EXERCISE 3.3

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Evaluate the given Limits

QNo.1. $\lim\limits_{x\to 0} \dfrac{e^x - e^{-x}}{\sin x}$.

Solution: We have.

$$\lim_{x\to 0} \frac{e^x - e^{-x}}{\sin x} \equiv \left(\frac{0}{0}\right).$$

Using L'H rule.

$$= \lim_{x\to 0} \frac{\frac{d}{dx}(e^x - e^{-x})}{\frac{d}{dx}\sin x} = \lim_{x\to 0} \frac{e^x + e^{-x}}{\cos x}$$

$$= \frac{e^0 + e^{-0}}{\cos 0} = \frac{1+1}{1} = 2 \quad \#$$

QNo.2. $\lim\limits_{x\to 0} \dfrac{e^{x^2} - 1}{\cos x - 1}$.

Solution: We have.

$$\lim_{x\to 0} \frac{e^{x^2} - 1}{\cos x - 1} \equiv \left(\frac{0}{0}\right)$$

Using L'H rule

$$= \lim_{x\to 0} \frac{\frac{d}{dx}(e^{x^2} - 1)}{\frac{d}{dx}(\cos x - 1)}$$

$$= \lim_{x\to 0} \frac{2x\, e^{x^2}}{-\sin x}.$$

$$= \lim_{x \to 0} \frac{2x e^{x^2}}{-\sin x} \equiv \left(\frac{0}{0}\right).$$

using L'H rule.

$$= \lim_{x \to 0} \frac{2[x.2xe^{x^2} + e^{x^2}(1)]}{-\cos x}.$$

$$= \lim_{x \to 0} \frac{2[2x^2 e^{x^2} + e^{x^2}]}{-\cos x} = \frac{2[0+1]}{-1}$$

$$= \frac{2}{-1} = -2 \quad \#$$

<u>QNo.3</u> $\lim_{x \to 0} \frac{x - \tan x}{x - \sin x}$ -

<u>Solution:</u> We have.

$$\lim_{x \to 0} \frac{x - \tan x}{x - \sin x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule.

$$= \lim_{x \to 0} \frac{\frac{d}{dx}(x - \tan x)}{\frac{d}{dx}(x - \sin x)}$$

$$= \lim_{x \to 0} \frac{1 - \sec^2 x}{1 - \cos x} \equiv \left(\frac{0}{0}\right)$$

again using L'H rule

$$= \lim_{x \to 0} \frac{\frac{d}{dx}(1 - \sec^2 x)}{\frac{d}{dx}(1 - \cos x)}.$$

$$= \lim_{x \to 0} \frac{-2\sec x \cdot \sec x \tan x}{\sin x}$$

$$= \lim_{x \to 0} \frac{-2\sec^2 x \tan x}{\sin x} \equiv \left(\frac{0}{0}\right).$$

using L'H rule.

$$= \lim_{x \to 0} \frac{\frac{d}{dx}\left[-2\sec^2 x \tan x\right]}{\frac{d}{dx} \sin x}$$

$$= \lim_{x \to 0} \frac{-2\left[\sec^2 x \sec^2 x + \tan x \cdot 2\sec x \sec x \tan x\right]}{\cos x}$$

$$= \lim_{x \to 0} \frac{-2\left[\sec^4 x + 2\sec^2 x \tan^2 x\right]}{\cos x}$$

$$= \frac{-2[1+0]}{1} = -2$$

Q No.4 $\lim_{x \to \pi} \frac{\sin^2 x}{\cos 3x + 1}$

Solution: We have.

$$\lim_{x \to \pi} \frac{\sin^2 x}{\cos 3x + 1} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to \pi} \frac{\frac{d}{dx}(\sin^2 x)}{\frac{d}{dx}(\cos 3x + 1)}$$

$$= \lim_{x \to \pi} \frac{2 \sin x \cos x}{-3 \sin 3x}.$$

$$= \lim_{x \to \pi} \frac{\sin 2x}{-3 \sin 3x.} \equiv \left(\frac{0}{0}\right).$$

using L'H rule.

$$= \lim_{x \to \pi} \frac{\frac{d}{dx}(\sin 2x)}{-3 \frac{d}{dx} \sin 3x.}$$

$$= \lim_{x \to \pi} \frac{2 \cos 2x}{-9 \cos 3x.} = \frac{2 \cos 2\pi}{-9 \cos 3\pi.}$$

$$= \frac{2(1)}{-9(-1)} = \frac{2}{9}$$ //

<u>Q.No.5</u> $\lim_{x \to 0} \frac{\sin x}{\sqrt{1 - \cos x.}}$

<u>Solution:</u> we have.

$$\lim_{x \to 0} \frac{\sin x}{\sqrt{1 - \cos x}} \equiv \left(\frac{0}{0}\right).$$

$$= \lim_{x \to 0} \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{\sqrt{2 \sin^2 x/2.}}$$

$$= \lim_{x \to 0} \frac{2 \sin x/2 \cos x/2}{\sqrt{2} \sin x/2}$$

$$= \lim_{x \to 0} \frac{2}{\sqrt{2}} \cos x/2 = \frac{\sqrt{2}\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$ //

Q.No.6 $\lim_{x \to 0} \frac{\tan x - \sin x}{x^2 \tan x}$.

Solution: We have.

$$\lim_{x \to 0} \frac{\tan x - \sin x}{x^2 \tan x} \equiv \left(\frac{0}{0}\right).$$

$$= \lim_{x \to 0} \frac{\tan x \left(1 - \frac{\sin x}{\tan x}\right)}{x^2 \tan x}$$

$$= \lim_{x \to 0} \frac{1 - \sin x \cot x}{x^2}$$

$$= \lim_{x \to 0} \frac{1 - \sin x \frac{\cos x}{\sin x}}{x^2}$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{x^2} \qquad \because 1 - \cos x = 2\sin^2\frac{x}{2}$$

using L'H rule.

$$= \lim_{x \to 0} \frac{0 - (-\sin x)}{2x}$$

$$= \lim_{x \to 0} \frac{\sin x}{2x} \equiv \left(\frac{0}{0}\right).$$

using L'H rule.

$$= \lim_{x \to 0} \frac{\cos x}{2} = \frac{1}{2} \quad \#$$

Q No. 7: $\lim_{x\to 1} \frac{nx^{n+1} - (n+1)x^n + 1}{(x-1)^2}$

Solution. We have

$$\lim_{x\to 1} \frac{nx^{n+1} - (n+1)x^n + 1}{(x-1)^2} \equiv \left(\frac{0}{0}\right)$$

using L'H rule.

$$= \lim_{x\to 1} \frac{n(n+1)x^n - (n+1)nx^{n-1} + 0}{2(x-1)}$$

$$= \lim_{x\to 1} \frac{n(n+1)x^n - (n+1)nx^{n-1}}{2(x-1)} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x\to 1} \frac{n(n+1)nx^{n-1} - (n+1)n(n-1)x^{n-2}}{2}$$

$$= \frac{n(n+1)n(1) - (n+1)n(n-1)(1)}{2}$$

$$= \frac{n^3 + n^2 - n(n^2-1)}{2}$$

$$= \frac{n^3 + n^2 - n^3 + n}{2}$$

$$= \frac{n^2+n}{2} = \frac{n(n+1)}{2} \quad \#$$

QNo.8 $\lim\limits_{x\to 0} \dfrac{e^x - 2\cos x + e^{-x}}{x\sin x}$

Solution: We have.

$$\lim_{x\to 0} \frac{e^x - 2\cos x + e^{-x}}{x\sin x} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{x\to 0} \frac{e^x + 2\sin x - e^{-x}}{x\cos x + \sin x} \equiv \left(\frac{0}{0}\right)$$

again using L'H. rule

$$= \lim_{x\to 0} \frac{e^x + 2\cos x + e^{-x}}{-x\sin x + \cos x + \cos x}.$$

$$= \frac{e^0 + 2\cos 0 + e^0}{0 + \cos 0 + \cos 0} = \frac{1+2+1}{1+1}$$

$$= \frac{4}{2} = 2$$

QNo9 $\lim\limits_{x\to 0} \dfrac{\ln(1-x^2)}{\ln\cos x}$

Solution: We have.

$$\lim_{x\to 0} \frac{\ln(1-x^2)}{\ln\cos x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{\frac{-2x}{1-x^2}}{-\frac{\sin x}{\cos x}}$$

$$= \lim_{x \to 0} \frac{2x\cos x}{(1-x^2)\sin x} \equiv \left(\frac{0}{0}\right)$$

