b + 0c + d = 0$ —— (3)

$0a - 4b + 0c + d = 0$ —— (4)

Subt. (2), (3) & (3), (4)

$$\left.\begin{aligned} -3a + 0b + 0c &= 0 \\ 3a + 4b + 0c &= 0 \end{aligned}\right]$$

$$\frac{a}{0-0} = \frac{-b}{0-0} = \frac{c}{-12-0}$$

$$\frac{a}{0} = \frac{b}{0} = \frac{c}{1}$$

Put values in (2)

$0 + 0 + 0 + d = 0$

$d = 0$

Put all values in (1)

$0x + 0y + z + 0 = 0$

, or $\boxed{z = 0}$ is req. eq.

(ii) <u>Eq. of plane face ABD</u>

let the req. eq. of plane is

$ax + by + cz + d = 0$ —— (1)

Since it passes through $(0,0,0)$, $(3,0,0)$ & $D(0,0,5)$

So $0a + 0b + 0c + d = 0$ —— (2)

$3a + 0b + 0c + d = 0$ —— (3)

$0a + 0b + 5c + d = 0$ —— (4)

Subt. (2), (3) & (3), (4)

$$\left.\begin{aligned} -3a + 0b + 0c &= 0 \\ 3a + 0b - 5c &= 0 \end{aligned}\right]$$

$$\frac{a}{0-0} = \frac{-b}{15-0} = \frac{c}{0-0}$$

$$\frac{a}{0} = \frac{b}{-15} = \frac{c}{0}$$

$$\frac{a}{0} = \frac{b}{1} = \frac{c}{0}$$

Put in eq. (2)

$$0a + 0b + 0c + d = 0$$

$$d = 0$$

Put all values in (1)

$$0x + y + 0z + 0 = 0$$

$\boxed{y = 0}$ is req. eq.

(iii) <u>Eq. of plane face ACD</u>

Let the eq. of req. plane is

$$ax + by + cz + d = 0 \quad \text{---} (1)$$

Since it passes through $(0,0,0)$, $(0,-4,0)$ & $(0,0,5)$

So $0a + 0b + 0c + d = 0$ ——— (2)

$0a - 4b + 0c + d = 0$ ——— (3)

$0a + 0b + 5c + d = 0$ ——— (4)

Subt. (2), (3) & (3), (4)

$$\left.\begin{aligned} 0a + 4b + 0c &= 0 \\ 0a - 4b - 5c &= 0 \end{aligned}\right\}$$

$$\frac{a}{-20+0} = \frac{-b}{0-0} = \frac{c}{0-0}$$

$$\frac{a}{1} = \frac{b}{0} = \frac{c}{0}$$

Put these values in (2)

$$0+0+0+d=0$$

$$d=0$$

Put all values in (1)

$$x+0y+0z+0=0$$

$\boxed{x=0}$ is req. eq.

(iv) <u>Eq. of plane face BCD</u>

Let the eq. of req. plane is

$$ax+by+cz+d=0 \quad \text{——— (1)}$$

Since it passes through $(3,0,0)$, $(0,-4,0)$ & $(0,0,5)$

So

$$3a+0b+0c+d=0 \quad \text{——— (2)}$$

$$0a-4b+0c+d=0 \quad \text{——— (3)}$$

$$0a+0b+5c+d=0 \quad \text{——— (4)}$$

Subt. (2), (3) & (3), (4)

$$\left.\begin{aligned} 3a+4b+0c &= 0 \\ 0a-4b-5c &= 0 \end{aligned}\right]$$

$$\frac{a}{-20+0} = \frac{-b}{-15-0} = \frac{c}{-12-0}$$

$$\frac{a}{-20} = \frac{b}{15} = \frac{c}{-12}$$

Put these values in (2)

$$-60+0b+0c+d=0$$

$$d=60$$

Put all values in (1)

$$-20x+15y-12z+60=0$$

$20x-15y+12z-60=0$ is req. eq.

Q11 Find the eq. of the plane through $(5,-1,4)$ & $\perp^{r}$ to each of the planes $x+y-2z-3=0$ & $2x-3y+z=0$

Sol. Let the eq. of req. plane is

$$ax+by+cz+d=0 \quad \text{———} (1)$$

Since it passes through $(5,-1,4)$, so

$$5a-b+4c+d=0 \quad \text{———} (2)$$

As plane (1) is $\perp$ to planes $x+y-2z-3=0$ & $2x-3y+z=0$ so by condition of perpendicularity

$$\left.\begin{aligned} a+b-2c &= 0 \\ 2a-3b+c &= 0 \end{aligned}\right]$$

$$\frac{a}{1-6} = \frac{-b}{1+4} = \frac{c}{-3-2}$$

$$\frac{a}{-5} = \frac{b}{-5} = \frac{c}{-5}$$

or $$\frac{a}{1} = \frac{b}{1} = \frac{c}{1}$$

Put these values in (1)

$$5-1+4+d=0$$

$$8+d=0$$

$$\boxed{d=-8}$$

Put all values in (1)

$x+y+z-8=0$ is req. eq.

