Exercise 2.2

General Maths ①
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Differentiate w.r.t x.

1 $y = \sqrt{a^2+x^2}$

$y = (a^2+x^2)^{1/2}$ differentiat w.r.t 'x'

$\frac{dy}{dx} = \frac{d}{dx}(a^2+x^2)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(a^2+x^2)^{\frac{1}{2}-1}\frac{d}{dx}(a^2+x^2)$

$= \frac{1}{2}(a^2+x^2)^{-1/2}(0+2x)$

$= \frac{1}{\cancel{2}}(\cancel{2}x).(a^2+x^2)^{-1/2}$

$= \frac{x}{(a^2+x^2)^{1/2}}$

$\frac{dy}{dx} = \frac{x}{\sqrt{a^2+x^2}}$

2. $y = \sqrt[3]{x^2+x+1}$

diff. w.r.t 'x'

$\frac{dy}{dx} = \frac{d}{dx}(x^2+x+1)^{1/3}$

$= \frac{1}{2}(x^2+x+1)^{\frac{1}{3}-1}\frac{d}{dx}(x^2+x+1)$

$= \frac{1}{3}(x^2+x+1)^{-2/3}(2x+1+0)$

$= \frac{1}{3(x^2+x+1)^{2/3}}(2x+1)$

$\frac{dy}{dx} = \frac{2x+1}{3(x^2+x+1)^{2/3}}$

3. $Y = \frac{\sqrt{a+x} - \sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$

$y = \frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}} \times \frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}$

$= \frac{(\sqrt{a+x})^2+(\sqrt{a-x})^2-2\sqrt{a+x}\sqrt{a-x}}{(\sqrt{a+x})^2-(\sqrt{a-x})^2}$

$= \frac{a+\cancel{x}+a-\cancel{x}-2\sqrt{(a+x)(a-x)}}{a+x-(a-x)}$

$= \frac{2a-2\sqrt{a^2-x^2}}{\cancel{a}+x-\cancel{a}+x}$

$y = \frac{2(a-\sqrt{a^2-x^2})}{2x}$

$y = \frac{a-\sqrt{a^2-x^2}}{x}$

differentiate w.r.t 'x'.

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a-\sqrt{a^2-x^2}}{x}\right)$

$= \frac{x\frac{d}{dx}(a-\sqrt{a^2-x^2}) - (a-\sqrt{a^2-x^2})\frac{d}{dx}(x)}{x^2}$

$\frac{dy}{dx} = \frac{x\left(0-\frac{1}{\cancel{2}}(a^2-x^2)^{-1/2}(-\cancel{2}x)\right)-(a-\sqrt{a^2-x^2})}{x^2}$

$\frac{dy}{dx} = \frac{x\left(\frac{1}{\sqrt{a^2-x^2}}\right)-a+\sqrt{a^2-x^2}}{x^2}$

$\frac{dy}{dx} = \frac{\frac{\cancel{x^2}-a\sqrt{a^2-x^2}+a^2-\cancel{x^2}}{\sqrt{a^2-x^2}}}{x^2}$

$\frac{dy}{dx} = \frac{a^2-a\sqrt{a^2-x^2}}{x^2\sqrt{a^2-x^2}}$

4. $Y = \frac{\sqrt{\sin x}}{\sin\sqrt{x}}$

differentiate w.r.t 'x'

$\frac{dy}{dx} = \frac{\sin\sqrt{x}\left(\frac{d}{dx}(\sin x)^{1/2}\right)-\sqrt{\sin x}\frac{d}{dx}(\sin\sqrt{x})}{(\sin\sqrt{x})^2}$

$= \frac{1}{(\sin\sqrt{x})^2}\left[\sin\sqrt{x}.\frac{1}{2\sqrt{\sin x}}\frac{d}{dx}(\sin x)-\sqrt{\sin x}.\cos\sqrt{x}\frac{d}{dx}\sqrt{x}\right]$

$= \frac{1}{\sin^2\sqrt{x}}\left[\frac{\sin\sqrt{x}}{2\sqrt{\sin x}}\cos x - \sqrt{\sin x}\cos\sqrt{x}.\frac{1}{2\sqrt{x}}\right]$

$= \frac{1}{\sin^2\sqrt{x}}\left[\frac{\sqrt{x}\sin\sqrt{x}\cos x-\sin x\cos\sqrt{x}}{2\sqrt{x}\sqrt{\sin x}}\right]$

$\frac{dy}{dx} = \frac{\sqrt{x}\sin\sqrt{x}\cos x-\sin x\cos\sqrt{x}}{2\sqrt{x}\sqrt{\sin x}\sin^2\sqrt{x}}$

$$y = \sqrt{\log_{10}(x^2+1)}$$

$$y = \sqrt{\frac{\log_e(x^2+1)}{\log_e 10}} \qquad \because \frac{\log_a x}{\log_a y} = \log_y x$$

$$y = \sqrt{\frac{\ln(x^2+1)}{\ln 10}} = \frac{1}{\sqrt{\ln 10}}\sqrt{\ln(x^2+1)}$$

differentiate w.r.t 'x'.

$$\frac{dy}{dx} = \frac{1}{\sqrt{\ln 10}}\frac{d}{dx}\sqrt{\ln(x^2+1)}$$

$$= \frac{1}{\sqrt{\ln 10}} \cdot \frac{1}{2\sqrt{\ln(x^2+1)}}\frac{d}{dx}\left(\ln(x^2+1)\right)$$

$$= \frac{1}{2\sqrt{\ln 10}\sqrt{\ln(x^2+1)}}\frac{1}{(x^2+1)}\frac{d}{dx}(x^2+1)$$

$$= \frac{1}{2(x^2+1)\sqrt{\ln 10}\sqrt{\ln(x^2+1)}} \cdot (2x+0)$$

$$\frac{dy}{dx} = \frac{2x}{2(x^2+1)\sqrt{\ln 10}\sqrt{\ln(x^2+1)}}$$

$$\frac{dy}{dx} = \frac{x}{(x^2+1)\sqrt{\ln 10}\sqrt{\ln(x^2+1)}}$$

6. $y = \tan(\sin x)$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan(\sin x)\right)$$

$$= \sec^2(\sin x)\frac{d}{dx}(\sin x)$$

$$= \sec^2(\sin x) \times \cos x$$

$$\frac{dy}{dx} = \cos x . \sec^2(\sin x)$$

7. $y = \tan^{-1}\left(\frac{x\sin\alpha}{1-x\cos\alpha}\right)$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}\left(\frac{x\sin\alpha}{1-x\cos\alpha}\right)\right)$$

$$= \frac{1}{1+\left(\frac{x\sin\alpha}{1-x\cos\alpha}\right)^2}\frac{d}{dx}\left(\frac{x\sin\alpha}{1-x\cos\alpha}\right)$$

$$= \frac{\sin\alpha}{\frac{(1-x\cos\alpha)^2+(x\sin\alpha)^2}{(1-x\cos\alpha)^2}}\frac{d}{dx}\left(\frac{x}{1-x\cos\alpha}\right)$$

$$\frac{dy}{dx} = \frac{\sin\alpha(1-x\cos\alpha)^2}{1+x^2\cos^2\alpha-2x\cos\alpha+x^2\sin^2\alpha}\frac{d}{dx}\frac{x}{1-x\cos\alpha}$$

