Zoobaria
BSc Part(I)

Exercise 2.3

(1)

Find Δy, dy, $\Delta y - dy$.

Method:

$$y = f(x)$$
$$y + \Delta y = f(x + \Delta x)$$
$$y + \Delta y - y = f(x+\Delta x) - f(x)$$
$$\boxed{\Delta y = f(x+\Delta x) - f(x)}$$

$$y = f(x)$$
$$\frac{dy}{dx} = f'(x)$$
$$\boxed{dy = f'(x)dx}$$

(i) $y = x^3 - 1$ $\quad x = 1$, $\Delta x = -0.5$

$$\Delta y = f(x+\Delta x) - f(x)$$
$$\Delta y = \left[(x+\Delta x)^3 - 1\right] - (x^3 - 1)$$
$$\Delta y = (x+\Delta x)^3 - 1 - x^3 + 1$$
$$= (x+\Delta x)^3 - x^3$$
$$\Delta y = (1 - 0.5)^3 - (1)^3$$
$$= 0.125 - 1$$
$$\Delta y = -0.875$$

$$y = x^3 - 1$$
$$\frac{dy}{dx} = 3x^2$$
$$dy = 3x^2 dx \qquad \because dx = \Delta x = -0.5$$
$$dy = 3(1)(-0.5)$$
$$dy = -1.5$$

$$\Delta y - dy = -0.875 - (-1.5)$$
$$= -0.875 + 1.5$$
$$\Delta y - dy = 0.625$$

(ii) $y = \sqrt{3x-2}$ $\quad x = 2$, $\Delta x = 0.3$

$$\Delta y = f(x+\Delta x) - f(x)$$
$$\Delta y = \sqrt{3(x+\Delta x)-2} - \sqrt{3x-2}$$
$$\Delta y = \sqrt{3(2+0.3)-2} - \sqrt{3(2)-2}$$
$$\Delta y = \sqrt{3(2.3)-2} - \sqrt{6-2}$$
$$\Delta y = \sqrt{4.9} - \sqrt{4}$$
$$\Delta y = 2.2136 - 2$$
$$\Delta y = 0.2136$$

$$y = \sqrt{3x-2}$$
$$\frac{dy}{dx} = \frac{1}{2\sqrt{3x-2}}(3)$$
$$dy = \frac{3}{2\sqrt{3x-2}} \cdot dx$$
$$dy = \frac{3}{2\sqrt{3(2)-2}} \cdot (0.3)$$
$$= \frac{3}{2\sqrt{6-2}} \cdot (0.3)$$
$$= \frac{3}{2(2)}(0.3)$$
$$dy = 0.225$$

$$\Delta y - dy = 0.2136 - 0.225$$
$$= -0.0114$$

2. Using differentials to approximate.

(i) $\sqrt{26.2}$

$$= \sqrt{25 + 1.2}$$
$$y = f(x) \quad \text{with } x = 25, \ \Delta x = 1.2$$
$$y = \sqrt{x}$$
$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$
$$dy = \frac{1}{2\sqrt{x}} dx$$
$$dy = \frac{1}{2\sqrt{25}}(1.2) = \frac{1}{2(5)}(1.2)$$
$$dy = 0.12$$

$$dy \approx \Delta y = y + \Delta y - y$$
$$dy \approx y + \Delta y - y$$
$$\boxed{dy + y \approx y + \Delta y} = f(x+\Delta x)$$

$$y + \Delta y \approx \sqrt{x} + 0.12.$$
$$f(x+\Delta x) \approx \sqrt{25} + 0.12$$
$$\sqrt{x+\Delta x} \approx 5 + 0.12$$
$$\sqrt{25+1.2} \approx 5.12$$
$$\sqrt{26.2} \approx 5.12$$

By calculator
$$\sqrt{26.2} \approx 5.1186$$
$$\text{error} = 5.12 - 5.1186 = 0.0014$$

(ii) $\sqrt{80.9} = \sqrt{81 - 0.1}$

$y = f(x) = \sqrt{x}$ with $x = 81$

$dy = \Delta x = -0.1$

$y = \sqrt{81}$

$\boxed{y = 9}$

$$dy = f'(x)dx$$
$$dy = \frac{1}{2\sqrt{x}} dx$$
$$= \frac{1}{2\sqrt{81}}(-0.1)$$
$$= -\frac{0.1}{2 \times 9}$$
$$\boxed{dy = -0.005556}$$

$$f(x+\Delta x) \approx y + \Delta y$$
$$\sqrt{x+\Delta x} \approx 9 - 0.00556$$
$$\sqrt{81+(-0.1)} \approx 8.994444$$
$$\sqrt{80.9} \approx 8.99444$$