Using L'H rule

$$= \lim_{x \to 0} \frac{-2x\sin x + 2\cos x}{(1-x^2)\cos x + (-2x)\sin x}$$

$$= \frac{0\sin 0 + 2\cos 0}{(1-0)\cos 0 + 0} = \frac{2(1)}{1} = 2$$

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Q.No.10 $\displaystyle \lim_{x \to 0} \frac{\cosh x - \cos x}{x\sin x}$

Solution: We have

$$\lim_{x \to 0} \frac{\cosh x - \cos x}{x\sin x} \equiv \left(\frac{0}{0}\right)$$

Using L'H rule

$$= \lim_{x \to 0} \frac{\sinh x + \sin x}{x\cos x + \sin x} \equiv \left(\frac{0}{0}\right)$$

Using L'H rule

$$= \lim_{x \to 0} \frac{\cosh x + \cos x}{-x\sin x + \cos x + \cos x}$$

$$= \frac{1+1}{0+1+1} = \frac{2}{2} = 1$$

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Q No 11:

$$\lim_{x\to 1} \frac{1-x+\ln x}{1-\sqrt{2x-x^2}}$$

Solution: We have.

$$\lim_{x\to 1} \frac{1-x+\ln x}{1-\sqrt{2x-x^2}} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{x\to 1} \frac{0-1+\frac{1}{x}}{0-\frac{1}{2\sqrt{2x-x^2}}(2-2x)}$$

$$= \lim_{x\to 1} \frac{\frac{1-x}{x}}{\frac{-2(1-x)}{2\sqrt{2x-x^2}}}$$

$$= \lim_{x\to 1} \frac{1-x}{x} \times -\frac{\sqrt{2x-x^2}}{(1-x)}$$

$$= \lim_{x\to 1} \frac{-\sqrt{2x-x^2}}{x} = \frac{-\sqrt{2(1)-1}}{1}$$

$$= -\sqrt{1} = -1 \quad \#$$

Q No. 12

$$\lim_{x\to 0} \frac{x\cos x - \ln(1+x)}{x^2}$$

Solution: We have

$$\lim_{x\to 0} \frac{x\cos x - \ln(1+x)}{x^2} \equiv \left(\frac{0}{0}\right).$$

$$= \lim_{x \to 0} \frac{-x\sin x + \cos x - \frac{1}{1+x}}{2x} \equiv \left(\frac{0}{0}\right).$$

$$= \lim_{x \to 0} \frac{-x\sin x + \cos x - (1+x)^{-1}}{2x}$$

using L'H rule.

$$= \lim_{x \to 0} \frac{-[x\cos x + \sin x] - \sin x + (1+x)^{-2}}{2}$$

$$= \frac{-[0+0] - 0 + (1+0)^{-2}}{2} = \frac{1}{2} \quad \#$$

__QNo. 13.__ $\lim_{x \to 0} \frac{\sin x - \ln(e^x \cos x)}{x \sin x}.$

__Solution:__ we have.

$$\lim_{x \to 0} \frac{\sin x - \ln(e^x \cos x)}{x \sin x} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{x \to 0} \frac{\cos x - \frac{1}{e^x \cos x}\left[-e^x \sin x + \cos x\, e^x\right]}{x\cos x + \sin x}$$

$$= \lim_{x \to 0} \frac{\cos x + \tan x - 1}{x\cos x + \sin x} \equiv \left(\frac{0}{0}\right).$$

using L'H Rule.

$$= \lim_{x \to 0} \frac{-\sin x + \sec^2 x - 0}{-x\sin x + \cos x + \cos x}.$$

$$= \frac{-\sin 0 + \sec^2(0)}{0 + \cos 0 + \cos 0} = \frac{0+1}{1+1}.$$

$$= \frac{1}{2} \quad \#$$

QNo.14 $\quad \lim_{x \to 0} \frac{1 + \sin x - \cos x + \ln(1-x)}{x\tan^2 x}$

Solution

We have

$$\lim_{x \to 0} \frac{1 + \sin x - \cos x + \ln(1-x)}{x\tan^2 x} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{x \to 0} \frac{0 + \cos x + \sin x + \frac{1}{1-x}}{2x\tan x\sec^2 x + \tan^2 x}$$

$$= \lim_{x \to 0} \frac{\cos x + \sin x + (1-x)^{-1}}{2x\tan x(1+\tan^2 x) + \tan^2 x}$$

$$= \lim_{x \to 0} \frac{\cos x + \sin x + (1-x)^{-1}}{2x\tan x + 2x\tan^3 x + \tan^2 x} \equiv \left(\frac{0}{0}\right).$$

$$= \lim_{x \to 0} \frac{-\sin x + \cos x - (1-x)^{-2}}{2x\sec^2 x + 2\tan x + 6x\tan^2 x\sec^2 x + 2\tan^3 x + 2\tan x\sec^2 x} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{x \to 0} \frac{-\cos x - \sin x - 2(1-x)^{-3}}{2\sec^2 x + 4x\sec^2 x \tan x + 2\sec^2 x + 6\tan^2 x \sec^2 x + 12\sec^2 x \tan^3 x + 12x\tan x \sec^4 x + 6\tan^2 x \sec^2 x + 4\sec^2 x \tan^2 x + 2\sec^4 x}$$

$$= \frac{-3}{6} = -\frac{1}{2} \; \#$$

Q.No.15 If $\lim_{x \to 0} \frac{\sin 2x + a\sin x}{x^3}$ exists, find the value of a and the limit.

Solution: We have

$$\lim_{x \to 0} \frac{\sin 2x + a\sin x}{x^3} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{x \to 0} \frac{2\cos 2x + a\cos x}{3x^2}$$

Now as $3x^2 \to 0$ as $x \to 0$

So, we calcude that

$$2\cos 2x + a\cos x \to 0 \quad \text{as} \quad x \to 0$$

$$\Rightarrow 2\cos 2(0) + a\cos(0) = 0$$

$$2 + a = 0$$

$$\Rightarrow a = -2.$$

So the given limit becomes

$$= \lim_{x \to 0} \frac{\sin 2x - 2\sin x}{x^3} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{2\cos 2x - 2\cos x}{3x^2} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{x \to 0} \frac{-4\sin 2x + 2\sin x}{6x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{-8\cos 2x + 2\cos x}{6}$$

$$= \frac{-8(1) + 2(1)}{6} = \frac{-6}{6} = -1$$

Q No. 16 $$\lim_{x \to 0} \frac{\ln(\sin 3x)}{\ln(\sin x)}$$

Solution We have.

$$\lim_{x \to 0} \frac{\ln(\sin 3x)}{\ln(\sin x)} \equiv \left(\frac{\infty}{\infty}\right).$$

using L'H rule.

$$= \lim_{x \to 0} \frac{\frac{3\cos 3x}{\sin 3x}}{\frac{\cos x}{\sin x}}$$

$$= \lim_{x \to 0} \frac{3\cos 3x \sin x}{\cos x \sin 3x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{3[-3\sin 3x \sin x + \cos 3x \cos x]}{-\sin x \sin 3x + \cos x\,(3\cos 3x)}$$

$$= \frac{3[0 + (1)(1)]}{0 + 3(1)} = \frac{3}{3} = 1$$

Q No. 17 $\lim_{x \to 0} \left[\frac{1}{x\sin^{-1}x} - \frac{1}{x^2}\right]$

Solution: We have.

$$\lim_{x \to 0} \left[\frac{1}{x\sin^{-1}x} - \frac{1}{x^2}\right]$$

Let $\sin^{-1}x = z \Rightarrow \sin z = x$

As $x \to 0$, $z \to 0$

$$= \lim_{z \to 0} \left[\frac{1}{z\sin z} - \frac{1}{\sin^2 z}\right]$$

$$= \lim_{z \to 0} \frac{\sin z - z}{z\sin^2 z} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{z \to 0} \frac{\cos z - 1}{z(2\sin z \cos z) + \sin^2 z}$$

$$= \lim_{z \to 0} \frac{\cos z - 1}{z \sin 2z + \sin^2 z} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{z \to 0} \frac{-\sin z - 0}{z(2\cos 2z) + \sin 2z + 2\sin z \cos z}$$

$$= \lim_{z \to 0} \frac{-\sin z}{2z\cos 2z + 2\sin 2z} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{z \to 0} \frac{-\cos z}{-4z\sin 2z + 2\cos 2z + 4\cos 2z}$$

$$= \frac{-\cos(0)}{0 + 2\cos(0) + 4\cos(0)}$$

$$= \frac{-1}{2+4}$$

$$= -\frac{1}{6}$$ #

Q No. 18: $\lim_{x \to a} \dfrac{\ln(x-a)}{\ln(e^x - e^a)}$

Solution. We have.