$$= \frac{\sin\alpha\,\cancel{(1-x\cos\alpha)^2}}{1+x^2(\cos^2\alpha+\sin^2\alpha)-2x\cos\alpha} \cdot \frac{(1-x\cos\alpha)(1)-x(0-\cos\alpha)}{\cancel{(1-x\cos\alpha)^2}}$$

$$= \frac{\sin\alpha}{1+x^2-2x\cos\alpha} \cdot (1-\cancel{x\cos\alpha}+\cancel{x\cos\alpha})$$

$$\frac{dy}{dx} = \frac{\sin\alpha}{1+x^2-2x\cos\alpha}$$

8. $y = \ln\left(\frac{x^2+x+1}{x^2-x+1}\right)$

$$y = \ln(x^2+x+1) - \ln(x^2-x+1)$$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left[\ln(x^2+x+1)\right] - \frac{d}{dx}\left[\ln(x^2-x+1)\right]$$

$$= \frac{1}{(x^2+x+1)}\frac{d}{dx}(x^2+x+1) - \frac{1}{(x^2-x+1)}\frac{d}{dx}(x^2-x+1)$$

$$\frac{dy}{dx} = \frac{2x+1}{x^2+x+1} - \frac{2x-1}{x^2-x+1}$$

$$\frac{dy}{dx} = \frac{(2x+1)(x^2-x+1)-(2x-1)(x^2+x+1)}{(x^2+x+1)(x^2-x+1)}$$

$$= \frac{\cancel{2x^3}-2x^2+\cancel{2x}+x^2-\cancel{x}+1-\cancel{2x^3}-2x^2-\cancel{2x}+x^2+\cancel{x}+1}{((x^2+1)+x)((x^2+1)-x)}$$

$$= \frac{-4x^2+2x^2+2}{(x^2+1+x)(x^2+1-x)}$$

$$= \frac{-2x^2+2}{(x^2+1+x)(x^2+1-x)}$$

$$\frac{dy}{dx} = \frac{2(1-x^2)}{(x^2+1-x)(x^2+1+x)}$$

9. $y = x^{x^2}$

$$\ln y = \ln x^{x^2}$$

$$\ln y = x^2\ln x$$

differentiate w.r.t 'x'

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x^2\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = x^2\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x^2)$$

$$\frac{d}{dx} = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$$

$$\frac{1}{y}\frac{dy}{dx} = x + 2x\ln x$$

$$\frac{dy}{dx} = y(x + 2x\ln x)$$

$$\frac{dy}{dx} = x^{x^2}(x + 2x\ln x)$$

$$\frac{dy}{dx} = x^{x^2}.x\ (1 + 2\ln x)$$

$$\frac{dy}{dx} = x^{x^2+1}(1 + 2\ln x)$$

10. $y = \ln(x^2 + x)$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\ln(x^2+x)\right]$$

$$\frac{dy}{dx} = \frac{1}{x^2+x}\ \frac{d}{dx}\ (x^2+x)$$

$$\frac{dy}{dx} = \frac{1}{x(x+1)}\quad(2x+1)$$

$$\frac{dy}{dx} = \frac{2x+1}{x(x+1)}$$

11. $y = (\sin^{-1}x)^{x^{1/x}}$

$$\ln y = \ln(\sin^{-1}x)^{x^{1/x}}$$

$$\ln y = x^{1/x}\ln(\sin^{-1}x)$$

differentiate w.r.t 'x'.

$$\frac{1}{y}\frac{dy}{dx} = x^{1/x}\frac{d}{dx}\left[\ln(\sin^{-1}x)\right] + \ln(\sin^{-1}x)\frac{d}{dx}x^{1/x}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{x^{1/x}}{\sin^{-1}x}\frac{d}{dx}\left[\sin^{-1}x\right] + \ln(\sin^{-1}x)\frac{d}{dx}(x)^{1/x}$$

$$= \frac{x^{1/x}}{\sin^{-1}x.\sqrt{1-x^2}} + \ln(\sin^{-1}x)\frac{d}{dx}\left[x^{1/x}\right] \quad (1)$$

Let $u = x^{1/x}$

$$\frac{du}{dx} = \frac{d}{dx}(x^{1/x}) \qquad \ln u = \ln(x)^{1/x}$$

$$\ln u = \frac{1}{x}\ln(x)$$

$$\frac{1}{u}\cdot\frac{du}{dx} = \frac{1}{x}\cdot\frac{d(\ln x)}{dx} + \ln(x)\left(\frac{-1}{x^2}\right)$$

$$\frac{1}{u}\frac{du}{dx} = \left(\frac{1}{x}\right)\left(\frac{1}{x}\right) - \frac{1}{x^2}\ln x$$

$$\frac{du}{dx} = u\left[\frac{1}{x^2} - \frac{1}{x^2}\ln x\right]$$

$$\frac{du}{dx} = x^{1/x}\cdot\frac{1}{x^2}\left[1-\ln x\right] \quad (3)$$

$$\frac{du}{dx} = x^{\frac{1}{x}-2}(1-\ln x)$$

put in (1)

$$\frac{1}{y}\frac{dy}{dx} = \frac{x^{1/x}}{\sin^{-1}x\sqrt{1-x^2}} + \ln(\sin^{-1}x)\,x^{\frac{1}{x}-2}(1-\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{x^{1/x}}{\sin^{-1}x\sqrt{1-x^2}} + \ln(\sin^{-1}x)x^{\frac{1}{x}-2}(\ln e - \ln x)$$

$$= \frac{x^{1/x}}{\sin^{-1}x\sqrt{1-x^2}} + \ln(\sin^{-1}x)x^{\frac{1}{x}-2}\ln\left(\frac{e}{x}\right)$$

$$\frac{dy}{dx} = y\left[\frac{x^{1/x}}{\sin^{-1}x\sqrt{1-x^2}} + x^{1/x-2}\ln\left(\frac{e}{x}\right)\ln(\sin^{-1}x)\right]$$

$$\frac{dy}{dx} = (\sin^{-1}x)^{x^{1/x}}\left[\frac{x^{1/x}}{\sin^{-1}x\sqrt{1-x^2}} + x^{\frac{1}{x}-2}\ln\left(\frac{e}{x}\right)\ln(\sin^{-1}x)\right]$$

13. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x+\sqrt{x+\sqrt{x}}}}\ \frac{d}{dx}\left(x+\sqrt{x+\sqrt{x}}\right)$$

$$= \frac{1}{\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1 + \frac{d}{dx}\sqrt{x+\sqrt{x}}\right)$$

$$= \frac{1}{\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1 + \frac{1}{\sqrt{x+\sqrt{x}}}\frac{d}{dx}(x+\sqrt{x})\right)$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1 + \frac{1}{2\sqrt{x+\sqrt{x}}}\left(1 + \frac{d}{dx}\sqrt{x}\right)\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1 + \frac{1}{2\sqrt{x+\sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)\right)$$

12. $y = |x^2 - 9|$

$$y = \begin{cases} x^2 - 9 & |x| \geq 3 \\ -(x^2-9) & |x| < 3 \end{cases}$$

$$\frac{dy}{dx} = 2x \qquad |x| \geq 3$$

$$\frac{dy}{dx} = -2x \qquad |x| < 3$$

14. $y=(x+|x|)^{1/2}$

$$y=\begin{cases}(x+x)^{1/2} & x>0\\ (x-x)^{1/2} & x<0\\ (0-0)^{1/2} & x=0\end{cases}$$

$$y=\begin{cases}(2x)^{1/2} & x>0\\ 0 & x\le 0\end{cases}$$

$$\frac{dy}{dx}=\begin{cases}\frac{1}{\cancel{2}}(2x)^{-1/2}(\cancel{2}) & x>0\\ 0 & x\le 0\end{cases}$$

$$\frac{dy}{dx}=\begin{cases}\frac{1}{\sqrt{2x}} & x>0\\ 0 & x\le 0\end{cases}$$

15. differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ w.r.t $\cos^{-1}x^2$.