Q12 Find the eq. of the plane each of whose pt. is equidistant from the pts. $A(2,-1,1)$ & $B(3,1,5)$

Sol. Since the req. plane is equidistant pts. $A(2,-1,1)$ & $B(3,1,5)$, so it should be a perpendicular bisector of the line AB & line AB will be a normal line to req. plane. 68

d.rs. of line AB are $3-2, 1+1, 5-1$ or $1, 2, 4$

Now mid pt. of AB $= \left(\frac{2+3}{2}, \frac{-1+1}{2}, \frac{1+5}{2}\right) = \left(\frac{5}{2}, 0, 3\right)$

This pt. will lie on req. plane

So req. eq. of plane through $\left(\frac{5}{2}, 0, 3\right)$ & having normal with d.rs. $1, 2, 4$ is

$$1\left(x-\frac{5}{2}\right)+2(y-0)+4(z-3)=0$$

$$x+2y+4z-\frac{5}{2}-12=0$$

$$2x+4y+8z-5-24=0$$

$$2x+4y+8z-29=0$$

is req. eq.

Q13 Find the eq. of the plane through the pt. $(3,-2,5)$ & perp. to the line

$$x=2+3t,\ y=1-6t,\ z=-2+2t$$

Sol:- Given that the req. plane passes through the pt. $(3,-2,5)$. & it is perpendicular to the given line $x=2+3t,\ y=1-6t,\ z=-2+2t$

D.rs. of given line are $3, -6, 2$

So d.rs. of normal to req. plane are $3, -6, 2$

Hence req. eq. of plane through $(3,-2,5)$ & having normal line with d.rs. $3,-6,2$ is

$$3(x-3)-6(y+2)+2(z-5)=0$$

$$3x-6y+2z-9-12-10=0$$

$$3x-6y+2z-31=0 \text{ is req. eq.}$$

Q14 Find the parametric eqs. of the line containing the pt. $(2,4,-3)$ & perp. to the plane $3x+3y-7z=9$

Sol.

Since the req. line is perp. to the plane

$$3x+3y-7z-9=0$$

So req. line is a normal line to given plane.

$\therefore$ d.rs. of this line are $3,3,-7$

Also the req. line passes through $(2,4,-3)$.

So eq. of req. line through $(2,4,-3)$ & d.rs. $3,3,-7$ is

$$\frac{x-2}{3}=\frac{y-4}{3}=\frac{z+3}{-7}=t$$

$$\Rightarrow \left.\begin{aligned} x &= 2+3t \\ y &= 4+3t \\ z &= -3-7t \end{aligned}\right\}$$

are req. parametric eqs. of line.

Q15 Write eq. of the family of all planes whose distance from the origin is 7. Find those members parallel to the plane. $x+y+z+5=0$

Sol:-
The eq. of family of all planes in normal form is
$$lx+my+nz=p$$
Here $p'=7$

So $\boxed{lx+my+nz=7}$ is the req. eq. of family of planes whose distance from origin is 7.

Here l, m, n are d.cs of normal to planes.

Eq. of given plane is
$$x+y+z+5=0$$
$$\text{or } x+y+z=-5$$
Dividing both sides by $\sqrt{1^2+1^2+1^2}=\sqrt{3}$
$$\frac{x}{\sqrt{3}}+\frac{y}{\sqrt{3}}+\frac{z}{\sqrt{3}}=-\frac{5}{\sqrt{3}}$$
$$\text{or } -\frac{x}{\sqrt{3}}-\frac{y}{\sqrt{3}}-\frac{z}{\sqrt{3}}=\frac{5}{\sqrt{3}} \quad \text{—— (1)}$$

A plane parallel to (1) has a normal vector with d.cs. $\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$ or $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

So there are two members of family parallel to (1) & these members are
$$-\frac{1}{\sqrt{3}}x-\frac{1}{\sqrt{3}}y-\frac{1}{\sqrt{3}}z=7 \quad \& \quad \frac{1}{\sqrt{3}}x+\frac{1}{\sqrt{3}}y+\frac{1}{\sqrt{3}}z=7$$

Q16 Find the eq. of the plane which passes through the pt. (3,4,5), has an x-intercept equal to −5 & is perpendicular to the plane $2x+3y-z=8$

Sol. Eq. of a plane in intercept form is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Here $a=-5$

Put in above eq.

