Techniques of Integration

EXERCISE 4.2

Q No. 1 $\boldsymbol{I=\int\frac{dx}{\sqrt{a^2+x^2}}}$

Put $x=atan\theta \Rightarrow dx=asec^2\theta d\theta$

$$I=\int\frac{asec^2\theta d\theta}{\sqrt{(a^2+a^2\tan^2\theta)}}$$

$$=\int\frac{asec^2\theta d\theta}{\sqrt{a^2(1+tan^2\theta)}}=\int\frac{asec^2\theta d\theta}{a\sqrt{(1+tan^2\theta)}}=$$

$$\int\frac{sec^2\theta d\theta}{\sqrt{(1+tan^2\theta)}}$$

$$=\int\frac{sec^2\theta d\theta}{sec\theta}=\int sec\theta d\theta=ln|sec\theta+tan\theta|$$

Now substitution returns:

$$I=ln\left|\sqrt{1+\tan^2\theta}+tan\theta\right|$$

$$I=ln\left|\sqrt{1+\left(\frac{x}{a}\right)^2}+\frac{x}{a}\right| = ln\left|\frac{\sqrt{x^2+a^2}+x}{a}\right|$$

Q No. 2 $\boldsymbol{I=\int\frac{dx}{\sqrt{x^2-a^2}}}$

Put $x=acosh\theta \Rightarrow dx=asinh\theta d\theta$

$$I=\int\frac{asinh\theta d\theta}{\sqrt{(a^2\cosh^2\theta-a^2)}}$$

$$=\int\frac{asinh\theta d\theta}{\sqrt{a^2(cosh^2\theta-1)}}=\int\frac{asinh\theta d\theta}{a\sqrt{(cosh^2\theta-1)}}$$

$$=\int\frac{sinh\theta d\theta}{\sqrt{(cosh^2\theta-1)}}=\int\frac{sinh\theta d\theta}{sinh\theta}$$

$$=\int d\theta=\theta$$

Now substitution returns:

$$=\cosh^{-1}\frac{x}{a}$$

Q No. 3 $\boldsymbol{I=\int tanxdx}$

$$\int tanxdx=\int\frac{sinx}{cosx}dx$$

$$=-\int\frac{-sinx}{cosx}dx=-\ln(cosx)=\ln(secx)$$

Q No. 4 $\boldsymbol{I=\int cotxdx}$

$$\int cotxdx=\int\frac{cosx}{sinx}dx=\ln(sinx)$$

Q No. 5 $\boldsymbol{I=\int secxdx}$

$$\int secxdx=\int\frac{secx(secx+tanx)}{(secx+tanx)}dx$$

$$=\int\frac{sec^2x+secxtanx)}{(secx+tanx)}dx=\ln(secx+tanx)$$

Q No. 6 $\boldsymbol{I=\int cscxdx}$

$$\int cscxdx=-\int\frac{-cscx(cscx+cotx)}{(cscx+cotx)}dx$$

$$=-\int\frac{-cosec^2x-cscxcotx)}{(cscx+cotx)}=-\ln(cscx+cotx)$$

by rationalizing this answer we can get another result

i.e $\ln(cscx-cotx)$

Q No. 7 $\boldsymbol{I=\int(ax^2+2bx+c)^2(ax+b)dx}$

$$I=\frac{1}{2}\int(ax^2+2bx+c)^2(2ax+2b)dx$$

$$I=\frac{(ax^2+2bx+c)^{2+1}}{2+1}$$

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Q No. 8 $I = \int \sqrt{\frac{1+x}{1-x}}dx$

By rationalizing we get, $\frac{1+x}{1-x} \times \frac{1+x}{1+x} = \frac{(1+x)^2}{1-x^2}$

So, $I = \int \frac{1+x}{\sqrt{1-x^2}}dx = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{xdx}{\sqrt{1-x^2}}$