(iii) $\sqrt[3]{123} = \sqrt[3]{125 + (-2)}$

$y = f(x) = \sqrt[3]{x}$ with $x = 25$

$dy = \Delta y = -2$

$$y = (x)^{1/3}$$

$y = (125)^{1/3}$

$\boxed{y = 5}$

$$\frac{dy}{dx} = \frac{1}{3} x^{-2/3}$$
$$dy = \frac{1}{3x^{2/3}} dx$$
$$dy = \frac{1}{3(125)^{2/3}}(-2)$$
$$= \frac{1}{3(5)^2}(-2)$$
$$\boxed{dy = -0.0267}$$

$$f(x+\Delta x) \approx y + \Delta y$$
$$\sqrt{x+\Delta x} \approx 5 + (-0.0267)$$
$$\sqrt{125+(-2)} \approx 4.9733$$
$$\sqrt{123} \approx 4.9733$$

(iv) $\cos 61^\circ = \cos(60+1)$

$y = f(x) = \cos(x)$ with $x = 60^\circ$

$\Delta x = 1^\circ$

in radian

$x = 60 \times \frac{\pi}{180}$

$\boxed{x = \frac{\pi}{3}}$

$\Delta x = 1 \times \frac{\pi}{180}$

$\boxed{\Delta x = 0.0174}$

$y = \cos x$

$y = \cos(\frac{\pi}{3})$

$y = + \frac{1}{2}$

$$\frac{dy}{dx} = -\sin x$$
$$dy = -\sin x \, dx$$
$$dy = -\sin(\tfrac{\pi}{3})(0.0174)$$
$$dy = -\frac{\sqrt{3}}{2}(0.0174)$$
$$dy = -0.01506$$

$$f(x+\Delta x) \approx y + dy$$
$$\cos(x+\Delta x) \approx \cos 60 + dy$$
$$\cos(60+1) \approx 0.5 + (0.01506)$$
$$\cos 61 \approx 0.5 - 0.01506$$
$$\cos 61 \approx 0.4849$$

(V) $(3.02)^4 = (3 + 0.02)^4$

$y = f(x) = x^4$ with $x = 3$

$dx = \Delta x = 0.02$

$$dy = 4x^3 dx$$
$$dy = 4(3)^3(0.02)$$
$$dy = 2.16$$

$y = x^4$

$y = 3^4$

$y = 81$

$$f(x+\Delta x) \approx y + dy$$
$$(x+\Delta x)^4 \approx 81 + 2.16$$
$$(3+0.2)^4 \approx 83.16$$

(Vi) $\tan 29 = \tan(30 + (-1))$

$y = f(x) = \tan x$, with $x = 30 = \frac{\pi}{6}$

$\Delta x = -1$

$\Delta x = -0.0174$

$$dy = \sec^2 x \, dx$$
$$dy = \sec^2(\tfrac{\pi}{6}).(-0.0174)$$
$$= (1.3329)(-0.0174)$$
$$= -0.02319$$

$y = \tan x$

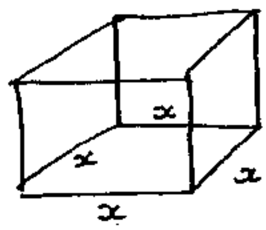
$y = \tan(\frac{\pi}{6})$

$y = 0.577$

$$f(x+\Delta x) \approx y + dy$$
$$\tan(x+\Delta x) \approx 0.5773 - 0.02319$$
$$\tan(30+(-1)) \approx 0.5541$$
$$\tan 29 \approx 0.5541$$

...e side of a cube is measured............. one face of cube. ③

Sol. Let 'x' be the side of cube



error in side of cube $= dx$.

percentage error $= \pm 2\%$

$$\frac{dx}{x} \times 100 = \pm 2\%$$

$$\frac{dx}{x} = \frac{\pm 2}{100} = \pm 0.02$$

surface Area of one face $= x^2$ $\quad \because A = \text{length} \times \text{width} = x \times x = x^2$

$$A = x^2$$

$$dA = 2x\,dx$$

we find percentage error in surface area

i.e. %age error in area $= \frac{dA}{A} \times 100$

$$= \frac{2x\,dx}{x^2} \times 100$$

$$= 2\frac{dx}{x} \times 100$$

$$= 2(\pm 0.02) \times 100$$

$$= \pm 4\%$$

percentage error in surface area $= \pm 4\%$.

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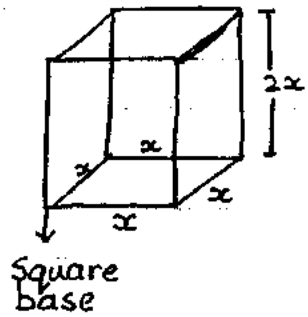
4. A box with square base .... volume of box.