$$\lim_{x \to a} \frac{\ln(x-a)}{\ln(e^x - e^a)} \equiv \left(\frac{\infty}{\infty}\right).$$

using L'H rule

$$= \lim_{x \to a} \frac{\frac{1}{x-a}(1)}{\frac{1}{e^x - e^a}\, e^x}$$

$$= \lim_{x \to a} \frac{e^x - e^a}{e^x(x-a)} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{x \to a} \frac{e^x}{e^x + (x-a)e^x}.$$

$$= \lim_{x \to a} \frac{1}{1 + x - a} = \frac{1}{1+0}$$

$$= \frac{1}{1} = 1$$

Q No. 19 $\lim_{x \to 0} \dfrac{\ln(\tan x)}{\ln x}$

Solution: We have

$$\lim_{x \to 0} \frac{\ln(\tan x)}{\ln x} \equiv \left(\frac{\infty}{\infty}\right).$$

using L'H rule

$$= \lim_{x\to 0} \frac{\frac{1}{\tan x}\sec^2 x}{\frac{1}{x}}$$

$$= \lim_{x\to 0} \frac{\frac{\cos x}{\sin x}\cdot\frac{1}{\cos^2 x}}{\frac{1}{x}}$$

$$= \lim_{x\to 0} \frac{x}{\sin x \cos x} = \lim_{x\to 0} \frac{2x}{2\sin x \cos x}$$

$$= \lim_{x\to 0} \frac{2x}{\sin 2x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x\to 0} \frac{2(1)}{2\cos 2x} = \lim_{x\to 0} \frac{1}{\cos 2x}$$

$$= \frac{1}{\cos 2(0)} = \frac{1}{1} = 1 \quad \#$$

QNo. 20

$$\lim_{x\to 0} \log_{\tan x}(\tan 2x)$$

Solution: We have.

$$\lim_{x\to 0} \log_{\tan x}(\tan 2x).$$

$$= \lim_{x\to 0} \frac{\ln \tan 2x}{\ln \tan x} \equiv \left(\frac{\infty}{\infty}\right).$$

using L'H rule.

$$= \lim_{x\to 0} \frac{\frac{2\sec^2 2x}{\tan 2x}}{\frac{\sec^2 x}{\tan x}}$$

$$= \lim_{x\to 0} \frac{\frac{2}{\cos^2 2x}\cdot\frac{\cos 2x}{\sin 2x}}{\frac{1}{\cos^2 x}\cdot\frac{\cos x}{\sin x}}$$

$$= \lim_{x\to 0} \frac{\frac{2}{\sin 2x \cos 2x}}{\frac{1}{\sin x \cos x}}$$

$$= \lim_{x\to 0} \frac{2\sin x \cos x}{\sin 2x \cos 2x}$$

$$= \lim_{x\to 0} \frac{2\sin 2x}{\sin 4x} \equiv \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{2\cos 2x\,(2)}{\cos 4x\,(4)}$$

$$= \lim_{x\to 0} \frac{\cos 2x}{\cos 4x}$$

$$= \frac{\cos 2(0)}{\cos 4(0)}$$

$$= \frac{1}{1} = 1 \quad \#$$

Q No. 21

$$\lim_{x\to a}(x-a)\operatorname{Cosec}\left(\frac{\pi x}{a}\right).$$

Solution: We have

$$\lim_{x\to a}(x-a)\operatorname{Cosec}\left(\frac{\pi x}{a}\right) \equiv (0\times\infty)$$

$$=\lim_{x\to a}\frac{x-a}{\sin\left(\frac{\pi x}{a}\right)} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$=\lim_{x\to a}\frac{1}{\frac{\pi}{a}\cos\left(\frac{\pi x}{a}\right)} = \frac{1}{\frac{\pi}{a}\cos(\pi)}$$

$$= \frac{1}{\frac{\pi}{a}(-1)} = -\frac{a}{\pi}$$

Q No. 22:

$$\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$$

Solution: We have

$$\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right) \equiv (0\times\infty).$$

$$=\lim_{x\to 1}\frac{1-x}{\operatorname{Cot}\left(\frac{\pi x}{2}\right)} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$=\lim_{x\to 1}\frac{-1}{-\operatorname{Cosec}^2\left(\frac{\pi x}{2}\right)\left(\frac{\pi}{2}\right)} = \lim_{x\to 1}\frac{\sin^2\left(\frac{\pi x}{2}\right)}{\frac{\pi}{2}}$$

$$= \frac{\sin^2\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}} = \frac{1}{\pi/2} = \frac{2}{\pi}$$

Q No. 23 $\lim_{x\to 0} x\ln(\tan x)$

Solution. We have

$$\lim_{x\to 0} x\ln(\tan x) \equiv (0\times\infty)$$

$$= \lim_{x\to 0} \frac{\ln(\tan x)}{\frac{1}{x}} \equiv \left(\frac{\infty}{\infty}\right)$$

using L'H rule

$$= \lim_{x\to 0} \frac{\frac{1}{\tan x}\sec^2 x}{-\frac{1}{x^2}} = \lim_{x\to 0} \frac{\frac{\cos x}{\sin x}\cdot\frac{1}{\cos^2 x}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0} -\frac{x^2}{\sin x\cos x}$$

$$= \lim_{x\to 0} -\frac{2x^2}{2\sin x\cos x} = \lim_{x\to 0} \frac{-2x^2}{\sin 2x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x\to 0} \frac{-4x}{2\cos 2x} = \frac{-4(0)}{2\cos 2(0)}$$

$$= 0$$

Q No. 24 $\lim_{x\to 0} x\tan\left(\frac{\pi}{2}-x\right)$

Solution: We have

$$\lim_{x\to 0} x\tan\left(\frac{\pi}{2}-x\right) \equiv (0\times\infty)$$

$$= \lim_{x \to 0} \frac{\tan\left(\frac{\pi}{2} - x\right)}{\frac{1}{x}} \equiv \left(\frac{\infty}{\infty}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{-\sec^2\left(\frac{\pi}{2} - x\right)}{-\frac{1}{x^2}} = \lim_{x \to 0} \frac{x^2}{\cos^2\left(\frac{\pi}{2} - x\right)}$$

using $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$$= \lim_{x \to 0} \frac{x^2}{\sin^2 x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{2x}{2\sin x \cos x} = \lim_{x \to 0} \frac{2x}{\sin 2x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{2}{2\cos 2x} = \lim_{x \to 0} \frac{1}{\cos 2x}$$

$$= \frac{1}{\cos 0} = \frac{1}{1} = 1$$

QNo. 25 $\quad \lim_{x \to \pi/2} \tan x . \ln(\sin x)$

Solution: We have

$$\lim_{x \to \pi/2} \tan x \ln(\sin x) \equiv (\infty \times 0)$$

$$= \lim_{x \to \pi/2} \frac{\ln(\sin x)}{\cot x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to \pi/2} \frac{\frac{\cos x}{\sin x}}{-\operatorname{cosec}^2 x} = \lim_{x \to \pi/2} -\frac{\cos x}{\sin x} \times \sin^2 x$$

$$= \lim_{x \to \pi/2} -\sin x . \cos x = -\sin(\pi/2)\cos(\pi/2)$$

$$= -(1)(0) = 0$$

Q No. 26

$$\lim_{x \to 0} \left[\frac{1}{x} - \frac{1}{e^x - 1}\right]$$

Solution We have.

$$\lim_{x \to 0} \left[\frac{1}{x} - \frac{1}{e^x - 1}\right] \equiv (\infty - \infty)$$

$$= \lim_{x \to 0} \frac{e^x - 1 - x}{x(e^x - 1)}$$

$$= \lim_{x \to 0} \frac{e^x - 1 - x}{xe^x - x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{e^x - 1}{xe^x + e^x - 1} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{e^x}{xe^x + e^x + e^x} = \frac{e^0}{0 + e^0 + e^0} = \frac{1}{1+1} = \frac{1}{2}$$