Let $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$

$u=\cos^{-1}x^2$ we find $\frac{dy}{du}$

$x^2=\cos u$

$$y=\tan^{-1}\left(\frac{\sqrt{1+\cos u}-\sqrt{1-\cos u}}{\sqrt{1+\cos u}+\sqrt{1-\cos u}}\right)$$

$$y=\tan^{-1}\left(\frac{\sqrt{2\cos^2\frac{u}{2}}-\sqrt{2\sin^2\frac{u}{2}}}{\sqrt{2\cos^2\frac{u}{2}}+\sqrt{2\sin^2\frac{u}{2}}}\right)$$

$$y=\tan^{-1}\left(\frac{\sqrt{2}\left(\sqrt{\cos^2\frac{u}{2}}-\sqrt{\sin^2\frac{u}{2}}\right)}{\sqrt{2}\left(\sqrt{\cos^2\frac{u}{2}}+\sqrt{\sin^2\frac{u}{2}}\right)}\right)$$

$$y=\tan^{-1}\left(\frac{\cos\frac{u}{2}-\sin\frac{u}{2}}{\cos\frac{u}{2}+\sin\frac{u}{2}}\right)$$

$$y=\tan^{-1}\left(\frac{\frac{\cos u/2}{\cos u/2}-\frac{\sin u/2}{\cos u/2}}{\frac{\cos u/2}{\cos u/2}+\frac{\sin u/2}{\cos u/2}}\right)$$

$$y=\tan^{-1}\left(\frac{1-\tan u/2}{1+\tan u/2}\right)$$

$$y=\tan^{-1}\left(\frac{\tan\pi/4-\tan(u/2)}{1+\tan(\frac{\pi}{4})\tan(\frac{u}{2})}\right)$$

$$y=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{u}{2}\right)\right)$$

4

$y=\frac{\pi}{4}-\frac{u}{2}$ differentiate w.r.t 'u'

$$\frac{dy}{du}=0-\frac{1}{2}$$

$$\frac{dy}{du}=-\frac{1}{2}$$

Find $\frac{dy}{dx}$. (Problem 16-20)

16. $y=x^{\sin y}$

taking logarithm on both sides

$\ln y=\ln(x)^{\sin y}$

$\ln y=\sin y\ln x$

diff. w.r.t 'x'

$$\frac{1}{y}\frac{dy}{dx}=\sin y.\frac{d}{dx}[\ln x]+\ln x.\frac{d}{dx}(\sin y)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{\sin y}{x}+\ln x\ \cos y.\frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx}-\cos y\ln x\frac{dy}{dx}=\frac{\sin y}{x}$$

$$\frac{dy}{dx}\left[\frac{1}{y}-\ln x.\cos y\right]=\frac{\sin y}{x}$$

$$\frac{dy}{dx}\left[\frac{1-y\ln x\cos y}{y}\right]=\frac{\sin y}{x}$$

$$\frac{dy}{dx}=\frac{y\sin y}{x(1-y\ln x\cos y)}$$

17. $x^y=e^{x-y}$

$\ln x^y=\ln e^{x-y}$

$y\ln x=(x-y)\ln e$

$y\ln x=x-y$

$y\ln x+y=x$

$y(1+\ln x)=x$

$y=\frac{x}{1+\ln x}$

$$y = \frac{x}{1+\ln x}$$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{(1+\ln x)(1) - (x)(0+\frac{1}{x})}{(1+\ln x)^2}$$

$$= \frac{(1+\ln x) - x\cdot\frac{1}{x}}{(1+\ln x)^2}$$

$$= \frac{1+\ln x - 1}{(1+\ln x)^2}$$

$$\frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$$

18. $y^x + x^y = c \longrightarrow (A)$

Let $u = y^x \rightarrow (1)$ $\quad$ $v = x^y \rightarrow (2)$

Solving (1)

$$\ln u = \ln y^x$$

$$\ln u = x\ln y$$

diff. w.r.t 'x'

$$\frac{1}{u}\frac{du}{dx} = \ln y + \frac{x}{y}\frac{dy}{dx}$$

$$\frac{du}{dx} = u\left[\ln y + \frac{x}{y}\frac{dy}{dx}\right]$$

$$\frac{du}{dx} = y^x\left[\ln y + \frac{x}{y}\frac{dy}{dx}\right]$$

Solving (2)

$$\ln v = \ln x^y$$

$$\ln v = y\ln x.$$

diff. w.r.t x

$$\frac{1}{v}\frac{dv}{dx} = \ln x\frac{dy}{dx} + \frac{y}{x}$$

$$\frac{dv}{dx} = v\left[\ln x\frac{dy}{dx} + \frac{y}{x}\right]$$

$$\frac{dv}{dx} = x^y\left[\ln x\frac{dy}{dx} + \frac{y}{x}\right]$$

(A) $\Rightarrow$

$$y^x + x^y = c$$

$$u + v = c$$

diff. w.r.t x

$$\frac{du}{dx} + \frac{dv}{dx} = 0$$

$$y^x\left[\ln y + \frac{x}{y}\frac{dy}{dx}\right] + x^y\left[\ln x\frac{dy}{dx} + \frac{y}{x}\right] = 0$$

$$y^x\ln y + xy^{x-1}\frac{dy}{dx} + x^y\ln x\,\frac{dy}{dx} + x^{y-1}y = 0$$

$$\frac{dy}{dx}\left(xy^{x-1} + x^y\ln x\right) = -x^{y-1}y - y^x\ln y$$

$$\frac{dy}{dx} = -\frac{y^x\ln y + yx^{y-1}}{xy^{x-1} + x^y\ln y}$$

19. $\dfrac{x+y}{x-y} = x^2+y^2$

differentiate w.r.t 'x'.

$$\frac{(x-y)(1+\frac{dy}{dx}) - (x+y)(1-\frac{dy}{dx})}{(x-y)^2} = 2x + 2y\frac{dy}{dx}$$

put $\frac{dy}{dx} = y'$

$$\frac{(x-y)(1+y') - (x+y)(1-y')}{(x-y)^2} = 2x + 2y\frac{dy}{dx}$$

$$\frac{x+xy'-y-y'y-x+xy'-y+yy'}{(x-y)^2} = 2x+2yy'$$

$$\frac{y'(x-\cancel{y}+x+\cancel{y}) + \cancel{x}-y-\cancel{x}-y}{(x-y)^2} = 2x+2yy'$$

$$\frac{2xy'-2y}{(x-y)^2} = 2(x+yy')$$

$$\cancel{2}(xy'-y) = \cancel{2}(x+yy')(x-y)^2$$

$$xy'-y = (x+yy')(x-y)^2$$

$$xy'-y = x(x-y)^2 + yy'(x-y)^2$$

$$xy' - yy'(x-y)^2 = y + x(x-y)^2$$

$$y'\left[x - y(x-y)^2\right] = y + x(x-y)^2$$

$$y' = \frac{y - x(x-y)^2}{x - y(x-y)^2}$$

20. $x + \sin^{-1}y = xy$

differentiate w.r.t x.

$$1 + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} - x\frac{dy}{dx} = y - 1$$

$$\frac{dy}{dx}\left[\frac{1}{\sqrt{1-y^2}} - x\right] = y - 1$$

$$\frac{dy}{dx}\left[\frac{1-x\sqrt{1-y^2}}{\sqrt{1-y^2}}\right] = y - 1$$

$$\frac{dy}{dx} = \frac{(y-1)\sqrt{1-y^2}}{1-x\sqrt{1-y^2}}$$

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Problem 21-30, find $f'(x)$.