$= sin^{-1}x + \int (1-x^2)^{-\frac{1}{2}}xdx$

$= sin^{-1}x - \frac{1}{2}\int (1-x^2)^{-\frac{1}{2}}(-2x)dx$

$= sin^{-1}x - \frac{1}{2}\frac{(1-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$

$= sin^{-1}x - \frac{1}{2}\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}}$

$= sin^{-1}x - \sqrt{1-x^2}$

Q No. 9 $\int \frac{dx}{a+\sqrt{bx+c}}$

(linear under square root)

Put $\sqrt{bx+c} = z$

$\Rightarrow \; bx + c = z^2$

$\Rightarrow \; b\,dx = 2zdz$

$I = \int \frac{2zdz/b}{a+z}$

$I = \frac{2}{b}\int \frac{zdz}{a+z}$

$I = \frac{2}{b}\int \left(1 - \frac{a}{a+z}\right)dz$

$I = \frac{2}{b}\int dz - \frac{2a}{b}\int \frac{dz}{a+z}$

$I = \frac{2}{b}z - \frac{2a}{b}ln(a+z)$

$I = \frac{2}{b}\sqrt{bx+c} - \frac{2a}{b}ln\,(a+\sqrt{bx+c})$

Q No. 10 $\int \frac{dx}{(1+x^2)tan^{-1}x}$

$I = \int \frac{1/(1+x^2)}{tan^{-1}x}dx \quad = \ln\,(tan^{-1}x)$

Q No. 11 $I = \int \frac{sinx+cosx}{sinx-cosx}dx$

$I = \int \frac{cosx-(-sinx)}{sinx-cosx}dx \; = \ln\,(sinx - cosx)$

Q No. 12 $I = \int \frac{sin\sqrt{x}}{\sqrt{x}}dx$

(Substitute the complicated angle)

Put $\sqrt{x} = z \;\Rightarrow\; x = z^2 \;\Rightarrow\; dx = 2zdz$

$I = \int \frac{sinz}{z}.2zdz \; = 2\int sinzdz \; = -2cosz$

$= -2cos\sqrt{x}$

Q No. 13 $I = \int \sqrt{e^{2x}+e^{3x}}dx$

$I = \int \sqrt{e^{2x}+e^{3x}}dx = \int \sqrt{e^{2x}(1+e^x)}dx$

$I = \int \sqrt{1+e^x}.e^xdx \; = \frac{(1+e^x)^{\frac{1}{2}+1}}{\frac{1}{2}+1}$

Q No. 14 $I = \int \frac{dx}{e^x+e^{-x}}$

I

$= \int \frac{e^xdx}{e^x(e^x+e^{-x})}$ $\quad (multiplied\ D^r and\ N^r by\ e^x$

$I = \int \frac{e^xdx}{e^{2x}+1} \; = \tan^{-1}(e^x)$

<u>Alternatively,</u>

Put $e^x = z \;\Rightarrow e^xdx = dz$

So $I = \int \frac{e^xdx}{e^{2x}+1} = \int \frac{dz}{z^2+1} = \tan^{-1}z = \tan^{-1}(e^x)$

Q No. 15 $I = \int \frac{e^{2x}dx}{\sqrt{e^x-1}}$

$$Put\ e^x = z \quad \Rightarrow e^x dx = dz$$

$$I = \int \frac{e^x.e^x dx}{\sqrt{e^x-1}} = \int \frac{zdz}{\sqrt{z-1}} = \int \frac{(z-1+1)dz}{\sqrt{z-1}}$$

$$I = \int \frac{(z-1)dz}{\sqrt{z-1}} + \int \frac{dz}{\sqrt{z-1}}$$

$$I = \int (z-1)^{1-\frac{1}{2}}dz + \int (z-1)^{\frac{-1}{2}}dz$$

$$I = \int (z-1)^{\frac{1}{2}}dz + \int (z-1)^{\frac{-1}{2}}dz$$

$$I = \frac{(z-1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{(z-1)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}$$

$$I = \frac{(z-1)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(z-1)^{\frac{1}{2}}}{\frac{1}{2}} = \frac{2}{3}(e^x-1)^{\frac{3}{2}} + 2\sqrt{e^x-1}$$

