Exercise 6.1

Examine whether each of the given equations represents two straight lines. If so, find an equation of each straight line.

Q.1
$$10xy + 8x - 15y - 12 = 0 \quad \text{—— (1)}$$

General equation of 2nd degree is
$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \text{—— (2)}$$

From (1) and (2)
$$a = 0,\ h = 5,\ b = 0,\ g = 4,\ f = -\frac{15}{2},\ c = -12$$

Now
$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 0 & 5 & 4 \\ 5 & 0 & -\frac{15}{2} \\ 4 & -\frac{15}{2} & -12 \end{vmatrix}$$

$$= 0 - 5(-60 - (-30)) + 4\left(-\frac{75}{2} - 0\right)$$
$$= -5(-60 + 30) - 2(75)$$
$$= -5(-30) - 150$$
$$= 150 - 150$$
$$= 0$$

i.e (1) represents two straight lines.

(1) $\Rightarrow$
$$2x(5y+4) - 3(5y+4) = 0$$
$$(2x-3)(5y+4) = 0$$
$$2x - 3 = 0, \quad 5y + 4 = 0$$
are the two straight lines.

Q#2
$$2x^2 - xy + 5x - 2y + 2 = 0 \quad \text{—— (1)}$$
$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \text{—— (2)}$$

Here $a = 2,\ h = -\frac{1}{2},\ b = 0,\ g = \frac{5}{2},\ f = -1,\ c = 2$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 2 & -\frac{1}{2} & \frac{5}{2} \\ -\frac{1}{2} & 0 & -1 \\ \frac{5}{2} & -1 & 2 \end{vmatrix}$$

$$= 2(0-1) + \frac{1}{2}\left(-1 + \frac{5}{2}\right) + \frac{5}{2}\left(\frac{1}{2} - 0\right)$$

$$= -2 + \frac{1}{2}\left(\frac{-2+5}{2}\right) + \frac{5}{4}$$

$$= -2 + \frac{3}{4} + \frac{5}{4} = \frac{-8+3+5}{4} = 0$$

Hence the given equation represents two straight lines.

1) => $\quad 2x^2 + x(5-y) + 2(1-y) = 0$

$$x = \frac{-(5-y) \pm \sqrt{(5-y)^2 - 4(2)(2-2y)}}{2(2)}$$

$$x = \frac{-(5-y) \pm \sqrt{(5-y)^2 - 8(2-2y)}}{4}$$

$$x = \frac{-(5-y) \pm \sqrt{25+y^2-10y-16+16y}}{4}$$

$$x = \frac{-(5-y) \pm \sqrt{9+y^2+6y}}{4}$$

$$x = \frac{-(5-y \pm \sqrt{(3+y)^2}}{4}$$

$$x = \frac{-(5-y) \pm (3+y)}{4}$$

$$x = \frac{-5+y+3+y}{4} \quad \text{and} \quad \frac{-5+y-3-y}{4}$$

$$x = \frac{-2+2y}{4} \quad \text{and} \quad \frac{-8}{4}$$

$$x = \frac{y-1}{2} \quad \text{and} \quad -\frac{8}{4}$$

$$\Rightarrow \quad x = \frac{y-1}{2} \quad \text{and} \quad x = \frac{-8}{4}$$

$$2x - y + 1 = 0 \quad \text{and} \quad 4x + 8 = 0$$

are the two straight lines.

Q#3 $6x^2-17xy-3y^2+22x+10y-8=0$ ——— (1)

$ax^2+2hxy+by^2+2gx+2fy+c=0$

Here $a=6,\ h=-\frac{17}{2},\ b=-3,\ g=11,\ f=5,\ c=-8$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 6 & -\frac{17}{2} & 11 \\ -\frac{17}{2} & -3 & 5 \\ 11 & 5 & -8 \end{vmatrix}$$

$$= 6(24-25)+\frac{17}{2}(68-55)+11\left(-\frac{85}{2}+33\right)$$

$$= -6+\frac{221}{2}-\frac{935}{2}+363$$

$$= \frac{221}{2}-\frac{935}{2}+357$$

$$= \frac{221-935+714}{2} = \frac{0}{2} = 0$$

Hence the given equation represents the two straight lines.

(1) $\Rightarrow$ $6x^2+x(22-17y)+(-3y^2+10y-8)=0$

$$x = \frac{-(22-17y) \pm \sqrt{(22-17y)^2-4(6)(-3y^2+10y-8)}}{2(6)}$$

$$x = \frac{-22+17y \pm \sqrt{484+289y^2-748y+72y^2-240y+192}}{12}$$

$$x = \frac{-22+17y \pm \sqrt{676+361y^2-988y}}{12}$$

$$x = \frac{-22+17y \pm \sqrt{(26-19y)^2}}{12}$$

$$x = \frac{-22+17y \pm (26-19y)}{12}$$

$\Rightarrow$ $x = \frac{-22+17y+26-19y}{12}$, $x = \frac{-22+17y-26+19y}{12}$

$x = \frac{4-2y}{12}$, $x = \frac{-48+36y}{12}$

$$x = \frac{2-y}{6}, \quad x = -4+3y$$

$$6x+y-2=0, \quad x-3y+4=0$$

are the two straight lines.