21. $f(x) = x^2\sqrt{2ax-x^2}$

differentiate w.r.t 'x'

$$f'(x) = x^2\frac{d}{dx}(2ax-x^2)^{1/2} + \sqrt{2ax-x^2}\,\frac{d}{dx}(x)^2$$

$$f'(x) = \frac{x^2}{2\sqrt{2ax-x^2}}\frac{d}{dx}(2ax-x^2) + \sqrt{2ax-x^2}\,(2x)$$

$$= \frac{x^2}{2\sqrt{2ax-x^2}}\cdot(2a-2x) + 2x\sqrt{2ax-x^2}$$

$$= \frac{x^2\cdot 2(a-x)}{2\sqrt{2ax-x^2}} + 2x\sqrt{2ax-x^2}$$

$$= \frac{x^2(a-x)}{\sqrt{2ax-x^2}} + 2x\sqrt{2ax-x^2}$$

$$= \frac{x^2(a-x) + 2x(2ax-x^2)}{\sqrt{2ax-x^2}}$$

$$= \frac{ax^2 - x^3 + 4ax^2 - 2x^3}{\sqrt{2ax-x^2}}$$

$$\frac{dy}{dx} = \frac{5ax^2 - 3x^3}{\sqrt{2ax-x^2}}$$

22. $f(x) = \ln\left(\frac{e^x}{1+e^x}\right)$

$f(x) = \ln e^x - \ln(1+e^x)$

differentiate w.r.t 'x'

$$f'(x) = \frac{d}{dx}(\ln e^x) - \frac{d}{dx}[\ln(1+e^x)]$$

$$= \frac{1}{e^x}\cdot\frac{d}{dx}e^x - \frac{1}{1+e^x}\,\frac{d}{dx}(1+e^x)$$

$$= \frac{1}{e^x}e^x - \frac{1}{1+e^x}\cdot(0+e^x)$$

$$= 1 - \frac{e^x}{1+e^x}$$

$$= \frac{1+e^x - e^x}{1+e^x}$$

$$f'(x) = \frac{1}{1+e^x}$$

23. $f(x) = x^{\ln x}$

$f(x) = x^{\ln x}$

Taking log

$$\ln(f(x)) = \ln(x)^{\ln x}$$

$$\ln(f(x)) = \ln x.\ln(x)$$

$$= [\ln(x)]^2$$

differentiate w.r.t 'x'

$$\frac{1}{f(x)}\,\frac{d}{dx}(f(x)) = 2(\ln(x))\,\frac{d}{dx}(\ln x)$$

$$\frac{f'(x)}{f(x)} = \frac{2\ln(x)}{x}$$

$$f'(x) = f(x)\cdot\frac{2\ln(x)}{x}$$

$$f'(x) = \frac{2}{x}\cdot x^{\ln x}\ln(x).$$

24. $f(x) = \ln\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)$

$f(x) = \ln(1+\sqrt{x}) - \ln(1-\sqrt{x})$

differentiate w.r.t 'x'

$$f'(x) = \frac{1}{1+\sqrt{x}}\,\frac{d}{dx}(1+\sqrt{x}) - \frac{1}{1-\sqrt{x}}\,\frac{d}{dx}(1-\sqrt{x})$$

$$f'(x) = \frac{1}{1+\sqrt{x}}\left(0+\frac{1}{2\sqrt{x}}\right) - \frac{1}{1-\sqrt{x}}\cdot\left(0-\frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{x}}\left[\frac{1}{1+\sqrt{x}} + \frac{1}{1-\sqrt{x}}\right]$$

$$= \frac{1}{2\sqrt{x}}\left[\frac{1-\sqrt{x}+1+\sqrt{x}}{(1-\sqrt{x})(1+\sqrt{x})}\right]$$

$$= \frac{1}{\cancel{2}\sqrt{x}}\cdot\frac{\cancel{2}}{(1-x)} \qquad = (1)^2-(\sqrt{x})^2 = (1-x)$$

$$f'(x) = \frac{1}{\sqrt{x}(1-x)}$$

25. $f(x) = e^{ax}\cos[b\tan^{-1}x]$

differentiate w.r.t 'x'

$$f'(x) = e^{ax}\frac{d}{dx}[\cos(b\tan^{-1}x)] + \cos(b\tan^{-1}x)\frac{d}{dx}e^{ax}$$

$$= -e^{ax}\sin(b\tan^{-1}x)\frac{d}{dx}(b\tan^{-1}x) + \cos(b\tan^{-1}x)\,ae^{ax}$$

$$f'(x) = -e^{ax}\sin(b\tan^{-1}x)\cdot\frac{b}{1+x^2} + ae^{ax}\cos(b\tan^{-1}x)$$ (7)

$$f'(x) = e^{ax}\left(-\frac{b\sin(b\tan^{-1}(x))}{1+x^2} + a\cos(b\tan^{-1}(x))\right]$$

$$f'(x) = \frac{e^{ax}}{1+x^2}\left[a(1+x^2)\cos(b\tan^{-1}x) - b\sin(b\tan^{-1}x)\right]$$

26. $$f(x) = \frac{1}{\sqrt{b^2-a^2}}\ln\frac{\sqrt{b+a}+\sqrt{b-a}\tan(\frac{x}{2})}{\sqrt{b+a}-\sqrt{b-a}\tan(\frac{x}{2})}$$

$$f(x) = \frac{1}{\sqrt{b^2-a^2}}\left[\left[\ln\left(\sqrt{b+a}+\sqrt{b-a}\tan(\tfrac{x}{2})\right) - \ln\left(\sqrt{b+a}-\sqrt{b-a}\tan(\tfrac{x}{2})\right)\right]\right]$$

differentiate w.r.t 'x'

$$f'(x) = \frac{1}{\sqrt{b^2-a^2}}\left[\frac{1}{\sqrt{b+a}+\sqrt{b-a}\tan(\frac{x}{2})}\cdot\left(0+\sqrt{b-a}\sec^2(\tfrac{x}{2})\cdot\tfrac{1}{2}\right) - \frac{1}{\sqrt{b+a}-\sqrt{b-a}\tan(\frac{x}{2})}\left(0-\sqrt{b-a}\sec^2(\tfrac{x}{2})\cdot\tfrac{1}{2}\right)\right.$$

$$f'(x) = \frac{\sqrt{b-a}\sec^2(\frac{x}{2})(\frac{1}{2})}{\sqrt{b^2-a^2}}\left[\frac{1}{\sqrt{b+a}+\sqrt{b-a}\tan(\frac{x}{2})} + \frac{1}{\sqrt{b+a}-\sqrt{b-a}\tan(\frac{x}{2})}\right]$$