Q No. 16 $I = \int \frac{\cos(\ln x)}{x}dx$

(Substitute the complicated angle)

Put $\ln x = z \Rightarrow \frac{1}{x}dx = dz$

$$I = \int coszdz = sinz = \sin(\ln x)$$

Q No. 17 $I = \int \frac{2x+5}{\sqrt{x^2+5x+7}}dx$

$$I = \int (x^2+5x+7)^{-\frac{1}{2}}.(2x+5)dx$$

$$I = \frac{(x^2+5x+7)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$$

Q No. 18 $I = \int \frac{x+2}{\sqrt{2x^2+8x+5}}dx$

$$I = \int (2x^2+8x+5)^{-\frac{1}{2}}.(x+2)dx$$

$$I = \frac{1}{4}\int (2x^2+8x+5)^{-\frac{1}{2}}.4(x+2)dx$$

$$I = \frac{1}{4}\int (2x^2+8x+5)^{-\frac{1}{2}}.(4x+8)dx$$

$$I = \frac{(2x^2+8x+5)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$$

Q No. 19 $I = \frac{\sqrt{x^2-a^2}}{x^4}dx$

$Put\ x = acosh\theta \quad \Rightarrow \quad dx = asinh\theta$

$$I = \int \frac{\sqrt{a^2\cosh^2\theta - a^2}}{a^4\cosh^4\theta} asinh\theta d\theta$$

$$I = \int \frac{a\sqrt{\cosh^2\theta - 1}}{a^4\cosh^4\theta} asinh\theta d\theta$$

$$I = \int \frac{\sqrt{\cosh^2\theta - 1}}{a^2\cosh^4\theta}\sqrt{\cosh^2\theta - 1}\,d\theta$$

$$I = \frac{1}{a^2}\int \frac{\cosh^2\theta - 1}{\cosh^4\theta}d\theta$$

$$I = \frac{1}{a^2}\int \frac{1}{\cosh^2\theta}d\theta - \frac{1}{a^2}\int \frac{1}{\cosh^4\theta}d\theta$$

$$I = \frac{1}{a^2}\int sech^2\theta d\theta - \frac{1}{a^2}\int sech^4\theta d\theta$$

$$I = \frac{1}{a^2}tanh\theta - \frac{1}{a^2}I_1 \text{-------------(1)}$$

$$I_1 = \int sech^4\theta d\theta$$

$$I_1 = \int sech^2\theta.sech^2\theta d\theta$$

$$I_1 = \int (1-tanh^2\theta).sech^2\theta d\theta$$

$$I_1 = \int sech^2\theta d\theta - \int tanh^2\theta sech^2\theta d\theta$$

$$I_1 = tanh\theta - \frac{tanh^3\theta}{3}$$

Putting in eq(1) we get,

$$I = \frac{1}{a^2}tanh\theta - \frac{1}{a^2}tanh\theta + \frac{tanh^3\theta}{3a^2}$$

$$I = \frac{tanh^3\theta}{3} = \frac{1}{3}.\left(\frac{sinh\theta}{cosh\theta}\right)^3 = \frac{1}{3}.\left(\frac{\sqrt{cosh^2\theta - 1}}{cosh\theta}\right)^3$$

$$I = \frac{1}{3a^2}\left(\frac{\sqrt{\frac{x^2}{a^2}-1}}{\frac{x}{a}}\right)^3 = \frac{(x^2-a^2)^{\frac{3}{2}}}{3a^2x^3}$$