Q #4 $10x^2-23xy-5y^2-29x+32y+21=0$ ——— ①

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

Here $a=10,\ h=-\frac{23}{2},\ b=-5,\ g=-\frac{29}{2},\ f=16,\ c=21$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 10 & -\frac{23}{2} & -\frac{29}{2} \\ -\frac{23}{2} & -5 & 16 \\ -\frac{29}{2} & 16 & 21 \end{vmatrix}$$

$$= 10(-105-256)+\frac{23}{2}\left(-\frac{483}{2}+232\right)-\frac{29}{2}\left(-184+\frac{145}{2}\right)$$

$$= -3610+\frac{23}{2}\left(\frac{-483+464}{2}\right)-\frac{29}{2}\left(\frac{-368+145}{2}\right)$$

$$= -3610+\frac{23}{2}\left(-\frac{19}{2}\right)-\frac{29}{2}\left(-\frac{513}{2}\right)$$

$$= -3610-\frac{437}{4}+\frac{14877}{4}$$

$$= \frac{-14440-437+14877}{4} = 0$$

The given equation represents the pair of lines.

$$① \Rightarrow \quad 10x^2+x(-29-23y)+(-5y^2+32y+21)=0$$

$$10x^2-x(23y+29)+(-5y^2+32y+21)=0$$

$$x = \frac{(23y+29)\pm\sqrt{(23y+29)^2-40(-5y^2+32y+21)}}{20}$$

$$= \frac{23y+29\pm\sqrt{529y^2+841+1334y+200y^2-1280y-840}}{20}$$

$$= \frac{23y+29\pm\sqrt{729y^2+54y+1}}{20}$$

$$= \frac{23y+29 \pm (27y+1)}{20}$$

$$= \frac{23y+29+27y+1}{20}, \quad \frac{23y+29-27y-1}{20}$$

$$= \frac{50y+30}{20}, \quad \frac{-4y+28}{20}$$

$$= \frac{5y+3}{2}, \quad \frac{-y+7}{5}$$

Hence the lines are

$$2x-5y-3=0, \quad 5x+y-7=0$$

Q#5. $\quad 6x^2-15y^2-xy+16x+24y=0 \quad \text{———} \ (1)$

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

Here $\quad a=6, \ h=-\frac{1}{2}, \ b=-15, \ g=8, \ f=12, \ c=0$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 6 & -\frac{1}{2} & 8 \\ -\frac{1}{2} & -15 & 12 \\ 8 & 12 & 0 \end{vmatrix}$$

$$= 6(-144)+\frac{1}{2}(-96)+8(-6+120)$$

$$= -864-48+912$$

$$= 0$$

i.e (1) Represents the two lines.

$(1) \Rightarrow \quad 6x^2+x(-y+16)+(-15y^2+24y)=0$

$$x = \frac{-(-y+16) \pm \sqrt{(-y+16)^2-4(6)(-15y^2+24y)}}{2(6)}$$

$$x = \frac{(y-16) \pm \sqrt{y^2+256-32y+360y^2-576y}}{12}$$

$$x = \frac{y-16 \pm \sqrt{361y^2-608y+256}}{12}$$

$$x = \frac{y-16 \pm (19y-16)}{12}$$

$$\Rightarrow \quad x = \frac{y-16+19y-16}{12}, \quad x = \frac{y-16-19y+16}{12}$$

$$12x - 20y + 32 = 0 \quad , \quad 12x + 18y = 0$$

$$3x - 5y + 8 = 0 \quad , \quad 2x + 3y = 0$$

Are the two straight lines.

For what value of λ will each of the following equations represent a pair of straight lines.

Q# 6.
$$\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$$

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + C = 0$$

Here $a = \lambda$, $h = -5$, $b = 12$, $g = \frac{5}{2}$, $f = -8$, $C = -3$

The given equation represents two straight lines if,

$$\begin{vmatrix} \lambda & -5 & \frac{5}{2} \\ -5 & 12 & -8 \\ \frac{5}{2} & -8 & -3 \end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda(-36 - 64) + 5(15 + 20) + \frac{5}{2}(40 - 30) = 0$$

$$-100\lambda + 175 + 25 = 0$$

$$-100\lambda = -200$$

$$-\lambda = -2$$

$$\lambda = 2$$

Hence to represent a pair of straight lines λ must be equal to 2.