$$= \frac{\sqrt{b-a}\sec^2(\frac{x}{2})}{2\sqrt{b^2-a^2}}\left(\frac{\sqrt{b+a}-\sqrt{b-a}\tan(\frac{x}{2})+\sqrt{b+a}+\sqrt{b-a}\tan(\frac{x}{2})}{\left[\sqrt{b+a}+\sqrt{b-a}\tan(\frac{x}{2})\right]\left[\sqrt{b+a}-\sqrt{b-a}\tan(\frac{x}{2})\right]}\right)$$

$$= \frac{\sqrt{b-a}\sec^2(\frac{x}{2})}{2\sqrt{b^2-a^2}}\left(\frac{2\sqrt{b+a}}{(\sqrt{b+a})^2-\left[\sqrt{b-a}\tan(\frac{x}{2})\right]^2}\right)$$

$$= \frac{\sqrt{b-a}\sec^2(x/2)}{2\sqrt{b^2-a^2}}\cdot\left(\frac{2\sqrt{a+b}}{b+a-(b-a)\tan^2(\frac{x}{2})}\right)$$

$$= \frac{\sqrt{b-a}}{2\sqrt{b^2-a^2}\cos^2(\frac{x}{2})}\left(\frac{2\sqrt{a+b}}{(b+a)-(b-a)\frac{\sin^2(x/2)}{\cos^2(x/2)}}\right)$$

$$= \frac{\sqrt{b-a}}{\sqrt{b^2-a^2}\cos^2(\frac{x}{2})}\left(\frac{\sqrt{a+b}}{\frac{(b+a)\cos^2(\frac{x}{2})-(b-a)\sin^2(\frac{x}{2})}{\cos^2(\frac{x}{2})}}\right)$$

$$= \frac{\sqrt{b-a}\,\cancel{\cos^2(\frac{x}{2})}}{\sqrt{b^2-a^2}\,\cancel{\cos^2(\frac{x}{2})}}\left[\frac{\sqrt{a+b}}{b\cos^2(\frac{x}{2})+a\cos^2(\frac{x}{2})-b\sin^2(\frac{x}{2})+a\sin^2(\frac{x}{2})}\right]$$

$$= \frac{\cancel{\sqrt{b-a}}}{\cancel{\sqrt{b-a}}\,\cancel{\sqrt{b+a}}}\cdot\frac{\cancel{\sqrt{a+b}}}{b\left(\cos^2(\frac{x}{2})-\sin^2(\frac{x}{2})\right]+a\left[\cos^2(\frac{x}{2})+\sin^2(\frac{x}{2})\right]}$$

$$= \frac{1}{b\cos 2\cdot(\frac{x}{2})+a(1)}$$

$$f(x) = \frac{1}{b\cos x+a}$$

27. $f(x) = x a^x \sinh x.$ differentiate w.r.t 'x'.

$$f'(x) = x.a^x \frac{d}{dx}(\sinh x) + x \sinh x \frac{d}{dx}(a^x) + a^x \sinh x \frac{d}{dx}(x)$$

$$= x.a^x \cosh x + x \sinh x . a^x \ln a + a^x \sinh x$$

$\because \frac{d}{dx} a^x = a^x \ln a.$

28. $$f(x) = \frac{-\cos x}{2\sin^2 x} + \frac{1}{2}\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

differentiate w.r.t 'x'.

$$f'(x) = -\frac{1}{2}\frac{d}{dx}\left(\frac{\cos x}{\sin^2 x}\right) + \frac{1}{2}\frac{d}{dx}\left[\ln\left(\tan\left(\frac{x}{2}\right)\right)\right]$$

$$= -\frac{1}{2}\left[\frac{\sin^2 x \frac{d}{dx}(\cos x) - \cos x \frac{d}{dx}(\sin^2 x)}{\sin^4 x}\right] + \frac{1}{2}\cdot\frac{1}{\tan\left(\frac{x}{2}\right)}\frac{d}{dx}\left(\tan\left(\frac{x}{2}\right)\right)$$

$$= -\frac{1}{2}\left[\frac{+\sin^2 x.(-\sin x) - \cos x . 2\sin x \frac{d}{dx}(\sin x)}{\sin^4 x}\right] + \frac{1}{2}\cdot\frac{1}{\tan\left(\frac{x}{2}\right)}\cdot \sec^2\left(\frac{x}{2}\right)\cdot\left(\frac{1}{2}\right)$$

$$= -\frac{1}{2}(-\sin x)\left[\frac{\sin^2 x + 2\cos x . \cos x}{\sin^4 x}\right] + \frac{1}{4}\frac{\sec^2\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}$$

$$= \frac{1}{2}\left[\frac{\sin^2 x + 2\cos^2 x}{\sin^3 x}\right] + \frac{1}{4}\cdot\frac{1}{\cos^2\left(\frac{x}{2}\right)}\cdot\frac{1}{\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}}$$

$$= \frac{\sin^2 x + 2\cos^2 x}{2\sin^3 x} + \frac{1}{4\cos\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)\right)}$$

$$= \frac{\sin^2 x + 2\cos^2 x}{2\sin^3 x} + \frac{1}{2\sin x}$$

$\because 2.2\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$
$= 2.\sin 2\left(\frac{x}{2}\right)$
$= 2\sin(x)$

$$= \frac{\sin^2 x + 2\cos^2 x + \sin^2 x}{2\sin^3 x}$$

$$= \frac{2\sin^2 x + 2\cos^2 x}{2\sin^3 x} = \frac{\cancel{2}(\sin^2 x + \cos^2 x)}{\cancel{2}\sin^3 x} = \frac{1}{\sin^3 x} = \operatorname{cosec}^3 x$$

$f(x) = \operatorname{cosec}^3 x.$

29. $f(x) = \sec^{-1}(\operatorname{cosec} x + \sqrt{x})$ $\quad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

differentiate w.r.t 'x'.

$$f'(x) = \frac{1}{(\operatorname{cosec} x + \sqrt{x})\sqrt{(\operatorname{cosec} x + \sqrt{x})^2 - 1}}\frac{d}{dx}(\operatorname{cosec} x + \sqrt{x})$$

$$f'(x) = \frac{1}{(\operatorname{cosec} x + \sqrt{x})\sqrt{\underbrace{\operatorname{cosec}^2 x + x + 2\sqrt{x}\operatorname{cosec} x - 1}}}\left(-\operatorname{cosec} x \cot x + \frac{1}{2\sqrt{x}}\right)$$

$\cot^2 x + 1 = \operatorname{cosec}^2 x$
$\cot^2 x = \operatorname{cosec}^2 x -$

$$= \frac{1}{(\operatorname{cosec} x + \sqrt{x})\sqrt{(\operatorname{cosec}^2 x - 1) + x + 2\sqrt{x}\operatorname{cosec} x}}\left(\frac{-2\sqrt{x}\operatorname{cosec} x \cot x + 1}{2\sqrt{x}}\right)$$

$$f'(x) = \frac{1 - 2\sqrt{x}\operatorname{cosec} x \cot x}{2\sqrt{x}(\operatorname{cosec} x + \sqrt{x})\sqrt{\cot^2 x + x + 2\sqrt{x}\operatorname{cosec} x}}$$

$$f(x) = \left(1+\frac{1}{x}\right)^{x^2}$$ taking $\ln$ on both sides.