Sol. Let 'x' be width of box.

then height of box $= 2(\text{width}) = 2x$

$x = 8.5$ inches



possible error in width $= dx = \pm 0.3$ inches

Volume of box $= \text{Length} \times \text{width} \times \text{height}$

$$= (x) \times (x) \times (2x)$$

$$V = 2x^3$$

we find error in volume i.e dV.

$$dV = 2 \cdot 3x^2\,dx = 6x^2\,dx$$

$$dV = 6(8.5)^2(\pm 0.3)$$

$$= \pm 130.05\ (\text{inches})^3$$

So error in volume of box is $= 130.05\ (\text{inches})^3$.

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5. Radius 'x' of the circle increases ....
.... %age change in area.

Sol. Let x be the radius of circle

$x = 10$, $\quad x + \Delta x = 10.1$

$$\Delta x = 10.1 - x$$

$$\Delta x = 10.1 - 10$$

$$dx = \Delta x = 0.1$$

$\because A = \pi r^2$

$A = \pi x^2$

$dA = 2\pi x dx$

Area of circle $= A = \pi x^2$

we find percentage change in area

$$\%\text{age change} = \frac{dA}{A} \times 100$$

$$= \frac{2\pi x dx}{\pi x^2} \times 100$$

$$= 2\frac{dx}{x} \times 100$$

$$= 2 \cdot \frac{0.1}{10} \times 100 = 2\%$$

%age change in area $= 2\%$

6. The diameter of plant .... plant change.

Sol. Let 'r' be radius of plant

(i) Let 'C' be circumference of plant then $C = 2\pi r$

Increase in Circumference $= dC = 2$ inches

$$C = 2\pi r$$

$$dC = 2\pi dr$$

$$2 = 2\pi dr \Rightarrow dr = \frac{1}{\pi}$$

increase in radius $= dr = \frac{1}{\pi}$

$\Rightarrow$ increase in diameter $= 2dr = \frac{2}{\pi}$

(ii) Area of cross-section of plant $= A = \pi r^2$

$$A = \pi r^2$$

$$dA = 2\pi r dr$$

$$dA = 2\pi(4)\left(\frac{1}{\pi}\right)$$

$$dA = 8$$

diameter of plant $= 8$ inch

radius of plant $= \frac{\text{diametr}}{2}$

$= \frac{8}{2}$

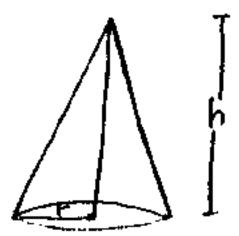
radius $= r = 4$ inches

$\Rightarrow$ change in area $= 8$ inches^2.

7. Sand pouring from a chute .... increase by $2cm^3$.

Sol.

Let 'r' be radius ; 'h' be heigth of conincal pile.



$r = 10cm$

given condition

altitude $=$ radius.

$\Rightarrow h = r$

Let V be voluеme then $dV = 2cm^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of conical pile $= V = \frac{1}{3}\pi r^2 (r)$

$$V = \frac{1}{3}\pi r^3$$

$$dV = \frac{1}{3}\pi(3r^2\,dr)$$

$$dV = \pi r^2 dr$$

5

put values;

$$2 = \pi(10)^2\,dr = 100\pi\,dr$$

$$dr = \frac{2}{100\pi}$$

$$dr = \frac{1}{50\pi}$$

change in radius $= \frac{1}{50\pi}$ cm

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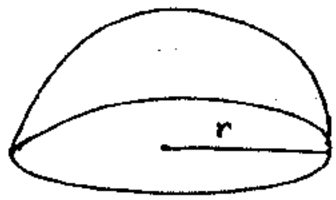
8. A dome is in the shape .... paint required.

Sol. Let radius of dome $= r = 60$ feet

Let V be volume of dome.

$\because$ Volume of hemisphere $= \frac{2}{3}\pi r^3$



So Volume of dome $= \frac{2}{3}\pi r^3$

we find dV, where $dr = 0.01$ inch

$dr = \frac{0.01}{12}$ feet

$\Rightarrow dr = \frac{1}{1200}$ feet

$$V = \frac{2}{3}\pi r^3$$

$$dV = \frac{2}{3}\pi(3r^2 dr)$$

$$dV = 2\pi r^2 dr$$

$$dV = \cancel{2}\pi(60)^2 \cdot \frac{1}{\cancel{1200}\,600}$$

$$= 6\cancel{0} \times 60\ \pi \cdot \frac{1}{60\cancel{0}}$$

$$dV = 6\pi\ ft^3.$$

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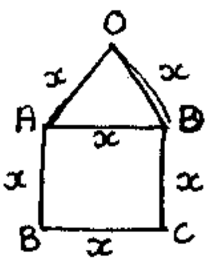
9. The side of building .... area of side.

Sol:

Let x be the length of side of base.