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Q No. 20 $I = \int \cos^6 x \sin^3 x dx$

$$I = \int \cos^6 x . \sin^2 x . \sin x \, dx$$

$$= \int \cos^6 x . (1 - \cos^2 x) . \sin x \, dx$$

$$= \int \cos^6 x \ . \sin x \, dx - \int \cos^8 x \ . \sin x \, dx$$

$$= -\int \cos^6 x \ . (-\sin x) dx + \int \cos^8 x \ . (-\sin x) \, dx$$

$$= -\frac{\cos^7 x}{7} + \frac{\cos^9 x}{9}$$

Q No. 21 $I = \int \tan^3 x \sec^3 x dx$

$$I = \int \tan^2 x . \sec^2 x . \sec x \tan x \, dx$$

$$I = \int (\sec^2 x - 1) . \sec^2 x . \sec x \tan x \, dx$$

$$I = \int \sec^4 x . \sec x \tan x \, dx - \int \sec^2 x . \sec x \tan x \, dx$$

$$I = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3}$$

Q No. 22 $I = \int \cot^3 x \csc^4 x dx$

$$I = \int \cot^2 x \csc^3 x . (\cot x \csc x) dx$$

$$I = -\int \cot^2 x \csc^3 x . (-\cot x \csc x) dx$$

$$I = -\int (\csc^2 x - 1) \csc^3 x . (-\cot x \csc x) dx$$

$$I = -\int \csc^5 x (-\cot x \csc x) dx + \int \csc^3 x (-\cot x \csc x) dx$$

$$I = \frac{-\csc^6 x}{6} + \frac{\csc^4 x}{4}$$

Alternatively,

$$I = \int \cot^3 x \csc^4 x dx$$

$$I = \int \cot^3 x \csc^2 x . (\csc^2 x) dx$$

$$I = -\int \cot^3 x \csc^2 x . (-\csc^2 x) dx$$

$$I = -\int \cot^3 x (\cot^2 x + 1) . (-\csc^2 x) dx$$

$$I = -\int \cot^5 x (-\csc^2 x) dx - \int \cot^3 x (-\csc^2 x) dx$$

$$I = -\frac{1}{6}\cot^6 x - \frac{1}{4}\cot^4 x$$

Q No. 23 $I = \int \frac{dx}{\sqrt{2x^2+3x+4}}$

$$I = \int \frac{dx}{\sqrt{2(x^2 + \frac{3}{2}x + 2)}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x^2 + \frac{3}{2}x + 2)}}$$

Completing square of

$$x^2 + \frac{3}{2}x + 2$$

$$= (x)^2 + 2\left(\frac{3}{4}\right)(x) + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 + 2$$

$$= \left(x + \frac{3}{4}\right)^2 - \frac{9}{16} + 2$$

$$= \left(x + \frac{3}{4}\right)^2 - \frac{9}{16} + \frac{32}{16}$$

$$= \left(x + \frac{3}{4}\right)^2 + \frac{23}{16}$$

$$= \left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2$$

So,

$$I = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2}}$$

$$I = \frac{1}{\sqrt{2}} \sinh^{-1}\left(\frac{x + \frac{3}{4}}{\frac{\sqrt{23}}{4}}\right) = \frac{1}{\sqrt{2}} \sinh^{-1}\left(\frac{4x+3}{\sqrt{23}}\right)$$

Q No. 24 $I=\sqrt{a^2-x^2}dx$

Put $x=asin\theta \Rightarrow dx=acos\theta d\theta$

$$I=\int\sqrt{a^2-a^2sin^2\theta}\,acos\theta\,d\theta$$

$$I=\int a\sqrt{1-sin^2\theta}\,acos\theta\,d\theta$$

$$I=a^2\int cos^2\theta d\theta$$

$$I=\frac{a^2}{2}\int(1+cos2\theta)d\theta \qquad as\ \ cos^2\theta=\frac{1+cos2\theta}{2}$$

$$I=\frac{a^2}{2}(\theta+\frac{sin2\theta}{2})$$

$$I=\frac{a^2}{2}(\theta+\frac{2sin\theta cos\theta}{2})$$

$$I=\frac{a^2}{2}(\theta+sin\theta\sqrt{1-sin^2\theta})$$

Substitution returned:

$$I=\frac{a^2}{2}\left(\sin^{-1}\frac{x}{a}+\frac{x}{a}\sqrt{1-\frac{x^2}{a^2}}\right)$$