$$\ln(f(x)) = \ln\left(1+\frac{1}{x}\right)^{x^2}$$

$$\ln(f(x)) = x^2 \ln\left(1+\frac{1}{x}\right)$$

differentiate w.r.t 'x'

$$\frac{1}{f(x)} \cdot f'(x) = x^2 \frac{d}{dx} \ln\left(1+\frac{1}{x}\right) + \ln\left(1+\frac{1}{x}\right)\frac{d}{dx}(x^2)$$

$$\frac{f'(x)}{f(x)} = \frac{x^2}{1+\frac{1}{x}} \frac{d}{dx}\left(1+\frac{1}{x}\right) + \ln\left(1+\frac{1}{x}\right)(2x)$$

$$\frac{f'(x)}{f(x)} = \frac{\cancel{x^2}}{\frac{x+1}{x}} \cdot \left(-\frac{1}{\cancel{x^2}}\right) + 2x\ln\left(1+\frac{1}{x}\right)$$

$$f'(x) = f(x)\left[-\frac{x}{x+1} + 2x\ln\left(1+\frac{1}{x}\right)\right]$$

$$f'(x) = \left(1+\frac{1}{x}\right)^{x^2}\left[2x\ln\left(1+\frac{1}{x}\right) - \frac{x}{x+1}\right]$$

31. **Differentiate w.r.t 'x'.**

$$y = \arctan\left(\frac{1+2x}{2-x}\right) = \tan^{-1}\left(\frac{1+2x}{2-x}\right)$$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}\left(\frac{1+2x}{2-x}\right)\right)$$

$$= \frac{1}{1+\left(\frac{1+2x}{2-x}\right)^2} \frac{d}{dx}\left(\frac{1+2x}{2-x}\right)$$

$$= \frac{1}{\frac{(2-x)^2+(1+2x)^2}{(2-x)^2}} \cdot \frac{(2-x)(2) - (1+2x)(-1)}{(2-x)^2}$$

$$= \frac{\cancel{(2-x)^2}}{4+x^2-\cancel{4x}+1+4x^2+\cancel{4x}} \cdot \frac{4-\cancel{2x}+1+\cancel{2x}}{\cancel{(2-x)^2}}$$

$$= \frac{5}{5+5x^2} = \frac{\cancel{5}}{\cancel{5}(1+x^2)}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

32. $y = \ln(\sin^{-1} e^x)$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{1}{\sin^{-1}e^x} \frac{d}{dx}(\sin^{-1}e^x) = \frac{1}{\sin^{-1}e^x} \cdot \frac{1}{\sqrt{1-(e^x)^2}} \frac{d}{dx}(e^x)$$

$$\frac{dy}{dx} = \frac{1}{\sin^{-1}e^x} \cdot \frac{1}{\sqrt{1-e^{x^2}}} \cdot e^x$$

$$\frac{dy}{dx} = \frac{e^x}{\sqrt{1-e^{x^2}}\,\sin^{-1}e^x}$$

$$y = (\sin^{-1}x^2)^{\pi}$$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \pi(\sin^{-1}x)^{\pi-1}\frac{d}{dx}(\sin^{-1}x^2)$$

$$\frac{dy}{dx} = \pi(\sin^{-1}x)^{\pi-1}\frac{1}{\sqrt{1-(x^2)^2}}\frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = \pi(\sin^{-1}x)^{\pi-1}\frac{1}{\sqrt{1-x^4}}\cdot 2x$$

$$\frac{dy}{dx} = \frac{2\pi x \cdot (\sin^{-1}x)^{\pi-1}}{\sqrt{1-x^4}}$$

34. $f\left(\frac{x^2+1}{x^2-1}\right) = y$

Let $u = \left(\frac{x^2+1}{x^2-1}\right)$

$$y = f(u)$$

$$\frac{dy}{du} = f'(u)$$

$$\frac{du}{dx} = \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2}$$

$$= \frac{2x\,(x^2-1-x^2-1)}{(x^2-1)^2}$$

$$\frac{du}{dx} = \frac{2x(-2)}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= f'(u) \times \frac{-4x}{(x^2-1)^2}$$

$$= f'\left(\frac{x^2+1}{x^2-1}\right)\cdot\frac{-4x}{(x^2-1)^2}$$

35. $y = \frac{1-\cosh x}{1+\cosh x}$

$$y = \frac{-\cancel{2}\sinh^2(\frac{x}{2})}{\cancel{2}\cosh^2(\frac{x}{2})}$$

$\because \cosh 2x = 2\sinh^2 x + 1$

$\cosh 2x = 2\cosh^2 x - 1$

$$y = -\tanh^2\left(\frac{x}{2}\right)$$

$$\frac{dy}{dx} = -2\tanh\left(\frac{x}{2}\right)\frac{d}{dx}\left(\tanh\left(\frac{x}{2}\right)\right)$$

$$-2\tanh\left(\frac{x}{2}\right)\operatorname{sech}^2\left(\frac{x}{2}\right)\frac{d}{dx}\left(\frac{x}{2}\right)$$

$$\frac{dy}{dx} = -2\tanh\left(\frac{x}{2}\right)\operatorname{sech}^2\left(\frac{x}{2}\right)\cdot\frac{1}{2}$$

$$\frac{dy}{dx} = -\tanh\left(\frac{x}{2}\right)\operatorname{sech}^2\left(\frac{x}{2}\right)$$

36. $y = \ln(\tanh 2x)$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{1}{\tanh 2x}\frac{d}{dx}(\tanh 2x)$$

$$\frac{dy}{dx} = \frac{1}{\tanh 2x}\cdot\operatorname{sech}^2(2x)\frac{d}{dx}(2x1)$$

$$\frac{dy}{dx} = \frac{2}{\frac{\sinh 2x}{\cancel{\cosh 2x}}}\cdot\frac{1}{\cosh^{\cancel{2}} 2x}$$

$$\frac{dy}{dx} = \frac{2}{\sinh 2x\cosh 2x}$$

$$\frac{dy}{dx} = \frac{2\times 2}{2\sinh 2x\cosh 2x}$$

$$\frac{dy}{dx} = \frac{4}{\sinh 2(2x)} = \frac{4}{\sinh(4x)}$$

$$\frac{dy}{dx} = 4\operatorname{cosech} 4x$$

37. $y = \log_{10}\left(\frac{x+1}{x}\right)$

$$y = \frac{\ln\left(\frac{x+1}{x}\right)}{\ln 10} = \frac{1}{\ln 10}\ln\left(\frac{x+1}{x}\right)$$

$$y = \frac{1}{\ln 10}\left(\ln(x+1) - \ln x\right)$$

differentiating w.r.t 'x'

$$\frac{dy}{dx} = \frac{1}{\ln 10}\left(\frac{1}{x+1}\frac{d}{dx}(x+1) - \frac{1}{x}\right)$$