Area of OABCD = Area of triangle OAD + Area of Square ABCD

Area of Square ABCD = length × width

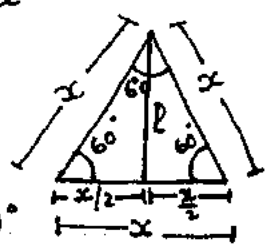
$= x^2$

Area of $\Delta OAD = \frac{1}{2}$(length × width)

$= \frac{1}{2}(l \times w)$

$= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} x \cdot x$

Area of $\Delta OAD = \frac{\sqrt{3}}{4}x^2$



$\sin 60° = \frac{l}{x}$

$l = x \sin 60°$

$l = \frac{\sqrt{3}}{2}x$

á of OABCD $= x^2 + \frac{\sqrt{3}}{4}x^2$

$A = x^2\left[1+\frac{\sqrt{3}}{4}\right]$

$dA = 2x\,dx\left[1+\frac{\sqrt{3}}{4}\right]$

$= 2x\cdot\frac{1}{100}x\left[1+\frac{\sqrt{3}}{4}\right]$

$= \frac{1}{50}x^2\left[1+\frac{\sqrt{3}}{4}\right]$

length of side of base $= x = 15$

error in length $= 1\%$

$\frac{dx}{x}\times 100 = 1$

$dx = \frac{1}{100}x$

%age error in area $= \frac{dA}{A}\times 100$

$$= \frac{\frac{1}{50}\cancel{x^2}\cancel{\left(1+\frac{\sqrt{3}}{4}\right)}}{\cancel{x^2}\cancel{\left[1+\frac{\sqrt{3}}{4}\right]}}\times 100$$

$$= \frac{100}{50}$$

%age error in area $= 2\%$

10. A boy makes paper ………………… capacity of cup.

Sol.

Let r be radius of base

" h be height of cup.

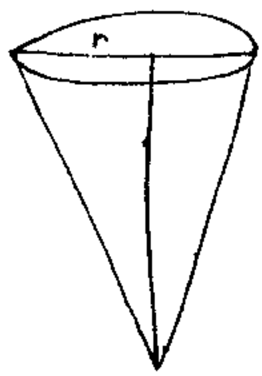
$h = 4r$

$r = 2\text{cm}$ $\qquad r+\Delta r = 1.5$

$\Delta r = 1.5 - r$

$= 1.5 - 2$

$dr = \Delta r = -0.5\text{ cm}$



Volume of cone $= \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2(4r) = \frac{4}{3}\pi r^3$

$dV = \frac{4}{3}\pi.\,3r^2dr$

$dV = 4\pi r^2 dr$

$dV = 4\pi(2)^2(-0.5)$

$dV = -8\pi$

decrease in capacity of cup $= dV = -8\pi\text{ cm}^3$.

11. To estimate the height ………………… height so found.

Sol.

Let 'x' be the height of Minar-e-pakistan.

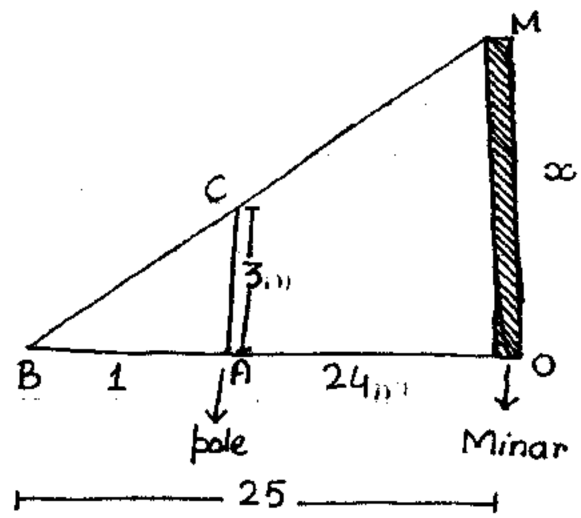
then from figure;

$$\frac{OM}{OB} = \frac{AC}{AB}$$

$$\Rightarrow \quad \frac{x}{25} = \frac{3}{1}$$

$$\Rightarrow \quad x = 75$$

height of minar-e-Pakistan $= 75$m.



(ii) if 'y' is actual length of shadow.

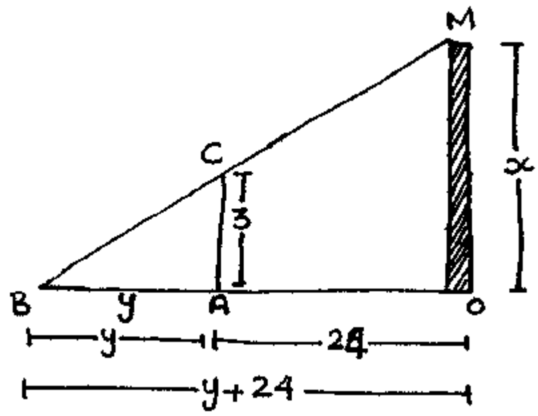
from figure;