(ii) #7
$$\lambda xy + 5x + 3y + 2 = 0 \quad \text{------} (1)$$

Here $a = 0$, $h = \frac{\lambda}{2}$, $b = 0$, $g = \frac{5}{2}$, $f = \frac{3}{2}$, $C = 2$

The given equation represents two straight lines if

$$\begin{vmatrix} 0 & \lambda/2 & 5/2 \\ \lambda/2 & 0 & 3/2 \\ 5/2 & 3/2 & 2 \end{vmatrix} = 0$$

$$-\lambda/2\left(\lambda - \frac{15}{4}\right) + 5/2\left(\frac{3\lambda}{4}\right) = 0$$

$$-\lambda^2/2 + 15\lambda/8 + 15\lambda/8 = 0$$

$$-4\lambda^2 + 15\lambda + 15\lambda = 0$$

$$-4\lambda^2 + 30\lambda = 0$$

$$-2\lambda^2 + 15\lambda = 0$$

$$\lambda(-2\lambda + 15) = 0 \Rightarrow \lambda = 0 \text{ or } \lambda = +15/2$$

If $\lambda = 0$, the given equation is linear and represent a straight line. Hence a point of lines $\lambda = \frac{15}{2}$ for (i) represen.

Q#8. $$4x^2 - 9y^2 - 2(8+\lambda)x - 18y = 29 + 2\lambda \quad \text{——— (1)}$$
$$4x^2 - 9y^2 - 16x - 2\lambda x - 18y - 29 - 2\lambda = 0$$
$$4x^2 - 9y^2 - 2x(8+\lambda) - 18y - 29 - 2\lambda = 0$$

Here $a = 4,\ h = 0,\ b = -9,\ g = -(8+\lambda),\ f = -9,\ c = -(29+2\lambda)$

$\because$ (1) Represents two straight lines if

$$\begin{vmatrix} 4 & 0 & -8-\lambda \\ 0 & -9 & -9 \\ -8-\lambda & -9 & -29-2\lambda \end{vmatrix} = 0$$

$$4(-9(-29-2\lambda) - 81) + (-8-\lambda)(-(-9(-8-\lambda))) = 0$$
$$4(261 + 18\lambda - 81) + (-8-\lambda)(-72 - 9\lambda) = 0$$
$$1044 + 72\lambda - 324 + 576 + 72\lambda + 72\lambda + 9\lambda^2 = 0$$
$$9\lambda^2 + 216\lambda + 1296 = 0$$
$$\lambda^2 + 24\lambda + 144 = 0$$
$$(\lambda + 12)^2 = 0$$
$$\lambda + 12 = 0$$
$$\lambda = -12$$

Hence $\lambda = -12$

Find the angle b/w each of the following pairs of lines.

Q#9 $$x^2 - 2xy\,\text{Tan}\,\theta - y^2 = 0 \quad \text{——— (1)}$$
$$ax^2 + 2hxy + by^2 = 0$$

Here $a = 1,\ h = -\text{Tan}\,\theta,\ b = -1$

$\because$ $a = 1,\ b = -1$

$\therefore$ $a + b = 0$

Lines generated by (1) are $\perp$ i.e the angle b/w the lines $= 90^\circ$

Q#10. $3x^2+7xy+2y^2=0$ ——— (1)

$ax^2+2hxy+by^2=0$

Here $a=3,\quad h=7/2,\quad b=2$

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$$

$$= \frac{2\sqrt{49/4-6}}{5}$$

$$= \frac{2\sqrt{\frac{49-24}{4}}}{5} = \frac{2\sqrt{25/4}}{5} = \frac{2\cdot 5/2}{5} = 1$$

$$\theta = \tan^{-1}(1)$$

$$\theta = \pi/4$$

i.e the angle b/w the lines is $\frac{\pi}{4}$.

Q #11. $11x^2+16xy-y^2=0$ ——— (1)

$a=11,\quad h=8,\quad b=-1$

$$\tan\theta = \frac{2\sqrt{64+11}}{11-1} = \frac{2\sqrt{75}}{10} = \frac{2\times5\sqrt{3}}{10}$$

$$\tan\theta = \sqrt{3}$$

$$\theta = \tan^{-1}(\sqrt{3})$$

$$\theta = 60^\circ$$

i.e the angle b/w the Curves is 60°.