$$\frac{x}{-5}+\frac{y}{b}+\frac{z}{c}=1 \quad \text{——— (1)}$$

As this plane is perpendicular to $2x+3y-z=8$

So by condition of perpendicularity

$$\frac{-1}{5}(2)+\frac{1}{b}(3)+\frac{1}{c}(-1)=0$$

$$-\frac{2}{5}+\frac{3}{b}-\frac{1}{c}=0$$

$$\text{or} \quad \frac{3}{b}-\frac{1}{c}=\frac{2}{5} \quad \text{——— (2)}$$

Also since plane (1) passes through (3,4,5)

So $\frac{3}{-5}+\frac{4}{b}+\frac{5}{c}=1$

$$\frac{4}{b}+\frac{5}{c}=1+\frac{3}{5}$$

$$\frac{4}{b}+\frac{5}{c}=\frac{8}{5} \quad \text{——— (3)}$$

& $\frac{15}{b}-\frac{5}{c}=2$ ——— (4) Multiply (2) by 5

Adding (3) & (4)

$$\frac{4}{b} + \frac{15}{b} = \frac{8}{5} + 2$$

$$\frac{19}{b} = \frac{18}{5}$$

$$\Rightarrow b = \frac{19 \times 5}{18}$$

$$\boxed{b = \frac{95}{18}}$$

Put value in (2)

$$\frac{3}{\frac{95}{18}} - \frac{1}{c} = \frac{2}{5}$$

$$\frac{54}{95} - \frac{1}{c} = \frac{2}{5}$$

$$\frac{1}{c} = \frac{54}{95} - \frac{2}{5}$$

$$= \frac{54 - 38}{95}$$

$$\frac{1}{c} = \frac{16}{95}$$

$$\Rightarrow \boxed{c = \frac{95}{16}}$$

Put, values in (1)

$$\frac{x}{-5} + \frac{y}{\frac{95}{18}} + \frac{z}{\frac{95}{16}} = 1$$

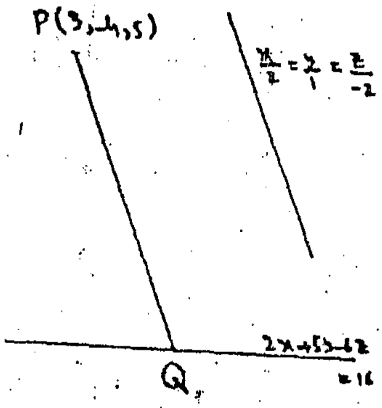
$$\frac{x}{-5} + \frac{18y}{95} + \frac{16z}{95} =$$

$$-19x + 18y + 16z = 95$$

$$\underline{19x - 18y - 16z + 95 = 0}$$

Q17 Show that the distance of the pt. $P(3,-4,5)$ from the plane $2x+5y-6z=16$ measured parallel to the line $\frac{x}{2}=\frac{y}{1}=\frac{z}{-2}$ is $\frac{60}{7}$.

Sol.



Since the line through $P(3,-4,5)$ is parallel to given line

So d.rs. of the line through P are

$2, 1, -2$

Hence eq. of line through $P(3,-4,5)$ & parallel to given line is

$$\frac{x-3}{2}=\frac{y+4}{1}=\frac{z-5}{-2}=t$$

$$\Rightarrow \left.\begin{array}{l} x=3+2t \\ y=-4+t \\ z=5-2t \end{array}\right\}$$

Any pt. on this line is $Q(3+2t,-4+t,5-2t)$

If $Q(3+2t,-4+t,5-2t)$ lies on plane $2x+5y-6z=16$

then $2(3+2t)+5(-4+t)-6(5-2t)=16$

$$6+4t-20+5t-30+12t=16$$

$$21t-44=16$$

$$21t=16+44$$

$$21t=60$$

$$t=\frac{60}{21} \Rightarrow \boxed{t=\frac{20}{7}}$$

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So Co-ords. of Q are $Q\left(3+2\left(\frac{20}{7}\right), -4+\frac{20}{7}, 5-2\left(\frac{20}{7}\right)\right)$

$$= Q\left(\frac{21+40}{7}, \frac{-28+20}{7}, \frac{35-40}{7}\right)$$

$$= Q\left(\frac{61}{7}, -\frac{8}{7}, -\frac{5}{7}\right)$$

Now the req. distance is $= |PQ|$

$$= \sqrt{\left(\frac{61}{7}-3\right)^2+\left(-\frac{8}{7}+4\right)^2+\left(-\frac{5}{7}-5\right)^2}$$

$$\sqrt{\left(\frac{61-21}{7}\right)^2+\left(\frac{-8+28}{7}\right)^2+\left(\frac{-5-35}{7}\right)^2}$$

$$= \sqrt{\left(\frac{40}{7}\right)^2+\left(\frac{20}{7}\right)^2+\left(-\frac{40}{7}\right)^2}$$

$$= \sqrt{\frac{1600}{49}+\frac{400}{49}+\frac{1600}{49}}$$

$$= \sqrt{\frac{3600}{49}}$$

$$= \frac{60}{7}$$

Q18 Show that the lines

$L: x = 3+2t, \; y = 2+t, \; z = -2-3t$

$M: x = -3+4s, \; y = 5-4s, \; z = 6-5s$

intersect. Find eq. of the plane containing these lines.