$$\frac{OM}{OB} = \frac{AC}{AB}$$

$$\frac{x}{y+24} = \frac{3}{y}$$

$$xy = 3(y+24)$$

$$xy = 3y + 72$$



differentiate;

$$x\,dy + y\,dx = 3dy + 0$$

$$y\,dx = 3dy - x\,dy = (3-x)dy$$

$$\frac{dy}{y} = \frac{dx}{(3-x)} \longrightarrow (1)$$

percentage error in length of shadow $= 1\%$

$$\frac{dy}{y} \times 100 = \pm 1\%$$

$$\frac{dy}{y} = \frac{1}{100} = \pm 0.01$$

$$(1) \Rightarrow \quad 0.01 = \frac{dx}{3-x} \Rightarrow \quad dx = \pm 0.01\,(3-x)$$

we find %age error in height of minar.

$$\%\text{age error} = \frac{dx}{x} \times 100$$

$$= \frac{\pm 0.01\,(3-x)}{75} \times 100$$

$$= \pm \frac{0.01\,(3-75)}{75} \times 100 = \pm \frac{1(-72)}{75} = \pm 0.96\%$$

%age error in height of minar-e-Pakistan $= \pm 0.96\%$

. Oil spilled from tanker .... 40 ft?

A: Let 'r' be radius of circle. $r = 40$ ft
rate of change of raduis $= \frac{dr}{dt} = 2$ ft/sec
we find rate of change of area; $\frac{dA}{dt} = ?$

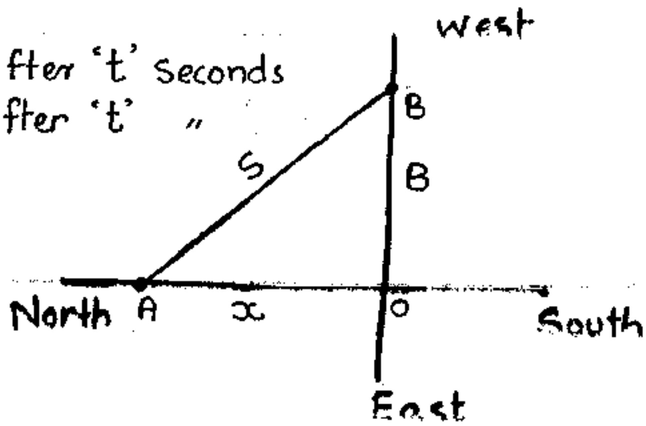
$$A = \pi r^2$$
$$\frac{dA}{dt} = \pi\left(2r\frac{dr}{dt}\right)$$
$$\frac{dA}{dt} = 2\pi\,(40)(2)$$
$$= 160\pi \text{ ft}^2/\text{sec}$$

area of circle increases at rate 160π ft²/Sec.

13. From a point 'O' .... after 5sec.

Sol.

Let A be position of car1 after 't' seconds
" B " " " car2 after 't' "



$OA = x = t^2+t$
$OB = y = t^2+3t$

By Pathagorus theorem (from fig)
$$S^2 = x^2+y^2$$
$$S^2 = (t^2+t)^2+(t^2+3t)$$
at $t = 5$ sec
$$S^2 = (5^2+5)^2+(5^2+3(5)) = (30)^2+(25+15)^2$$
$$S^2 = 30^2+40^2 = 2500$$
$$S = 50$$

$$S^2 = x^2+y^2 \rightarrow (1)$$

we find rate of change of distance at 5sec. i.e $\left.\frac{ds}{dt}\right|_{t=5}$

diff (1) w.r.t 't'

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$x = t^2+t$; $y = t^2+3t$
$\frac{dx}{dt} = 2t+1$; $\frac{dy}{dt} = 2t+3$

$$2(S)\frac{ds}{dt} = 2\left[\left(x\frac{dx}{dt}\right)+y\frac{dy}{dt}\right]$$
$$\cancel{2}S\frac{ds}{dt} = \cancel{2}\left[(t^2+t)(2t+1)+(t^2+3t)(2t+3)\right]$$
at $t=5$
$$(50)\left.\frac{ds}{dt}\right|_{t=5} = \left[(5^2+5)(2.5+1)+(5^2+3.5)(2.5+3)\right]$$

$$50 \cdot \left.\frac{ds}{dt}\right|_{t=5} = 1\left[(30)(11) + (40)(13)\right]$$

$$\left.\frac{ds}{dt}\right|_{t=5} = \frac{1}{50}\left[330 + 520\right] = \frac{\overset{17}{\cancel{850}}}{\cancel{50}}$$

$$\left.\frac{ds}{dt}\right|_{t=5} = 17$$

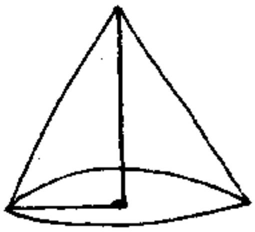
distance between cars changes at the rate of $17\ ft/sec^2$.