Q No. 27 $\lim_{x \to 0}\left[\frac{a}{x} - \cot\left(\frac{x}{a}\right)\right]$

Solution We have

$$\lim_{x \to 0}\left[\frac{a}{x} - \cot\left(\frac{x}{a}\right)\right] \equiv (\infty - \infty)$$

$$= \lim_{x \to 0}\left[\frac{a}{x} - \frac{1}{\tan\left(\frac{x}{a}\right)}\right]$$

$$= \lim_{x \to 0} \frac{a\tan\left(\frac{x}{a}\right) - x}{x\tan\left(\frac{x}{a}\right)} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{x \to 0} \frac{a\frac{1}{a}\sec^2\left(\frac{x}{a}\right) - 1}{\frac{1}{a}x\sec^2\left(\frac{x}{a}\right) + \tan\left(\frac{x}{a}\right)}$$

$$= \lim_{x \to 0} \frac{\sec^2\left(\frac{x}{a}\right) - 1}{\frac{1}{a}x\sec^2\left(\frac{x}{a}\right) + \tan\left(\frac{x}{a}\right)} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{\frac{1}{a}2\sec^2\left(\frac{x}{a}\right)\tan\left(\frac{x}{a}\right) - 0}{\frac{1}{a}\left[\frac{1}{a}2x\sec^2\left(\frac{x}{a}\right)\tan\left(\frac{x}{a}\right) + \sec^2\left(\frac{x}{a}\right)\right] + \frac{1}{a}\sec^2\left(\frac{x}{a}\right)}$$

$$= \lim_{x \to 0} \frac{\frac{2}{a}\sec^2\left(\frac{x}{a}\right)\tan\left(\frac{x}{a}\right)}{\frac{1}{a^2}2x\sec^2\left(\frac{x}{a}\right)\tan\left(\frac{x}{a}\right) + \frac{1}{a}\sec^2\left(\frac{x}{a}\right) + \frac{1}{a}\sec^2\left(\frac{x}{a}\right)}$$

$$= \frac{0}{0 + \frac{1}{a} + \frac{1}{a}} = 0$$

Q No. 28

$$\lim_{x\to 1}\left[\frac{x}{x-1}-\frac{1}{\ln x}\right]$$

Solution We have

$$\lim_{x\to 1}\left[\frac{x}{x-1}-\frac{1}{\ln x}\right]\equiv(\infty-\infty)$$

$$=\lim_{x\to 1}\left[\frac{x\ln x-x+1}{(x-1)\ln x}\right]\equiv\left(\frac{0}{0}\right).$$

using L'H rule

$$=\lim_{x\to 1}\frac{x\frac{1}{x}+\ln x-1+0}{(x-1)\frac{1}{x}+\ln x}$$

$$=\lim_{x\to 1}\frac{\ln x}{1-\frac{1}{x}+\ln x}\equiv\left(\frac{0}{0}\right).$$

using L'H rule

$$=\lim_{x\to 1}\frac{\frac{1}{x}}{0+\frac{1}{x^2}+\frac{1}{x}}$$

$$=\frac{\frac{1}{1}}{\frac{1}{1}+\frac{1}{1}}$$

$$=\frac{1}{1+1}=\frac{1}{2}$$

Q.No. 29 $\lim_{x \to \pi/2} (\sec x - \tan x)$

Solution: We have

$$\lim_{x \to \pi/2} (\sec x - \tan x) \equiv (\infty - \infty)$$

$$= \lim_{x \to \pi/2} \left[\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right]$$

$$= \lim_{x \to \pi/2} \left[\frac{1 - \sin x}{\cos x}\right] \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to \pi/2} \frac{-\cos x}{-\sin x} = \frac{\cos \pi/2}{\sin \pi/2}$$

$$= \frac{0}{1} = 0 \quad \#$$

Q.No. 30 $\lim_{x \to 1} \left[\frac{2}{x^2-1} - \frac{1}{x-1}\right]$

Solution: We have

$$\lim_{x \to 1} \left[\frac{2}{x^2-1} - \frac{1}{x-1}\right] \equiv (\infty - \infty)$$

$$= \lim_{x \to 1} \frac{2 - (x+1)}{x^2-1}$$

$$= \lim_{x \to 1} \frac{1-x}{x^2-1} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{x \to 1} \frac{-1}{2x} = -\frac{1}{2} \quad \#$$

Q No. 31 $\lim_{x \to \infty} \left(\sqrt{x^2+5x} - x\right)$

Solution We have

$$\lim_{x \to \infty} \left(\sqrt{x^2+5x} - x\right) \equiv (\infty - \infty)$$

Multiplying and dividing by $\sqrt{x^2+5x} + x$

$$= \lim_{x \to \infty} \left(\sqrt{x^2+5x} - x\right) \times \frac{\left(\sqrt{x^2+5x} + x\right)}{\sqrt{x^2+5x} + x}$$

$$= \lim_{x \to \infty} \frac{\left(\sqrt{x^2+5x}\right)^2 - (x)^2}{\sqrt{x^2+5x} + x}$$

$$= \lim_{x \to \infty} \frac{x^2+5x-x^2}{\sqrt{x^2+5x} + x} = \lim_{x \to \infty} \frac{5x}{\sqrt{x^2+5x} + x}$$

$$= \lim_{x \to \infty} \frac{5x}{x\sqrt{1+\frac{5}{x}} + x} = \lim_{x \to \infty} \frac{5x}{x\left(\sqrt{1+\frac{5}{x}} + 1\right)}$$

$$= \lim_{x \to \infty} \frac{5}{\sqrt{1+5/x} + 1} = \frac{5}{\sqrt{1+0} + 1}$$

$$= \frac{5}{1+1} = \frac{5}{2} \quad \#$$

Q No. 32
$$\lim_{x\to\infty}\left(e^{x}+e^{-x}\right)^{2/x}$$

Solution:

We have

$$\lim_{x\to\infty}\left(e^{x}+e^{-x}\right)^{2/x} \equiv (\infty^{0})$$

Let $y=\left(e^{x}+e^{-x}\right)^{\frac{2}{x}}$

Taking ln both sides

$$\ln y=\ln\left(e^{x}+e^{-x}\right)^{2/x}$$

$$\therefore \ln y=\frac{2}{x}\ln\left(e^{x}+e^{-x}\right)$$

$$\ln y=\frac{2\ln\left(e^{x}+e^{-x}\right)}{x}$$

$$\therefore \lim_{x\to\infty}\ln y=\lim_{x\to\infty}\frac{2\ln\left(e^{x}+e^{-x}\right)}{x}\equiv\left(\frac{\infty}{\infty}\right)$$

using L'H rule

$$\lim_{x\to\infty}\ln y=\lim_{x\to\infty}\frac{\frac{2}{e^{x}+e^{-x}}\left(e^{x}-e^{-x}\right)}{1}$$

$$\lim_{x\to\infty}\ln y=\lim_{x\to\infty}\frac{2\left(e^{x}-e^{-x}\right)}{e^{x}+e^{-x}}$$

$$=\lim_{x\to\infty}\frac{2\left(e^{x}-\frac{1}{e^{x}}\right)}{e^{x}+\frac{1}{e^{x}}}$$

$$=\lim_{x\to\infty}\frac{2\left(\left(e^{x}\right)^{2}-1\right)}{\left(e^{x}\right)^{2}+1}$$

$$= \lim_{x\to\infty} \frac{2(e^{2x}-1)}{e^{2x}+1} \equiv \left(\frac{\infty}{\infty}\right)$$

using L'H rule

$$= \lim_{x\to\infty} \frac{2(2e^{2x})}{2e^{2x}} = \lim_{x\to\infty} 2.$$

$$\Rightarrow \lim_{x\to\infty} \ln y = 2.$$

$$\lim_{x\to\infty} y = e^{2}$$

$$\Rightarrow \lim_{x\to\infty} (e^{x}+e^{-x})^{2/x} = e^{2} \quad \#$$

<u>QNo.33</u> $\lim_{x\to 0} \left(\frac{1}{x}\right)^{\tan x}$

<u>Solution</u> We have

$$\lim_{x\to 0} \left(\frac{1}{x}\right)^{\tan x} \equiv (\infty^{0}).$$

Let $y = \left(\frac{1}{x}\right)^{\tan x}$

Taking ln on both sides

$$\ln y = \ln\left(\frac{1}{x}\right)^{\tan x}$$

$$\ln y = \tan x . \ln\left(\frac{1}{x}\right)$$

$$\Rightarrow \ln y = \tan x \left[\ln 1 - \ln x\right]$$

$$\ln y = \tan x \left[0 - \ln x \right]$$

$$\Rightarrow \ln y = \frac{-\ln x}{\cot x}$$

$$\therefore \lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{-\ln x}{\cot x} \equiv \left(\frac{\infty}{\infty}\right)$$

$$= -\lim_{x \to 0} \frac{\frac{1}{x}}{-\operatorname{cosec}^2 x} \quad \because \text{using L'H rule}$$

$$= \lim_{x \to 0} \frac{\sin^2 x}{x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{2 \sin x \cos x}{1}$$

$$\Rightarrow \lim_{x \to 0} \ln y = \frac{2 \sin(0) \cos(0)}{1}$$

$$\lim_{x \to 0} \ln y = 0$$

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$$\Rightarrow \lim_{x \to 0} y = e^0$$

$$\Rightarrow \lim_{x \to 0} y = 1$$

$$\therefore \lim_{x \to 0} \left(\frac{1}{x}\right)^{\tan x} = 1 \quad \#$$

Q No. 34

$$\lim_{x\to\pi/2} (\cos x)^{-x+\pi/2}$$

Solution

We have.