$$\frac{dy}{dx} = \frac{1}{\ln 10}\left(\frac{1}{x+1} - \frac{1}{x}\right)$$

$$= \frac{1}{\ln 10}\left(\frac{x-(x+1)}{x(x+1)}\right)$$

$$= \frac{1}{\ln 10}\left(\frac{\cancel{x}-\cancel{x}-1}{x(x+1)}\right)$$

$$= \frac{1}{\ln 10}\left(\frac{-1}{x(x+1)}\right)$$

$$\frac{dy}{dx} = \frac{-1}{\ln 10 \cdot x(x+1)}$$

d. $y = \cos^{-1}(\sqrt{1-x^2})$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(\sqrt{1-x^2})^2}}\,\frac{d}{dx}(\sqrt{1-x^2})$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(1-x^2)}}\cdot\frac{1}{2\sqrt{1-x^2}}\cdot\frac{d}{dx}(1-x^2)$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-1+x^2}}\cdot\frac{1}{\cancel{2}\sqrt{1-x^2}}\cdot(-\cancel{2}x)$$

$$= \frac{-1}{\sqrt{x^2}}\cdot\frac{1}{\sqrt{1-x^2}}\cdot(-x)$$

$$\frac{dy}{dx} = \frac{x}{|x|\sqrt{1-x^2}}$$

39. $y = \sec^{-1}(\sinh x)$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{1}{\sinh x\sqrt{\sinh^2 x-1}}\,\frac{d}{dx}(\sinh x)$$

$$= \frac{1}{\sinh x\sqrt{\sinh^2 x-1}}\,(\cosh x)$$

$$\frac{dy}{dx} = \frac{\cosh x}{\sinh x\sqrt{\sinh^2 x-1}}$$

40. $y = \sin^{-1}(\cot^{-1}\ln x)$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin^{-1}\left(\cot^{-1}(\ln x)\right)\right]$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\cot^{-1}(\ln x))^2}}\cdot\frac{d}{dx}(\cot^{-1}(\ln x))$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\cot^{-1}(\ln x))^2}}\cdot\frac{-1}{(1+(\ln x)^2)}\cdot\frac{d}{dx}(\ln x)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\cot^{-1}(\ln x))^2}}\;\frac{-1}{(1+(\ln x)^2)}\cdot\frac{1}{x}$$

$$\frac{dy}{dx} = \frac{-1}{x(1+\ln^2 x)\sqrt{1-(\cot^{-1}(\ln x))^2}}$$

41. $y = \cosh^{-1}(1+x^2)$

differentiate w.r.t 'x'.

$$\frac{dy}{dx} = \frac{1}{\sqrt{(1+x^2)^2-1}}\;\frac{d}{dx}(1+x^2)$$

$$= \frac{1}{\sqrt{\cancel{1}+x^4+2x^2-\cancel{1}}}\cdot(0+2x)$$

$$= \frac{2x}{\sqrt{x^4+2x^2}} = \frac{2x}{\sqrt{x^2(x^2+2)}}$$

$$\frac{dy}{dx} = \frac{2x}{|x|\sqrt{x^2+2}}$$

42. $y = \sinh^{-1}(\tanh x)$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{1}{\sqrt{\tanh^2 x+1}}\;\frac{d}{dx}(\tanh x)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\tanh^2 x+1}}\cdot\operatorname{sech}^2 x$$

$$\frac{dy}{dx} = \frac{\operatorname{sech}^2 x}{\sqrt{\tanh^2 x+1}}$$

In Problem 43-54. find $\frac{dy}{dx}$.

43. $\sqrt{x}+\sqrt{y}=\sqrt{a}$

differentiate w.r.t 'x'.

$$\frac{d}{dx}\sqrt{x}+\frac{d}{dx}\sqrt{y} = \frac{d}{dx}\sqrt{a}$$

$$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\frac{1}{2\sqrt{y}}\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{2\sqrt{y}}{2\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$$

$$\therefore\ xy^2 - 2xy + x = 1$$

differentiate w.r.t 'x'

$$x\frac{d}{dx}y^2 + y^2\frac{d}{dx}(x) - 2\left[x\frac{dy}{dx} + y\frac{dx}{dx}\right] + 1 = 0$$

$$x(2y)\frac{dy}{dx} + y^2 - 2x\frac{dy}{dx} - 2y + 1 = 0$$

$$\frac{dy}{dx}\left[2xy - 2x\right] = -y^2 + 2y - 1$$

$$\frac{dy}{dx} = \frac{-y^2 + 2y - 1}{2xy - 2x}$$

$$\frac{dy}{dx} = \frac{-y + 2y - 1}{2x(y-1)}$$

45. $x^3 + y^3 - 3axy = 0$

differentiating w.r.t 'x' we get

$$3x^2 + 3y^2\frac{dy}{dx} - 3a\left[x\frac{dy}{dx} + y\right] = 0$$

$$3x^2 + 3y^2\frac{dy}{dx} - 3ax\frac{dy}{dx} - 3ay = 0$$

$$\frac{dy}{dx}\left[3y^2 - 3ax\right] = 3ay - 3x^2$$

$$\frac{dy}{dx} = \frac{3(ay - x^2)}{3y^2 - 3ax}$$

$$\frac{dy}{dx} = \frac{3(ay - x^2)}{3(y^2 - ax)}$$

$$\frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$

46. $(x^2 + y^2)^3 = y$

differentiating w.r.t 'x'

$$3(x^2+y^2)^2\frac{d}{dx}(x^2+y^2) = \frac{dy}{dx}$$

$$3(x^2+y^2)^2\left(2x + 2y\frac{dy}{dx}\right) = \frac{dy}{dx}$$

$$3(2x)(x^2+y^2)^2 + 3(2y)(x^2+y^2)^2\frac{dy}{dx} = \frac{dy}{dx}$$

$$6x(x^2+y^2)^2 + 6y(x^2+y^2)^2\frac{dy}{dx} = \frac{dy}{dx}$$

$$6x(x^2+y^2)^2 = \frac{dy}{dx} - 6y(x^2+y^2)^2\frac{dy}{dx}$$

$$6x(x^2+y^2)^2 = \frac{dy}{dx}\left[1 - 6y(x^2+y^2)^2\right]$$

$$\frac{dy}{dx} = \frac{6x(x^2+y^2)^2}{1 - 6y(x^2+y^2)^2}$$

47. $\tan^{-1}\left(\frac{y}{x}\right) + yx^2 = 1$

differentiate w.r.t 'x'

$$\frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{d}{dx}\left(\frac{y}{x}\right) + y(2x) + x^2\frac{dy}{dx} = 0$$

$$\frac{1}{\frac{x^2+y^2}{x^2}}\left[\frac{x\frac{dy}{dx} - y}{x^2}\right] + 2xy + x^2\frac{dy}{dx} = 0$$

$$\frac{\cancel{x^2}}{x^2+y^2}\left[\frac{x\frac{dy}{dx} - y}{\cancel{x^2}}\right] + 2xy + x^2\frac{dy}{dx} = 0$$

$$\frac{x\frac{dy}{dx} - y}{x^2+y^2} + 2xy + x^2\frac{dy}{dx} = 0$$

$$\frac{x}{x^2+y^2}\frac{dy}{dx} - \frac{y}{x^2+y^2} + 2xy + x^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left[\frac{x}{x^2+y^2} + x^2\right] = \frac{y}{x^2+y^2} - 2xy$$

$$\frac{dy}{dx}\left[\frac{x + x^2(x^2+y^2)}{x^2+y^2}\right] = \frac{y - 2xy(x^2+y^2)}{x^2+y^2}$$

$$\frac{dy}{dx}\left[\frac{x(1 + x(x^2+y^2))}{\cancel{x^2+y^2}}\right] = \frac{y(1 - 2x(x^2+y^2))}{\cancel{x^2+y^2}}$$