Sol. Given lines are

$L: x = 3+2t, \; y = 2+t, \; z = -2-3t$

$M: x = -3+4s, \; y = 5-4s, \; z = 6-5s$

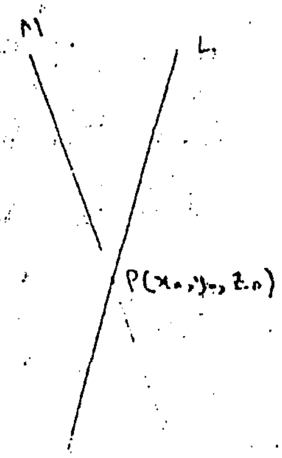
Suppose these lines intersect at pt. $P(x_0, y_0, z_0)$ 75

So this pt will lie on both lines

$\Rightarrow x_0 = 3+2t, \quad y_0 = 2+t, \quad z_0 = -2-3t$

& $x_0 = -3+4s, \quad y_0 = 5-4s, \quad z_0 = 6-5s$

$$\Rightarrow \begin{aligned} 3+2t &= -3+4s \\ 2+t &= 5-4s \\ -2-3t &= 6-5s \end{aligned}$$



or
$$2t-4s = -6 \quad \text{——— ①}$$
$$t+4s = 3 \quad \text{——— ②}$$
$$-3t+5s = 8 \quad \text{——— ③}$$

Adding ① & ②

$3t = -3$

$\boxed{t=-1}$

Put in ①

$2(-1)-4s = -6$

$-2-4s = -6$

$-4s = -6+2$

$-4s = -4$

$\boxed{s=1}$

We see that these values of t & s satisfy eq. ③

So given lines intersect

Now we find eq. of plane containing given lines L & M.

Eq. of lines L & M in symmetric forms are

$$L: \frac{x-3}{2} = \frac{y-2}{1} = \frac{z+2}{-3}$$

As the req. plane Contains both lines so it will Contain every pt. of both lines 76

A pt. on the line L is $(3,2,-2)$

If a,b,c are d.rs. of req. plane then eq. of plane through $(3,2,-2)$ is

$$a(x-3)+b(y-2)+c(z+2)=0 \quad \text{——} \quad (A)$$

Since this plane Contains both lines so the normal line of plane is perp. to both the lines

Hence
$$\left.\begin{aligned} 2a+b-3c&=0\\ 4a-4b-5c&=0 \end{aligned}\right]$$

$$\frac{a}{-5-12}=\frac{-b}{-10+12}=\frac{c}{-8-4}$$

$$\frac{a}{-17}=\frac{b}{-2}=\frac{c}{-12}$$

or $$\frac{a}{17}=\frac{b}{2}=\frac{c}{12}$$

Put these values in (A)

$$17(x-3)+2(y-2)+12(z+2)=0$$

$$17x+2y+12z-51-4+24=0$$

$$17x+2y+12z-31=0 \text{ is req. eq.}$$

Q19 If a,b,c are intercepts of a plane on the Co-ord. axes & r is the distance of the origin from the plane, prove that

$$\frac{1}{r^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

Sol. Since a, b, c are the intercepts of a plane π on Co-ord. axes, so eq. of a plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Now the distance of the plane from origin is

$$r=\frac{|0+0+0-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

$$r=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

or $\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}=\frac{1}{r}$

sq. both sides

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{r^2}$$

or $\frac{1}{r^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$

Q2. Find eqs. of two planes whose distances from the origin are 3 units each & which are perpendicular to the line through the pts. $A(7,3,1)$ & $B(6,4,-1)$.

Sol:- Since both req. planes are perp. to the line through $A(7,3,1)$ & $B(6,4,-1)$

So line AB is a normal to both the req.

planes.

Now d.rs. of line AB are $7-6, 3-4, 1+1$

or $1, -1, 2$

Now $\sqrt{1+1+4} = \sqrt{6}$

So d.cs. of line AB are $\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$

Eqs. of planes at distance 3 units each from the origin are

$$lx + my + nz = \pm 3$$

where l, m, n are d.cs. of normals to the two planes

Since AB is normal to the two planes

So $l, m, n = \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$

Put in above eq.

$$\frac{1}{\sqrt{6}}x - \frac{1}{\sqrt{6}}y + \frac{2}{\sqrt{6}}z = \pm 3$$

or $x - y + 2z = \pm 3\sqrt{6}$ are eqs. of req. planes.