$$\lim_{x\to\pi/2} (\cos x)^{-x+\pi/2} \equiv (0^0)$$

Let $y = (\cos x)^{\pi/2 - x}$

Taking ln on both sides

$$\ln y = \ln (\cos x)^{\pi/2 - x}$$

$$\Rightarrow \ln y = \left(\frac{\pi}{2} - x\right) \ln(\cos x)$$

Now

$$\lim_{x\to\pi/2} \ln y = \lim_{x\to\pi/2} (\pi/2 - x) \ln(\cos x)$$

$$\lim_{x\to\pi/2} \ln y = \lim_{x\to\pi/2} \frac{\ln(\cos x)}{\frac{1}{\frac{\pi}{2} - x}} \equiv \left(\frac{\infty}{\infty}\right)$$

using L'H rule

$$= \lim_{x\to\pi/2} \frac{\frac{-\sin x}{\cos x}}{(\pi/2 - x)^{-2}}$$

$$= \lim_{x\to\pi/2} \frac{-\tan x}{(\pi/2 - x)^{-2}}$$

$$\lim_{x\to\pi/2} \ln y = \lim_{x\to\pi/2} -\frac{(\pi/2-x)^2}{\cot x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x\to\pi/2} \frac{-2(\pi/2-x)(-1)}{-\operatorname{cosec}^2 x}$$

$$= \lim_{x\to\pi/2} \frac{-2(\pi/2-x)}{\operatorname{cosec}^2 x}$$

$$= \frac{0}{1}$$

$$\therefore \lim_{x\to\pi/2} \ln y = 0$$

$$\Rightarrow \lim_{x\to\pi/2} y = e^0$$

$$\Rightarrow \lim_{x\to\pi/2} (\cos x)^{-x+\pi/2} = 1$$

Q No. 35 $\lim_{x\to\infty} \left(\frac{x+a}{x-a}\right)^x$

Solution: We have

$$\lim_{x\to\infty} \left(\frac{x+a}{x-a}\right)^x$$

Let $y = \left(\frac{x+a}{x-a}\right)^x$

Taking ln on both sides

$$\ln y = \ln \left(\frac{x+a}{x-a}\right)^x$$

$$\ln y = x \ln \left(\frac{x+a}{x-a}\right)$$

$$\because \lim_{x\to\infty} \ln y = \lim_{x\to\infty} x \ln\left(\frac{x+a}{x-a}\right) \equiv (\infty * 0)$$

Let $x = \frac{1}{t}$

As $x \to \infty$, then $t \to 0$

So, $$\lim_{x\to\infty} \ln y = \lim_{t\to 0} \frac{1}{t} \ln\left(\frac{\frac{1}{t}+a}{\frac{1}{t}-a}\right)$$

$$= \lim_{t\to 0} \frac{1}{t}\left[\ln\left(\frac{1+at}{1-at}\right)\right]$$

$$= \lim_{t\to 0} \frac{1}{t}\left[\ln(1+at) - \ln(1-at)\right]$$

$$= \lim_{t\to 0} \frac{\ln(1+at) - \ln(1-at)}{t} \equiv \left(\frac{0}{0}\right)$$

using L'H rule.

$$= \lim_{t\to 0} \frac{\frac{a}{1+at} - \frac{-a}{1-at}}{1}$$

$$= \lim_{t\to 0} \left(\frac{a}{1+at} + \frac{a}{1-at}\right)$$

$$= \frac{a}{1+0} + \frac{a}{1-0} = 2a$$

$$\lim_{x\to\infty} \ln y = 2a$$

$$\Rightarrow \lim_{x\to\infty} y = e^{2a}$$

$$\therefore \lim_{x\to\infty} \left(\frac{x+a}{x-a}\right)^{x} = e^{2a} \quad \#$$

Q No. 36 $\displaystyle \lim_{x\to 0} \left(\frac{\sinh x}{x}\right)^{\frac{1}{x^2}}$

Solution:

we have.

$$\lim_{x\to 0} \left(\frac{\sinh x}{x}\right)^{\frac{1}{x^2}}$$

Let $y = \left(\frac{\sinh x}{x}\right)^{\frac{1}{x^2}}$

Taking ln both sides

$$\ln y = \ln \left(\frac{\sinh x}{x}\right)^{1/x^2}$$

$$\ln y = \frac{1}{x^2} \ln\left(\frac{\sinh x}{x}\right)$$

$$\therefore \lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{1}{x^2} \ln\left(\frac{\sinh x}{x}\right) \equiv (\infty \times 0)$$

$$= \lim_{x\to 0} \left[\frac{\ln\left(\frac{\sinh x}{x}\right)}{x^2}\right] \equiv \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{\ln(\sinh x) - \ln x}{x^2}$$

using L'H rule

$$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{\frac{1}{\sinh x}\cosh x - \frac{1}{x}}{2x}$$

$$= \lim_{x\to 0} \frac{\frac{x\cosh x - \sinh x}{x\sinh x}}{2x}$$

$$= \lim_{x\to 0} \frac{x\cosh x - \sinh x}{2x^2 \sinh x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x\to 0} \frac{x\sinh x + \cosh x - \cosh x}{2x^2\cosh x + 4x\sinh x}$$

$$= \lim_{x\to 0} \frac{x.\sinh x}{x(2x\cosh x + 4\sinh x)}$$

$$= \lim_{x\to 0} \frac{\sinh x}{2x\cosh x + 4\sinh x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x\to 0} \frac{\cosh x}{2x\sinh x + 2\cosh x + 4\cosh x}$$

$$= \lim_{x\to 0} \frac{\cosh x}{2x\sinh x + 6\cosh x}$$

$$\therefore \lim_{x\to 0} \ln y = \frac{1}{2(0)+6(1)} = \frac{1}{6}$$

$$\lim_{x\to 0} y = e^{1/6}$$

$$\therefore \lim_{x\to 0}\left(\frac{\sinh x}{x}\right)^{\frac{1}{x^2}} = e^{1/6}$$

Q No. 37 $\quad \lim_{x\to 0}(\tan x)^{\sin 2x}$

Solution: We have

$$\lim_{x\to 0}(\tan x)^{\sin 2x} \equiv (0^0)$$

Let $y = (\tan x)^{\sin 2x}$

Taking ln on both sides

$$\ln y = \ln(\tan x)^{\sin 2x}$$

$$\ln y = \sin 2x \cdot \ln(\tan x)$$

$$\therefore \lim_{x\to 0}\ln y = \lim_{x\to 0}\sin 2x \ln(\tan x) \equiv (0\times\infty)$$

$$= \lim_{x\to 0}\frac{\ln(\tan x)}{\operatorname{cosec} 2x} \equiv \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to 0}\frac{\ln\left(\frac{\sin x}{\cos x}\right)}{\operatorname{cosec} 2x}$$

$$= \lim_{x\to 0}\frac{\ln\sin x - \ln\cos x}{\operatorname{cosec} 2x}$$

Using L'H rule

$$= \lim_{x\to 0}\frac{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}{-\operatorname{cosec} 2x\cot 2x\,(2)}$$

$$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}}{-2\frac{1}{\sin 2x}\frac{\cos 2x}{\sin 2x}}$$

$$= \lim_{x\to 0} \frac{\frac{1}{\sin x \cos x}}{\frac{-2\cos 2x}{\sin^2 2x}}$$

$$= \lim_{x\to 0} \frac{1}{\sin x \cos x} \times \frac{\sin^2 2x}{-2\cos 2x}$$

$$= \lim_{x\to 0} \frac{1}{-2\sin x \cos x} \times \frac{\sin^2 2x}{\cos 2x}$$

$$= \lim_{x\to 0} \frac{1}{-\sin 2x} \times \frac{\sin^2 2x}{\cos 2x}$$

$$= \lim_{x\to 0} -\frac{\sin 2x}{\cos 2x}$$

$$\therefore \lim_{x\to 0} \ln y = 0$$

$$\Rightarrow \lim_{x\to 0} y = e^0$$

$$\Rightarrow \lim_{x\to 0} (\tan x)^{\sin 2x} = 1$$

QNo. 38: $\lim_{x\to 0} (1+\sin x)^{\cot x}$

Solution: We have

$$\lim_{x\to 0} (1+\sin x)^{\cot x} \equiv (1)^\infty.$$

Let $y = (1+\sin x)^{\cot x}$

Taking $\ln$ on both sides

$\ln y = \ln (1+\sin x)^{\cot x}$

$\ln y = \cot x \ln (1+\sin x)$

$\therefore \lim_{x\to 0} \ln y = \lim_{x\to 0} \cot x \ln (1+\sin x) \equiv (\infty \times 0)$