14. Sand falls from .... pile is 5ft high.

Sol.

Let 'r' be radius of pile.

" h " height " " $= h = 5ft$



$$h = 2r$$

$$\Rightarrow r = \frac{h}{2}$$

$$\text{volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\text{volume of pile} = V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{12}\pi h^3 \rightarrow (1)$$

$\because$ Sand falls from container at rate $= \frac{dV}{dt} = 10ft^3/min$

we find ~~in~~ rate of increase in height $= \frac{dh}{dt} = ?$

diff. (1) w.r.t 't'.

$$\frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2\frac{dh}{dt} = \frac{1}{4}\pi h^2\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt}$$

$$\frac{dh}{dt} = \frac{4}{\pi(5)^2}\cdot(10)$$

$$\frac{dh}{dt} = \frac{\overset{2}{\cancel{10}}\times 4}{\underset{5}{\cancel{25}}\pi} = \frac{8}{5\pi} = 0.509\ ft/min$$

height of pile is changing at rate of $0.51ft/min$

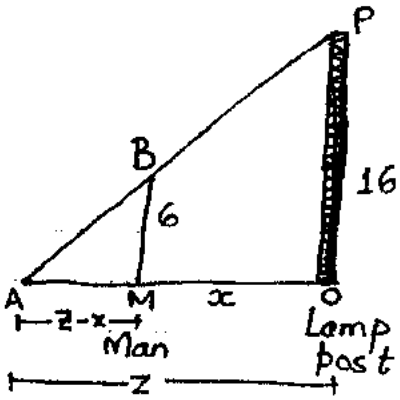
15. A 6ft tall man .... shadow changing.

Sol.

Let distance of man from lamp post $= OM = x$

Let distance of tip of shadow from $O = Z$

from triangles,



$$\frac{OP}{OA} = \frac{BM}{AM}$$

$$\frac{16}{z} = \frac{6}{z-x}$$

$$16z - 16x = 6z$$

$$16z - 6z - 16x = 0 \Rightarrow 10z - 16x = 0$$

$$2(5z - 8x) = 0 \Rightarrow 5z - 8x = 0$$

$$5z = 8x$$

Diff. w.r.t. 't'

$$5\frac{dz}{dt} = 8\frac{dx}{dt} \qquad \because \frac{dx}{dt} = 5 \quad (\text{Speed})$$

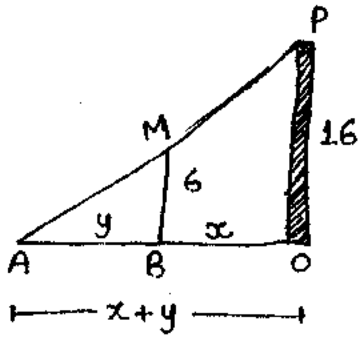
$$5\left(\frac{dz}{dt}\right) = 8(5) = 40$$

$$\frac{dz}{dt} = \frac{40}{5} \Rightarrow \boxed{\frac{dz}{dt} = 8}$$

Tip of man's shadow moves at the rate of 8ft/sec.

if 'y' is length of shadow

AB = y



from fig;

$$\frac{OP}{OA} = \frac{BM}{AB}$$

$$\frac{16}{x+y} = \frac{6}{y}$$

$$16y = \cancel{1}6x + \cancel{1}6y$$

$$\Rightarrow 16y - 6x - 6y = 0$$

$$\Rightarrow 10y - 6x = 0 \Rightarrow 2(5y - 3x) = 0$$

$$\Rightarrow 5y - 3x = 0$$

differentating w.r.t 'x'

$$5\frac{dy}{dt} - 3\frac{dx}{dt} = 0$$

$$5\frac{dy}{dt} - 3(5) = 0 \Rightarrow 5\frac{dy}{dt} = 15$$

$$\frac{dy}{dt} = \frac{15}{5}$$

$$\frac{dy}{dt} = 3$$

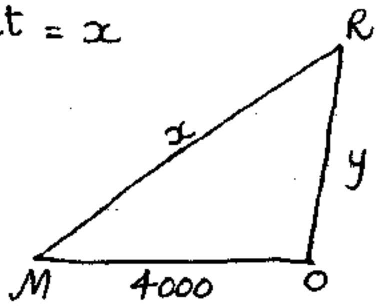
Shadow is changing at the rate of 3 ft/sec

...At a distance of .... at this instant?