$$I=\frac{a^2}{2}\left(\sin^{-1}\frac{x}{a}+\frac{x}{a}\sqrt{\frac{a^2-x^2}{a^2}}\right)$$

$$I=\frac{a^2}{2}\sin^{-1}\frac{x}{a}+\frac{a^2}{2}.\frac{x}{a}.\frac{\sqrt{a^2-x^2}}{a}$$

$$I=\frac{x}{2}.\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$$

Q No. 25 $I=\int(2x+3)\sqrt{2x+1}\,dx$

$$I=\int(2x+1+2)\sqrt{2x+1}dx$$

$$I=\int(2x+1)\sqrt{2x+1}dx+2\int\sqrt{2x+1}\,dx$$

$$I=\int(2x+1)^{1+\frac{1}{2}}dx+2\int(2x+1)^{\frac{1}{2}}dx$$

$$I=\frac{1}{2}\int(2x+1)^{\frac{3}{2}}.2dx+\int(2x+1)^{\frac{1}{2}}2.dx$$

$$I=\frac{1}{2}.\frac{(2x+1)^{\frac{3}{2}+1}}{\frac{3}{2}+1}+\frac{(2x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1}$$

Q No. 26 $I=\int(1+x^2)^{-\frac{3}{2}}dx$

Put $x=tan\theta \Rightarrow dx=sec^2\theta d\theta$

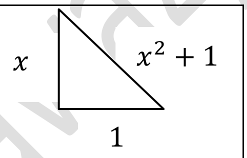
$$I=\int(1+tan^2\theta)^{-\frac{3}{2}}.sec^2\theta\,d\theta$$

$$I=\int(sec^2\theta)^{-\frac{3}{2}}.sec^2\theta\,d\theta$$

$$I=\int\frac{sec^2\theta}{sec^3\theta}d\theta=\int cos\theta d\theta=sin\theta$$

In right triangle :

$tan\theta=\frac{x}{1}$



By Pythagorean's theorem we can find the Hyp. So

$sin\theta=\frac{x}{x^2+1}$

Hence $I=\frac{x}{x^2+1}$

Q No. 27 $I=\int\frac{x^2}{\sqrt{x^2+1}}dx$

$$I=\int\frac{x^2+1-1}{\sqrt{x^2+1}}dx=\int\left(\frac{x^2+1}{\sqrt{x^2+1}}-\frac{1}{\sqrt{x^2+1}}\right)dx$$

$$I=\int\sqrt{x^2+1}dx-\int\frac{dx}{\sqrt{x^2+1}}$$

$$I=\left[\frac{x}{2}\sqrt{x^2+1}+\frac{1}{2}\sinh^{-1}x\right]-\sinh^{-1}x$$

$$I=\frac{x}{2}\sqrt{x^2+1}-\frac{1}{2}\sinh^{-1}x$$

Q No. 28 $I=\int(2x+4)\sqrt{2x^2+3x+1}dx$

$$I=\frac{1}{2}\int(2x^2+3x+1)^{\frac{1}{2}}.(4x+8)dx$$

$$I=\frac{1}{2}\int(2x^2+3x+1)^{\frac{1}{2}}.(4x+3+5)dx$$

$$I=\frac{1}{2}\int(2x^2+3x+1)^{\frac{1}{2}}.(4x+3)dx+\frac{1}{2}\int(2x^2+3x+1)^{\frac{1}{2}}.5dx$$

$$I=\frac{1}{2}\frac{(2x^2+3x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{5}{2}.\sqrt{2}\int\left(x^2+\frac{3}{2}x+\frac{1}{2}\right)^{\frac{1}{2}}dx$$

$$I=\frac{1}{3}(2x^2+3x+1)^{\frac{3}{2}}+\frac{5}{\sqrt{2}}\int\sqrt{\left(x+\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2}dx$$