$$= \lim_{x\to 0} \frac{\ln(1+\sin x)}{\tan x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x\to 0} \frac{\frac{\cos x}{1+\sin x}}{\sec^2 x}$$

$$= \lim_{x\to 0} \frac{\cos^3 x}{1+\sin x}$$

$$\Rightarrow \lim_{x\to 0} \ln y = \frac{\cos^3(0)}{1+\sin(0)}$$

$\lim_{x\to 0} \ln y = 1$

$\lim_{x\to 0} y = e^1$

$\therefore \lim_{x\to 0} (1+\sin x)^{\cot x} = e$. #

Q No. 39 $\lim_{x \to \pi/2} (\sec x)^{\cot x}$

Solution We have.

$$\lim_{x \to \pi/2} (\sec x)^{\cot x}$$

Let $y = (\sec x)^{\cot x}$

Taking ln both sides

$$\ln y = \ln (\sec x)^{\cot x}$$

$$\therefore \ln y = \cot x \cdot \ln(\sec x)$$

$$\therefore \lim_{x \to \pi/2} \ln y = \lim_{x \to \pi/2} \cot x \ln(\sec x) \equiv (0 \times \infty)$$

$$= \lim_{x \to \pi/2} \frac{\ln \sec x}{\tan x} \equiv \left(\frac{\infty}{\infty}\right)$$

using L'H rule

$$= \lim_{x \to \pi/2} \frac{\frac{1}{\sec x} \sec x \tan x}{\sec^2 x}$$

$$= \lim_{x \to \pi/2} \frac{\tan x}{\sec^2 x} \equiv \left(\frac{\infty}{\infty}\right)$$

using L'H rule

$$= \lim_{x \to \pi/2} \frac{\sec^2 x}{2 \sec x \sec x \tan x}.$$

$$\lim_{x\to\pi/2} \ln y = \lim_{x\to\pi/2} \frac{1}{2\tan x}$$

$$\Rightarrow \lim_{x\to\pi/2} \ln y = \frac{1}{2\tan \pi/2} = 0$$

$$\Rightarrow \lim_{x\to\pi/2} y = e^0 = 1$$

$$\therefore \lim_{x\to\pi/2} (\sec x)^{\cot x} = 1$$

QNo.40 $$\lim_{x\to 1} (1-x^2)^{\frac{1}{\ln(1-x)}} \equiv (0^\infty)$$

Solution We have

$$\lim_{x\to 1} (1-x^2)^{\frac{1}{\ln(1-x)}}$$

Let $y = (1-x^2)^{\frac{1}{\ln(1-x)}}$

Taking ln on both sides

$$\ln y = \ln (1-x^2)^{\frac{1}{\ln(1-x)}}$$

$$\ln y = \frac{1}{\ln(1-x)} \cdot \ln(1-x^2)$$

$$\therefore \lim_{x\to 1} \ln y = \lim_{x\to 1} \frac{\ln(1-x^2)}{\ln(1-x)} \equiv \left(\frac{\infty}{\infty}\right)$$

using L'H rule

$$= \lim_{x\to 1} \frac{\frac{-2x}{1-x^2}}{\frac{-1}{1-x}}$$

$$\lim_{x\to 1} \ln y = \lim_{x\to 1} \frac{2x}{1+x} = \frac{2(1)}{1+1}$$

$$\lim_{x\to 1} \ln y = 1$$

$$\Rightarrow \lim_{x\to 1} y = e^1$$

$$\therefore \lim_{x\to 1} (1-x^2)^{\frac{1}{\ln(1-x)}} = e$$

#

Q.No.41

$$\lim_{x\to 1} \left[\tan\left(\frac{x\pi}{4}\right)\right]^{\tan\left(\frac{\pi x}{2}\right)}$$

Solution

We have

$$\lim_{x\to 1} \left[\tan\left(\frac{x\pi}{4}\right)\right]^{\tan\left(\frac{\pi x}{2}\right)} \equiv (1)^\infty$$

Let $y = \left[\tan\left(\frac{x\pi}{4}\right)\right]^{\tan\left(\frac{\pi x}{2}\right)}$

Taking ln on both sides

$$\ln y = \ln\left[\tan\left(\frac{x\pi}{4}\right)\right]^{\tan\left(\frac{\pi x}{2}\right)}$$

$$\ln y = \tan\left(\frac{\pi x}{2}\right) \ln\left[\tan\left(\frac{x\pi}{4}\right)\right]$$

$$\therefore \lim_{x\to 1} \ln y = \lim_{x\to 1} \tan\left(\frac{\pi x}{2}\right) \ln\left[\tan\left(\frac{x\pi}{4}\right)\right] \equiv (\infty \times 0)$$

$$= \lim_{x\to 1} \frac{\ln\left[\tan\left(\frac{x\pi}{4}\right)\right]}{\cot\left(\frac{x\pi}{2}\right)} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x\to 1} \frac{\frac{1}{\tan\left(\frac{\pi x}{4}\right)} \sec^2\left(\frac{\pi x}{4}\right)\frac{\pi}{4}}{-\operatorname{cosec}^2\left(\frac{\pi x}{2}\right)\frac{\pi}{2}}$$

$$= \lim_{x\to 1} \frac{\frac{-\cos\left(\frac{\pi x}{4}\right)}{\sin\left(\frac{\pi x}{4}\right)} \cdot \frac{1}{\cos^2\left(\frac{\pi x}{4}\right)}}{2 \cdot \frac{1}{\sin^2\left(\frac{\pi x}{2}\right)}}$$

$$= \lim_{x\to 1} \frac{-\sin^2\left(\frac{\pi x}{2}\right)}{2\sin\left(\frac{\pi x}{4}\right)\cos\left(\frac{\pi x}{4}\right)}$$

$$= \lim_{x\to 1} \frac{-\sin^2\left(\frac{\pi x}{2}\right)}{\sin\frac{\pi x}{2}} \qquad \because \sin 2x = 2\sin x\cos x$$

$$\Rightarrow \lim_{x\to 1} \ln y = \lim_{x\to 1} -\sin\left(\frac{\pi x}{2}\right) = -\sin\frac{\pi}{2}$$

$$\Rightarrow \lim_{x\to 1} \ln y = -1$$

$$\Rightarrow \lim_{x\to 1} y = e^{-1} = \frac{1}{e}$$

$$\Rightarrow \lim_{x\to 1} \left[\tan\left(\frac{\pi x}{4}\right)\right]^{\tan\left(\frac{\pi x}{2}\right)} = \frac{1}{e}$$

#

Q.No. 42 $\quad \lim_{x\to \pi/2} (1-\sin x)^{\cos x}$

Solution $\quad$ We have.

$$\lim_{x\to \pi/2} (1-\sin x)^{\cos x} \equiv (0)^{0}$$

Let $y = (1-\sin x)^{\cos x}$

Taking ln both sides

$$\ln y = \ln (1-\sin x)^{\cos x}$$

$$\ln y = \cos x \ln (1-\sin x)$$

$$\therefore \lim_{x\to \pi/2} \ln y = \lim_{x\to \pi/2} \cos x \ln (1-\sin x) \equiv (0\times\infty)$$

$$= \lim_{x\to \pi/2} \frac{\ln(1-\sin x)}{\sec x} \equiv \left(\frac{\infty}{\infty}\right)$$

using L'H rule

$$= \lim_{x\to \pi/2} \frac{\frac{-\cos x}{1-\sin x}}{\sec x \tan x}$$

$$= \lim_{x\to \pi/2} \frac{-\cos x}{(1-\sin x)\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}}$$

$$= \lim_{x\to \pi/2} \frac{-\cos^3 x}{\sin x(1-\sin x)} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to \pi/2} \frac{3\cos^2 x \sin x}{\sin x(-\cos x) + \cos x(1-\sin x)}$$

$$= \lim_{x \to \pi/2} \frac{3\sin x \cos x}{-\sin x + 1 - \sin x}$$

$$\therefore \lim_{x \to \pi/2} \ln y = \lim_{x \to \pi/2} \frac{0}{-1+1-1} = 0$$