Sol: Let 'y' be the altitude of rocket $= 3000ft$

Let Distance b/w man and rocket $= x$

By pathagourus theorem,



$$x^2 = y^2 + 4000^2 \longrightarrow (1)$$

$$x^2 = 3000^2 + 4000^2$$

$$x^2 = 9000000 + 16000000$$

$$x^2 = 25000000$$

$$x = \sqrt{25000000}$$

$$x = 5000$$

diff. (1) w.r.t 't'

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt} + 100$$

$$x\frac{dx}{dt} = y\frac{dy}{dt}$$

$\because y = 3000$

$\frac{dy}{dt} = 600$ (speed)

$$5000\frac{dx}{dt} = (600)(3000)$$

$$\frac{dx}{dt} = \frac{(\cancel{600})(3000)}{\cancel{8000}} \quad 120$$

$$\frac{dx}{dt} = 360$$

distance b/w rocket and man is changing at rate of 360 ft/sec.

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17 A aeroplane flying horizontally at an altitude of .... increasing after 30 seconds.

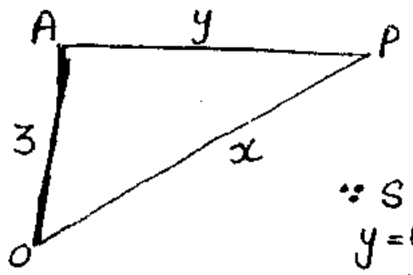
Sol: Let O be the observer on ground.

Let P be the plane.

Let $OP = x$

$AP = y$

Altitude $= OA = 3$ miles.



$\because S = vt$

$y = (V)(t)$

$= \frac{\cancel{480} \times \cancel{30}}{\cancel{3600}}$ (4, 12)

By pathagourus theorem

$$x^2 = 3^2 + y^2$$

$$x^2 = 3^2 + 4^2 = 9 + 16 = 25 \qquad \because y = 4, \ t = 30 \qquad \boxed{y = 4}$$

$$x = 5$$

diff. w.r.t 't' ; $\quad 2x\frac{dx}{dt} = 0 + 2y\frac{dy}{dt}$

$$x\,dx/dt = y\,dt/dt$$

rate of change of distance from observer to plane $= \frac{dx}{dt} = ?$

$$\frac{dy}{dt} = 480$$

$$x\frac{dx}{dt} = y\frac{dy}{dt}$$

$$(5)\frac{dx}{dt} = (4)(480)$$

$$\frac{dx}{dt} = \frac{\cancel{1920}}{\cancel{5}}\ 384$$

$$\frac{dx}{dt} = 384 \text{ miles/hr}$$

---

18. *A boy flies kite .... released in 70m.*

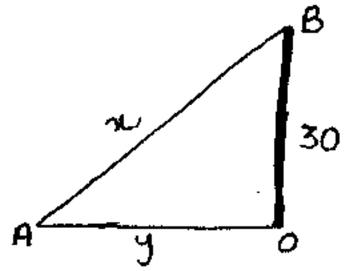
Let 'x' be the length of string.

altitude $= OB = 30m$

$OA = y$

$\frac{dy}{dt} = 2$ m/sec

$\frac{dx}{dt} = ?$



from figure

$$x^2 = y^2 + 30^2$$

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt} = 0$$

$$x\frac{dx}{dt} = y\frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{y}{x}\cdot\frac{dy}{dt}$$

$$= \frac{2\cancel{0}\sqrt{10}}{7\cancel{0}} \times (2)$$

$$\frac{dx}{dt} = \frac{4\sqrt{10}}{7}$$

$x = 70m$

$x^2 = y^2 + 30^2$

$y^2 = x^2 - 30^2$

$y^2 = 70^2 - 30^2 = 4900 - 900$

$y^2 = 4000 = 400(10)$

$y^2 = 20^2(10)$

$y = 20\sqrt{10}$

Thus string is being let out at the rate of $\frac{4\sqrt{10}}{7}$ m/sec.

---

19. A water tank .... is half way up?

Sol.

Let $BO = x$

height of frustum of cone = 6

So height of cone = AO = AB + OB = $x+6$

Let CP be the water level

and $CP = r$

Let $BC = y$

from $\triangle AOQ$ and $\triangle BOR$

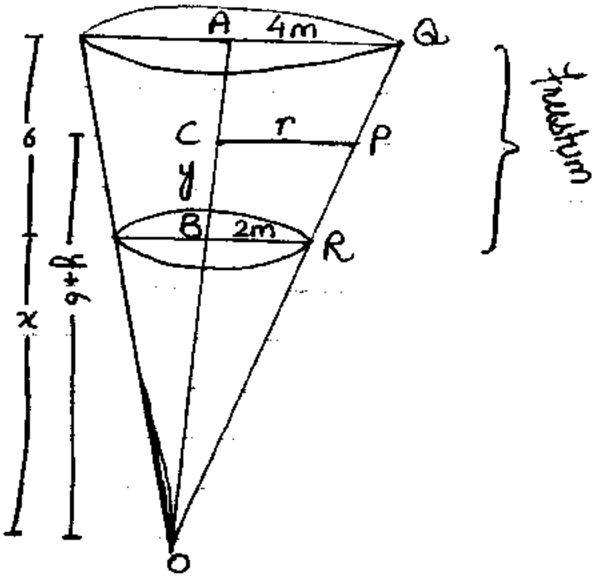
$$\frac{AQ}{AO} = \frac{BR}{OB}$$

$$\frac{4}{x+6} = \frac{2}{x}$$

$$4x = 2x + 12$$

$$\Rightarrow 4x - 2x = 12 \Rightarrow 2x = 12$$

$$\Rightarrow \boxed{x = 6}$$



from $\triangle BOR$ and COP.