Completing square of

$$x^2+\frac{3}{2}x+\frac{1}{2}$$

$$=(x)^2+2\left(\frac{3}{4}\right)(x)+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2+\frac{1}{2}$$

$$=\left(x+\frac{3}{4}\right)^2-\frac{9}{16}+\frac{1}{2}$$

$$=\left(x+\frac{3}{4}\right)^2-\frac{9}{16}+\frac{8}{16}$$

$$=\left(x+\frac{3}{4}\right)^2-\frac{1}{16}$$

$$=\left(x+\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2$$

Hence,

$$I=\frac{1}{3}(2x^2+3x+1)^{\frac{3}{2}}+\frac{5}{\sqrt{2}}\left[\frac{x+\frac{3}{4}}{2}\sqrt{\left(x+\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2}-\frac{(1/4)^2}{2}\cosh^{-1}\frac{x+\frac{3}{4}}{\frac{1}{4}}\right]$$

$$I=\frac{1}{3}(2x^2+3x+1)^{\frac{3}{2}}+\frac{5}{\sqrt{2}}\left[\frac{4x+3}{8}.\frac{\sqrt{(4x+3)^2-(1)^2}}{4}-\frac{1}{32}\cosh^{-1}(4x+3)\right]$$

$$I=\frac{1}{3}(2x^2+3x+1)^{\frac{3}{2}}+\frac{5}{32\sqrt{2}}\left[4x+\sqrt{(4x+3)^2-1}-\cosh^{-1}(4x+3)\right]$$

Q No. 29 $I=\int\frac{dx}{3sinx+4cosx}$

Let $3=rsint$ and $4=rcost$

Squaring and adding, we get,	Dividing , we get
$3^2+4^2=r^2sin^2t+r^2cos^2t$ $25=r^2$ $r=5$	$\frac{rsint}{rcost}=\frac{3}{4}$ $t=\tan^{-1}(\frac{3}{4})$

$$I=\int\frac{dx}{rsintsinx+rcostcosx}$$

$$I=\frac{1}{r}\int\frac{dx}{\cos(x-t)}$$

$$I=\frac{1}{r}\int\sec(x-t)\,dx$$

$$I=\frac{1}{r}ln|\sec(x-t)+\tan(x-t)|$$

$$I=\frac{1}{5}ln\left|\sec\left(x-\tan^{-1}\frac{3}{4}\right)+tan\left(x-\tan^{-1}\frac{3}{4}\right)\right|$$

Q No. 30 $I=\int\frac{tanxdx}{cosx+secx}$

$$I=\int\frac{\frac{sinx}{cosx}}{cosx+\frac{1}{cosx}}dx$$

$$I=\int\frac{sinx}{\cos^2x+1}dx$$

$$I=-\int\frac{-sinx}{\cos^2x+1}dx$$

$$I=\tan^{-1}(cosx)$$

Q No. 31 $I=\int\frac{dx}{sin(x-a)sin(x-b)}$

$$1=\frac{\sin(a-b)}{\sin(a-b)}=\frac{\sin(a-b+x-x)}{\sin(a-b)}$$

$$=\frac{\sin(x-b-x+a)}{\sin(a-b)}$$

$$=\frac{\sin[(x-b)-(x-a)]}{\sin(a-b)}$$

$$=\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(a-b)}$$

So,

$$I=\frac{1}{\sin(a-b)}\int\left(\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\right)$$

$$I=\frac{1}{\sin(a-b)}\int\left[\frac{\cos(x-a)}{\sin(x-a)}-\frac{\cos(x-b)}{\sin(x-b)}\right]dx$$

$$I=\frac{1}{\sin(a-b)}[lnsin(x-a)-lnsin(x-b)]$$

$$I=\frac{1}{\sin(a-b)}ln\frac{sin(x-a)}{sin(x-b)}$$

Q No. 32 $I = \int tanx ln(secx)\, dx$

Put $lnsecx = z \Rightarrow dz = \frac{1}{secx}.secx.tanx.dx$

$\Rightarrow dz = tanxdx$

$I = zdz = \frac{z^2}{2} = \frac{(lnsecx)^2}{2}$

Q No. 33 $I = \int \frac{dx}{(3tanx+1)cos^2x}$

$I = \int \frac{\sec^2 xdx}{3tanx + 1} = \frac{1}{3}\int \frac{3\sec^2 xdx}{3tanx + 1} = \frac{1}{3}\ln(3tanx + 1)$