$$\Rightarrow \lim_{x \to \pi/2} y = e^0 = 1$$

$$\Rightarrow \lim_{x \to \pi/2} (1-\sin x)^{\cos x} = 1$$

Q No. 43 $\lim_{x \to 0} (\cot x)^{\sin 2x}$

Solution: We have

$$\lim_{x \to 0} (\cot x)^{\sin 2x} \equiv (\infty)^0$$

Let $y = (\cot x)^{\sin 2x}$

Taking ln on both sides

$$\ln y = \ln (\cot x)^{\sin 2x}$$

$$\Rightarrow \ln y = \sin 2x \ln(\cot x)$$

$$\therefore \lim_{x \to 0} \ln y = \lim_{x \to 0} \sin 2x \ln(\cot x) \equiv (0 \times \infty)$$

$$= \lim_{x \to 0} \frac{\ln(\cot x)}{\operatorname{cosec} 2x} \equiv \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x \to 0} \frac{\ln\left(\frac{\cos x}{\sin x}\right)}{\operatorname{cosec} 2x}.$$

$$= \lim_{x \to 0} \frac{\ln \cos x - \ln \sin x.}{\operatorname{cosec} 2x} \equiv \left(\frac{\infty}{\infty}\right)$$

Using L'H rule

$$= \lim_{x \to 0} \frac{\frac{-\sin x}{\cos x} - \frac{\cos x}{\sin x}}{-\operatorname{cosec} 2x \cot 2x (2)}$$

$$= \lim_{x \to 0} \frac{-\left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)}{-2 \frac{1}{\sin 2x} \cdot \frac{\cos 2x}{\sin 2x}}$$

$$= \lim_{x \to 0} \frac{\frac{1}{\sin x \cos x}.}{\frac{2 \cos 2x}{\sin^2 2x}}$$

$$= \lim_{x \to 0} \frac{\sin^2 2x}{2 \sin x \cos x \cos 2x}$$

$$= \lim_{x \to 0} \frac{\sin^2 2x}{\sin 2x \cos 2x.}$$

$$= \lim_{x \to 0} \frac{\sin 2x}{\cos 2x}$$

$$\therefore \lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{\sin 2x}{\cos 2x} = \frac{0}{1}$$

$$\lim_{x \to 0} y = e^0$$

$$\Rightarrow \lim_{x \to 0} (\cot x)^{\sin 2x} = 1 \quad \#$$

Q.No. 44 $\lim_{x \to 0} \frac{\tanh x - \sinh x}{x^2}$.

Solution

We have

$$\lim_{x \to 0} \frac{\tanh x - \sinh x}{x^2} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{x \to 0} \frac{\operatorname{sech}^2 x - \cosh x}{2x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{2\operatorname{sech} x . \operatorname{sech} x \tanh x - \sinh x}{2}.$$

$$= \lim_{x \to 0} \frac{2\operatorname{sech}^2 x \tanh x - \sinh x}{2}.$$

$$= \frac{2\operatorname{sech}^2(0)\tanh(0) - \sinh(0)}{2}$$

$$= \frac{2(1)(0) - 0}{2} = \frac{0}{2} = 0$$

Q.No. 45 $\lim_{x \to 0} \frac{\sqrt{x} - \sqrt{\sin x}}{x^{5/2}}$.

Solution:

We have.

$$\lim_{x \to 0} \frac{\sqrt{x} - \sqrt{\sin x}}{x^{5/2}}$$

$$= \lim_{x \to 0} \frac{\sqrt{x} - \left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right]^{1/2}}{x^{5/2}}$$

$$= \lim_{x \to 0} \frac{\sqrt{x} - \sqrt{x}\left[1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots\right]^{1/2}}{x^{5/2}}$$

$$= \lim_{x \to 0} \frac{\sqrt{x}\left\{1 - \left[1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots\right]^{1/2}\right\}}{x^{5/2}}$$

$$= \lim_{x \to 0} \frac{1 - \left[1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots\right]^{1/2}}{x^2}$$

$$= \lim_{x \to 0} \frac{1 - \left[1 - \left(\frac{x^2}{3!} - \frac{x^4}{5!} + \cdots\right)\right]^{1/2}}{x^2}$$

$$= \lim_{x \to 0} \frac{1 - \left[1 - \frac{1}{2}\left(\frac{x^2}{6} - \frac{x^4}{120} + \cdots\right)\right]}{x^2}$$

$$= \lim_{x \to 0} \frac{1 - 1 + \frac{1}{2}\left(\frac{x^2}{6} - \frac{x^4}{120} + \cdots\right)}{x^2}$$

$$= \lim_{x \to 0} \frac{\frac{1}{2}\left(\frac{x^2}{6} - \frac{x^4}{120} + \cdots\right)}{x^2}$$

$$= \lim_{x \to 0} \frac{1}{2}\left(\frac{1}{6} - \frac{x^2}{120} + \cdots\right)$$

$$= \frac{1}{2}\left(\frac{1}{6}\right) - 0 + 0 + \cdots = \frac{1}{12}.$$

Q.No. 46 $\lim_{x\to 0} \frac{\sinh x - \sin x}{\sin^3 x}$

Solution We have

$$\lim_{x\to 0} \frac{\sinh x - \sin x}{\sin^3 x} \equiv \left(\frac{0}{0}\right)$$

using $\sin 3x = 3\sin x - 4\sin^3 x$

$$\Rightarrow \sin^3 x = \frac{1}{4}\left(3\sin x - \sin 3x\right)$$

$$= \lim_{x\to 0} \frac{\sinh x - \sin x}{\frac{1}{4}\left(3\sin x - \sin 3x\right)}$$

using L'H rule

$$= \lim_{x\to 0} \frac{\cosh x - \cos x}{\frac{1}{4}\left(3\cos x - 3\cos 3x\right)} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x\to 0} \frac{\sinh x + \sin x}{\frac{1}{4}\left(-3\sin x + 9\sin 3x\right)} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x\to 0} \frac{\cosh x + \cos x}{\frac{1}{4}\left(-3\cos x + 27\cos 3x\right)}$$

$$= \frac{1+1}{\frac{1}{4}(-3+27)} = \frac{2}{\frac{24}{4}} = \frac{2}{6} = \frac{1}{3}$$

#

Q.No. 47 $\lim_{x\to 0} \dfrac{(1+x)^{1/x} - e + \frac{ex}{2}}{x^2}$.

Solution We have

$$\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}} - e + \frac{ex}{2}}{x^2} \quad \text{--- (1)}$$

Let $y = (1+x)^{\frac{1}{x}}$

Taking ln both sides

$\ln y = \ln (1+x)^{\frac{1}{x}}$

$\ln y = \frac{1}{x} \ln(1+x)$

using series $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$

$\ln y = \frac{1}{x}\left(x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots\right)$

$\ln y = 1 - \frac{1}{2}x + \frac{1}{3}x^2 - \cdots$

$\Rightarrow y = e^{\left(1 - \frac{1}{2}x + \frac{1}{3}x^2 - \cdots\right)}$

using series $e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$

$y = e^1 \cdot e^{\left(-\frac{1}{2}x + \frac{1}{3}x^2 - \cdots\right)}$

$y = e\left[1 + \left(-\frac{x}{2} + \frac{1}{3}x^2 - \cdots\right) + \frac{1}{2!}\left(-\frac{x}{2} + \frac{1}{3}x^2 - \cdots\right)^2 + \cdots\right]$

$y = e\left[1 - \frac{x}{2} + \frac{1}{3}x^2 - \cdots + \frac{1}{8}x^2 + \frac{1}{18}x^4 + \cdots\right]$

$$y = e\left[1 - \frac{x}{2} + \frac{11}{24}x^2 + \cdots\right]$$

Equation ① becomes

$$\lim_{x\to 0} \frac{e\left[1 - \frac{x}{2} + \frac{11}{24}x^2 + \cdots\right] - e + \frac{ex}{2}}{x^2}$$

$$= \lim_{x\to 0} \frac{e - \frac{ex}{2} + \frac{11}{24}ex^2 + \cdots - e + \frac{ex}{2}}{x^2}$$

$$= \lim_{x\to 0} \frac{\frac{11}{24}ex^2 + \cdots}{x^2}$$

$$= \lim_{x\to 0} \left(\frac{11}{24}e + \cdots\right) = \frac{11}{24}e \quad \#$$

<u>QNo.48</u> $$\lim_{x\to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$$