$$\frac{BR}{OB} = \frac{CP}{CO}$$

$$\frac{2}{x} = \frac{r}{y+x}$$

$$\frac{\cancel{2}}{{}_3\cancel{6}} = \frac{r}{y+6}$$

$$r = \frac{y+6}{3}$$

Volume of frustum.

$$V = \frac{1}{3}\pi r^2 (y+6) - \frac{1}{3}\pi (2)^2 (6)$$

$$= \frac{1}{3}\pi \left(\frac{y+6}{3}\right)^2 (y+6) - \frac{1}{\cancel{3}}\pi\, 4 \times \cancel{6}^{2}$$

$$= \frac{1}{3\times 9}\pi (y+6)^3 - 8\pi$$

$$V = \frac{1}{27}\pi (y+6)^3 - 8\pi$$

$$\because \frac{dV}{dt} = 20 \text{ m}^3/\text{min}$$

$$\frac{dV}{dt} = \frac{1}{\cancel{27}_9}\pi \cdot \cancel{3}(y+6)^2 \left(\frac{dy}{dt}\right) - 0$$

$$20 = \frac{1}{9}\pi (y+6)^2 \frac{dy}{dt} \quad \rightarrow (1)$$

$\because$ water is half way up $\Rightarrow y = \frac{1}{2}$ (height of frustum)

$$y = \frac{1}{2}(6) = 3$$

$$y = 3\text{m}$$

(1) $\Rightarrow$

$$20 = \frac{1}{9}\pi (3+6)^2 \frac{dy}{dt}$$

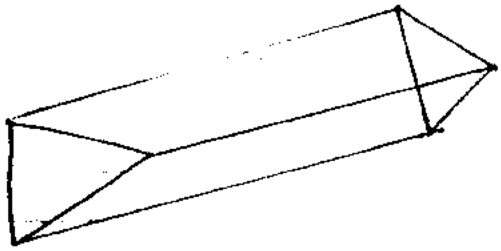
$$20 = \frac{1}{\cancel{9}}\pi \cdot 9^{\cancel{2}} \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{20}{9\pi} \text{ m/min}$$

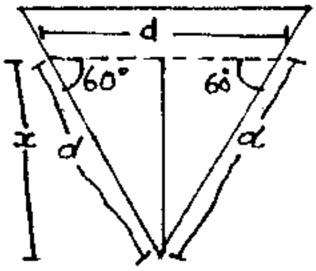
A 20m long water.................. water is $1\frac{1}{2}$ m?

Sol.

depth of water = x



Volume of water = Length of trough $\times$ Area of triangle



cross-sectional area:

Area of triangle = $\frac{1}{2}$ width $\times$ height

$$= \frac{1}{2}\left(\frac{2x}{\sqrt{3}}\right) \times x$$

$$= \frac{x^2}{\sqrt{3}}$$

$$\sin 60 = \frac{x}{d}$$

$$d = \frac{x}{\sin 60} = \frac{x}{\sqrt{3}/2}$$

$$d = \frac{2x}{\sqrt{3}}$$

Volume $= 12 \times \frac{x^2}{\sqrt{3}}$

$$V = \frac{12}{\sqrt{3}}x^2$$

$$\frac{dV}{dt} = \frac{12}{\sqrt{3}} \cdot 2x\frac{dx}{dt}$$

$$\frac{dV}{dt} = \frac{24}{\sqrt{3}} \cdot x\frac{dx}{dt}$$

Water level $= x = \frac{3}{2}$, $\frac{dV}{dt} = 4$

$$4 = \frac{\cancel{24}^{12}}{\sqrt{3}}\left(\frac{3}{\cancel{2}}\right)\frac{dx}{dt}$$

$$\frac{\cancel{4}\sqrt{3}}{\cancel{36}\,9} = \frac{dx}{dt}$$

$$\frac{\sqrt{3}}{9} = \frac{dx}{dt}$$

$$\frac{\sqrt{3}}{3 \times 3} = \frac{dx}{dt}$$

$$\frac{\cancel{\sqrt{3}}}{3 \cdot \sqrt{3} \times \cancel{\sqrt{3}}} = \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{1}{3\sqrt{3}}$$

water level is rising at the rate of $\frac{1}{3\sqrt{3}}$ m/m