Q No. 34 $I = \int e^{sinx} cosx\, dx$

Put $sinx = z \Rightarrow cosxdx = dz$

$I = \int e^z dz = e^z = e^{sinx}$

Q No. 35 $I = \int \sqrt{1 + 3cos^2x}\, sin2xdx$

Put $\cos^2 x = z \Rightarrow 2cosx(-sinx)dx = dz$

$\Rightarrow -2sinx.cosxdx = dz \;\; or \;\; sin2xdx = -dz$

$I = -\int (1 + 3z)^{\frac{1}{2}} dz$

$I = -\frac{1}{3}\int (1 + 3z)^{\frac{1}{2}}.3dz$

$I = -\frac{1}{3}.\frac{(1 + 3z)^{\frac{1}{2}+1}}{\frac{1}{2} + 1}$

$I = -\frac{2}{9}(1 + 3z)^{\frac{3}{2}}$

$I = -\frac{2}{9}(1 + 3\cos^2 x)^{\frac{3}{2}}$

Q No. 36 $I = \int \frac{sin2xdx}{\sqrt{1+cos^2x}}$

Put $\cos^2 x = z \Rightarrow 2cosx(-sinx)dx = dz$

$\Rightarrow -2sinx.cosxdx = dz \;\; or \;\; sin2xdx = -dz$

$I = -\int (1 + z)^{\frac{-1}{2}} dz$

$I = -\frac{(1 + z)^{\frac{-1}{2}+1}}{\frac{-1}{2} + 1}$

$I = -2\sqrt{1 + \cos^2 x}$

Q No. 37 $I = \int \frac{dx}{2sin^2x+3cos^2x}$

Divide N^r *and* D^r *by* $\cos^2 x$

$I = \int \frac{\sec^2 xdx}{2\tan^2 x + 3}$

Put $\tan x = z \quad \sec^2 xdx = dz$

$I = \int \frac{dz}{2z^2 + 3} = \frac{1}{2}\int \frac{dz}{z^2 + \frac{3}{2}}$

$I = \frac{1}{2}.\sqrt{\frac{2}{3}}\tan^{-1}\frac{\sqrt{2}z}{\sqrt{3}} \qquad as\ (\frac{1}{a}\tan^{-1}\frac{x}{a})$

Q No. 38 $I = \int \frac{1}{\sqrt{x}} sec\sqrt{x}tan\sqrt{x}\, dx$

Put $\sqrt{x} = z \Rightarrow \frac{1}{2\sqrt{x}}dx = dz \Rightarrow \frac{dx}{\sqrt{x}} = 2dz$

$I = \int secztanz.2dz = 2secz = 2sec\sqrt{x}$

Q No. 39 $I = \int [\pi^{sinx} + (sinx)^{\pi}]cosxdx$

Put $sinx = z \Rightarrow cosxdx = dz$

$I = \int \pi^z dz + \int z^{\pi} dz$

$I = \frac{\pi^z}{ln\pi} + \frac{z^{\pi+1}}{\pi + 1}$

$I = \frac{\pi^{sinx}}{ln\pi} + \frac{(sinx)^{\pi+1}}{\pi + 1}$

Q No. 40 $I = \int \frac{cosxdx}{3sinx+4\sqrt{sinx}}$

Put $\sqrt{sinx} = z \Rightarrow sinx = z^2 \Rightarrow cosxdx = 2zdz$

$I = \int \frac{2zdz}{3z^2 + 4z} = 2\int \frac{dz}{3z + 4} = \frac{2}{3}\int \frac{3dz}{3z + 4}$

$I = \frac{2}{3}\ln(3z + 4)$

$I = \frac{2}{3}\ln(3\sqrt{sinx} + 4)$