<u>Solution:</u> We have

$$\lim_{x\to 0} \frac{e^x - e^{\sin x}}{x - \sin x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x\to 0} \frac{e^x - e^{\sin x}\cos x}{1 - \cos x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x\to 0} \frac{e^x - e^{\sin x}\cos^2 x - e^{\sin x}(-\sin x)}{0 + \sin x}$$

$$= \lim_{x \to 0} \frac{e^x - e^{\sin x}\cos^2 x + e^{\sin x}\sin x}{\sin x} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{x \to 0} \frac{e^x - \left(e^{\sin x}\cos^3 x + e^{\sin x} 2\cos x(-\sin x)\right) + \left(e^{\sin x}\cos x \sin x + e^{\sin x}\cos x\right)}{\cos x}.$$

$$= \frac{1 - (1+0) + (0+1)}{1} = \frac{1-1+1}{1}$$

$$= 1.$$

QNo.49 Use L'Hospital's Rule to prove that

$$\lim_{x \to \infty} \left[\frac{a^{1/x} + b^{1/x}}{2}\right]^x = \sqrt{ab}$$

Solution

Let $y = \left[\frac{a^{\frac{1}{x}} + b^{\frac{1}{x}}}{2}\right]^x$

Taking ln on both sides

$$\ln y = \ln\left[\frac{a^{1/x} + b^{1/x}}{2}\right]^x$$

$$\ln y = x \ln\left[\frac{a^{1/x} + b^{1/x}}{2}\right]$$

$$\ln y = x\left[\ln(a^{1/x} + b^{1/x}) - \ln 2\right]$$

$$\therefore \lim_{x \to \infty} \ln y = \lim_{x \to \infty} x\left[\ln(a^{1/x} + b^{1/x}) - \ln 2\right]$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(a^{1/x}+b^{1/x}) - \ln 2}{\frac{1}{x}} \equiv \left(\frac{0}{0}\right).$$

using L'H rule

$$= \lim_{x\to\infty} \frac{\frac{1}{a^{1/x}+b^{1/x}} \frac{d}{dx}(a^{1/x}+b^{1/x}) - 0}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{a^{1/x}+b^{1/x}} \left[a^{1/x}\ln a\left(-\frac{1}{x^2}\right) + b^{1/x}\ln b\left(-\frac{1}{x^2}\right)\right]}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{a^{1/x}+b^{1/x}}\left(-\frac{1}{x^2}\right)\left[a^{1/x}\ln a + b^{1/x}\ln b\right]}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{1}{a^{1/x}+b^{1/x}}\left(a^{1/x}\ln a + b^{1/x}\ln b\right)$$

$$= \frac{1}{a^{1/\infty}+b^{1/\infty}}\left(a^{1/\infty}\ln a + b^{1/\infty}\ln b\right)$$

$$= \frac{1}{a^{0}+b^{0}}\left(a^{0}\ln a + b^{0}\ln b\right)$$

$$\therefore \lim_{x\to\infty} \ln y = \frac{\ln a + \ln b}{2}.$$

$$\lim_{x\to\infty} \ln y = \frac{\ln ab}{2}$$

$$\lim_{x\to\infty} \ln y = \frac{1}{2}\ln(ab)$$

$$\Rightarrow \lim_{x\to\infty} \ln y = \ln(ab)^{1/2}$$

$$\Rightarrow \lim_{x\to\infty} y = \sqrt{ab}$$

$$\therefore \lim_{x\to\infty}\left[\frac{a^{1/x}+b^{1/x}}{2}\right]^{x} = \sqrt{ab}$$

Q No. 50 If f is a thrice differentiable function, prove, by using L'Hospital's Rule, that

(i) $\lim_{h\to 0} \dfrac{f(x+h)-f(x-h)}{2h} = f'(x)$

(ii) $\lim_{h\to 0} \dfrac{f(x+h)-2f(x)+f(x-h)}{h^2} = f''(x)$

(iii) $\lim_{h\to 0} \dfrac{f(x+h)-f(x)-hf'(x)-\frac{h^2}{2}f''(x)}{h^3} = \dfrac{f'''(x)}{6}$.

Solution:

(i) Consider the limit

$$\lim_{h\to 0} \frac{f(x+h)-f(x-h)}{2h} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{h\to 0} \frac{f'(x+h)-f'(x-h)(-1)}{2}$$

$$= \lim_{h \to 0} \frac{f'(x+h) + f'(x-h)}{2}$$

$$= \frac{f'(x+0) + f'(x-0)}{2} = \frac{f'(x) + f'(x)}{2}$$

$$= \frac{2f'(x)}{2} = f'(x)$$

$$\therefore \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h} = f'(x).$$

(ii) Consider the limit

$$\lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{h \to 0} \frac{f'(x+h) - 2(0) + f'(x-h)(-1)}{2h}$$

$$= \lim_{h \to 0} \frac{f'(x+h) - f'(x-h)}{2h} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{h \to 0} \frac{f''(x+h) - f''(x-h)(-1)}{2}$$

$$= \lim_{h \to 0} \frac{f''(x+h) + f''(x-h)}{2}$$

$$= \frac{f''(x+0) + f''(x-0)}{2} = \frac{f''(x) + f''(x)}{2}$$

$$= \frac{2f''(x)}{2} = f''(x)$$

$$\therefore \lim_{h\to 0} \frac{f(x+h)-2f(x)+f(x-h)}{h^2} = f''(x).$$

(iii) Consider the limit

$$\lim_{h\to 0} \frac{f(x+h)-f(x)-hf'(x)-\frac{h^2}{2}f''(x)}{h^3} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{h\to 0} \frac{f'(x+h)-0-f'(x)-\frac{2h}{2}f''(x)}{3h^2}.$$

$$= \lim_{h\to 0} \frac{f'(x+h)-f'(x)-hf''(x)}{3h^2} \equiv \left(\frac{0}{0}\right)$$

using L'H rule

$$= \lim_{h\to 0} \frac{f''(x+h)-0-f''(x)}{6h}$$

$$= \lim_{h\to 0} \frac{f''(x+h)-f''(x)}{6h} \equiv \left(\frac{0}{0}\right)$$

using L'H rule.

$$= \lim_{h\to 0} \frac{f'''(x+h) - 0}{6}$$

$$= \frac{f'''(x+0)}{6} = \frac{f'''(x)}{6}$$

$$\therefore \lim_{h\to 0} \frac{f(x+h) - f(x) - hf'(x) - \frac{h^2}{2} f''(x)}{h^3} = \frac{f'''(x)}{6}$$

QNo. 51 Determine a, b, c, d and e such that

$$\lim_{x\to 0} \frac{\cos ax + bx^3 + cx^2 + dx + e}{x^4} = \frac{2}{3}$$

Solution:

We have

$$\lim_{x\to 0} \frac{\cos ax + bx^3 + cx^2 + dx + e}{x^4} = \frac{2}{3}$$

If the function indeterminate form $\left(\frac{0}{0}\right)$, then

$$\cos a(0) + e = 0$$

$$\Rightarrow \cos 0 + e = 0 \Rightarrow \boxed{e = -1}$$

Now

$$\lim_{x\to 0} \frac{\cos ax + bx^3 + cx^2 + dx - 1}{x^4} = \frac{2}{3}$$

using L'H rule.

$$\lim_{x\to 0} \frac{-a\sin ax+3bx^2+2cx+d}{4x^3} = \frac{2}{3}$$

It will be of the form $\left(\frac{0}{0}\right)$

$\therefore \quad 0+0+0+d=0$

$\Rightarrow \boxed{d=0}$

$$\therefore \lim_{x\to 0} \frac{-a\sin ax+3bx^2+2cx}{4x^3} = \frac{2}{3}$$

using L'H rule

$$\lim_{x\to 0} \frac{-a^2\cos ax+6bx+2c}{12x^2} = \frac{2}{3}$$

It will be of the form $\left(\frac{0}{0}\right)$

$-a^2+2c=0$ —— (1)

$$\lim_{x\to 0} \frac{a^3\sin ax+6b}{24x} = \frac{2}{3}$$

for $\left(\frac{0}{0}\right) \quad \therefore \quad 6b=0 \Rightarrow \boxed{b=0}$

$$\Rightarrow \lim_{x\to 0} \frac{a^4\cos ax}{24} = \frac{2}{3} \Rightarrow \frac{a^4}{24} = \frac{2}{3}$$

$a^4 = \frac{48}{3} \Rightarrow a^4=16$

$\boxed{a=\pm 2}$

(1) $\Rightarrow -(\pm 2)^2+2c=0 \Rightarrow -4+2c=0$

$\boxed{c=2}$

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