Q #12. $x^2+4xy+y^2-6x-3=0$

$a=1,\quad h=2,\quad b=1$

$$\tan\theta = \frac{2\sqrt{4-1}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\tan\theta = \sqrt{3}$$

$$\theta = 60^\circ$$

i.e the angle b/w the lines $= 60^\circ$

Q#13. $6x^2 + xy - y^2 - 21x - 8y + 9 = 0$

Here $a = 6$, $b = -1$, $h = \frac{1}{2}$

$$\tan\theta = \frac{2\sqrt{1/4 + 6}}{6-1} = \frac{2\sqrt{\frac{1+24}{4}}}{5}$$

$$= \frac{\sqrt{25}}{5} = \frac{5}{5} = 1$$

$$\theta = \tan^{-1}(1)$$
$$\theta = 45^\circ$$

i.e the angle b/w the lines is 45°.

Q#14 Show that $16xy - 6x + 8y - 3 = 0$ represents a pair of straight lines. Also prove that this together with the coordinates axes form a rectangle and find the area enclosed by the rectangle.

Soln.

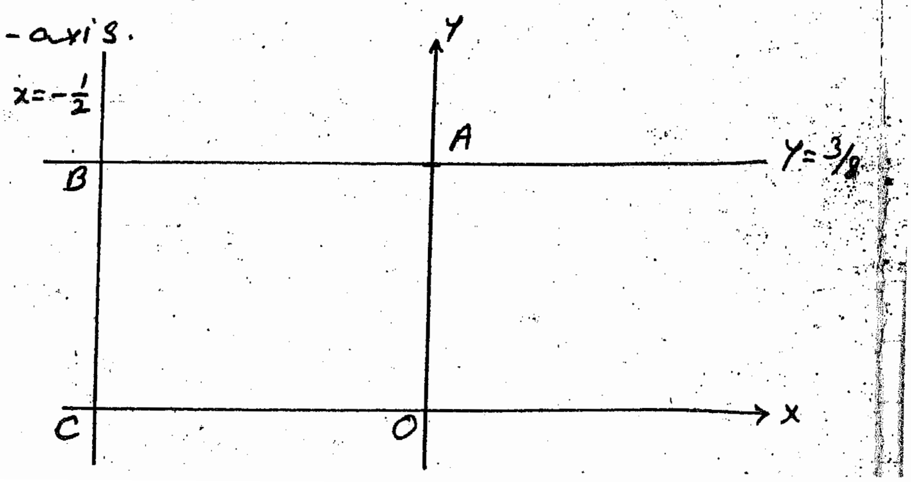
$$16xy - 6x + 8y - 3 = 0 \quad \text{---} \quad (1)$$
$$2x(8y-3) + (8y-3) = 0$$
$$(2x+1)(8y-3) = 0$$
$$\Rightarrow \quad 2x+1 = 0, \quad 8y-3 = 0$$

$\because$ (1) has been factorized into two linear facts

$\therefore$ The resulting equations represents the two straight lines.

We can write $2x+1=0$ and $8y-3=0$ in the form $x = -\frac{1}{2}$, slope // to y-axis and $y = 3/8$ st line // to x-axis.

Area of the Triangle
$= |OC||OA|$
$= |-1/2||3/8|$
$= 3/16$ Sq. Unit



Q#15. Show that an equation of rectangular hyperbola $x^2-y^2=1$ reffered to it's asymptotes as axis is $x'y'=-\frac{1}{2}$

Soln

The given rectangular hyperbola is

$$x^2-y^2=1 \quad \text{——— (1)}$$

We know that the equations of the transformations are

$$x = x'\cos\theta - y'\sin\theta \quad \text{——— (2)}$$

$$y = x'\sin\theta + y'\cos\theta \quad \text{——— (3)}$$

For the eq. refered to its asymptotes, we know that $\theta = \pi/4$

Angle b/w the asymptotes and the axes is $\pi/4$

$$2) \Rightarrow \quad x = x'\cos(\pi/4) - y'\sin(\pi/4)$$

$$x = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$$

$$x = \frac{1}{\sqrt{2}}(x'-y') \quad \text{——— (4)}$$

$$3) \Rightarrow \quad y = x'\sin(\pi/4) + y'\cos(\pi/4)$$

$$y = x'\frac{1}{\sqrt{2}} + y'\frac{1}{\sqrt{2}}$$

$$y = \frac{1}{\sqrt{2}}(x'+y') \quad \text{——— (5)}$$

putting these values in (1), we have

$$\frac{1}{2}(x'-y')^2 - \frac{1}{2}(x'+y')^2 = 1$$

$$x'^2+y'^2-2x'y' - x'^2-y'^2-2x'y' = 2$$

$$-4x'y' = 2$$

$$x'y' = -1/2$$

Note. The asymptotes $y=\pm x$ or regarded as axes. Keeping these axes in view the equation $x^2-y^2=1$ is reduced to $x'y'=-\frac{1}{2}$ by proceeding as above.

Analyze and graph the conic represented by each of the following equations.

Q# 16 - Q#25

For Q.16 - Q.25

See the graph Book.

Written by
Shahid Javed