$$\frac{dy}{dx}\left(x(1 + x^3 + xy^2)\right) = y(1 - 2x^3 - 2xy^2)$$

$$\frac{dy}{dx} = \frac{y(1 - 2x^3 - 2xy^3)}{x(1 + x^3 + xy^2)}$$

48. $\tan^{-1}(x+y) = \sin^{-1}(e^y + x)$

differentiating w.r.t x

$$\frac{1}{1+(x+y)^2}\cdot\frac{d}{dx}(x+y) = \frac{1}{\sqrt{1-(e^y+x)^2}}\cdot\frac{d}{dx}(e^y + x)$$

$$\frac{1}{1+(x+y)^2}\left(1 + \frac{dy}{dx}\right) = \frac{1}{\sqrt{1-(e^y+x)^2}}\left(e^y\frac{dy}{dx} + 1\right)$$

$$\therefore \frac{1}{1+(x+y)^2} + \frac{1}{1+(x+y)^2}\frac{dy}{dx} = \frac{e^y}{\sqrt{1-(e^y+x)^2}}\frac{dy}{dx} + \frac{1}{\sqrt{1-(e^y+x)^2}}$$

$$\frac{dy}{dx}\cdot\frac{1}{1+(x+y)^2} - \frac{e^y}{\sqrt{1-(e^y+x)^2}}\frac{dy}{dx} = \frac{1}{\sqrt{1-(e^y+x)^2}} - \frac{1}{1+(x+y)^2}$$

$$\frac{dy}{dx}\left[\frac{1}{1+(x+y)^2} - \frac{e^y}{\sqrt{1-(e^y+x)^2}}\right] = \frac{1+(x+y)^2-\sqrt{1-(e^y+x)^2}}{\sqrt{1-(e^y+x)^2}\,(1+(x+y)^2)}$$

$$\frac{dy}{dx}\left[\frac{\sqrt{1-(e^y+x)^2}-e^y(1+(x+y)^2)}{\cancel{\sqrt{1-(e^y+x)^2}\,(1+(x+y)^2)}}\right] = \frac{1+(x+y)^2-\sqrt{1-(e^y+x)^2}}{\cancel{\sqrt{1-(e^y+x)^2}\,(1+(x+y)^2)}}$$

$$\frac{dy}{dx} = \frac{1+(x+y)^2-\sqrt{1-(e^y+x)^2}}{\sqrt{1-(e^y+x)^2}-e^y(1+(x+y)^2)}$$

49. $y = \sin^{-1}(\ln x) - \ln(\tan^{-1}x)$

differentiate w.r.t 'x'

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\ln x)^2}}\frac{d}{dx}(\ln x) - \frac{1}{\tan^{-1}x}\frac{d}{dx}(\tan^{-1}x)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\ln x)^2}}\cdot\frac{1}{x} - \frac{1}{\tan^{-1}x}\cdot\frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{x\sqrt{1-(\ln x)^2}} - \frac{1}{(1+x^2)\tan^{-1}x}$$

50. $y\sin^{-1}x - x\tan^{-1}y = 1$

differentiate w.r.t 'x'

$$y\frac{d}{dx}(\sin^{-1}x) + \sin^{-1}x\frac{dy}{dx} - \left[x\frac{d}{dx}\tan^{-1}y + \tan^{-1}y\frac{d}{dx}(x)\right] = \frac{d}{dx}(1)$$

$$y\cdot\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x\frac{dy}{dx} - \left[x\cdot\frac{1}{1+y^2}\frac{dy}{dx} + \tan^{-1}y\right] = 0$$

$$\frac{y}{\sqrt{1-x^2}} + \sin^{-1}x\frac{dy}{dx} - \frac{x}{1+y^2}\frac{dy}{dx} - \tan^{-1}y = 0$$

$$\frac{dy}{dx}\left[\sin^{-1}x - \frac{x}{1+y^2}\right] = \tan^{-1}y - \frac{y}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx}\left[\frac{(1+y^2)\sin^{-1}x - x}{1+y^2}\right] = \frac{\sqrt{(1-x^2)}\tan^{-1}y - y}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{(1+y^2)(\sqrt{1-x^2}\tan^{-1}y - y)}{\sqrt{1-x^2}\,((1+y^2)\sin^{-1}x - x)}$$

51. $\sin^{-1}(\ln xy) = x + y^2$

differentiate w.r.t 'x'

$$\frac{1}{\sqrt{1-(\ln xy)^2}}\frac{d}{dx}[\ln(xy)] = 1 + 2y\frac{dy}{dx}$$

$$\frac{1}{\sqrt{1-(\ln xy)^2}}\cdot\frac{1}{xy}\frac{d}{dx}[xy] = 1 + 2y\frac{dy}{dx}$$

$$\frac{1}{\sqrt{1-(\ln xy)^2}}\cdot\frac{1}{xy}\left[x\frac{dy}{dx} + y\right] = 1 + 2y\frac{dy}{dx}$$

$$\frac{\cancel{x}}{\cancel{x}y\sqrt{1-(\ln xy)^2}}\frac{dy}{dx}+\frac{\cancel{y}}{x\cancel{y}\sqrt{1-(\ln xy)^2}} = 1+2y\frac{dy}{dx}$$

$$\frac{1}{y\sqrt{1-(\ln xy)^2}}\frac{dy}{dy}-2y\frac{dy}{dx} = 1-\frac{1}{x\sqrt{1-(\ln xy)^2}}$$

$$\frac{dy}{dx}\left[\frac{1}{y\sqrt{1-(\ln xy)^2}}-2y\right] = \frac{x\sqrt{1-(\ln xy)^2}-1}{x\sqrt{1-(\ln xy)^2}}$$

$$\frac{dy}{dx}\left[\frac{1-2y^2\sqrt{1-(\ln xy)^2}}{y\cancel{\sqrt{1-(\ln xy)^2}}}\right] = \frac{x\sqrt{1-(\ln xy)^2}-1}{x\cancel{\sqrt{1-(\ln xy)^2}}}$$

$$\frac{dy}{dx} = \frac{y(x\sqrt{1-(\ln xy)^2}-1)}{x(1-2y^2\sqrt{1-(\ln xy)^2})}$$

52. $\sec^{-1}(x^2+y)-e^x = \dfrac{1}{x+y}$

differentiate w.r.t 'x'.

$$\frac{1}{(x^2+y)\sqrt{(x^2+y)^2-1}}\frac{d}{dx}(x^2+y) - e^x = -\frac{1}{(x+y)^2}\frac{d}{dx}(x+y)$$

$$\frac{1}{(x^2+y)\sqrt{(x^2+y)^2-1}}\left(2x+\frac{dy}{dx}\right)-e^x = \frac{-1}{(x+y)^2}\left(1+\frac{dy}{dx}\right)$$

$$\frac{2x}{(x^2+y)\sqrt{(x^2+y)^2-1}}+\frac{1}{(x^2+y)\sqrt{(x^2+y)^2-1}}\frac{dy}{dx}-e^x = \frac{-1}{(x+y)^2}-\frac{1}{(x+y)^2}\frac{dy}{dx}$$

$$\frac{1}{(x^2+y)\sqrt{(x^2+y)^2-1}}\frac{dy}{dx}+\frac{1}{(x+y)^2}\frac{dy}{dx} = e^x-\frac{2x}{(x^2+y)\sqrt{(x^2+y)^2-1}}-\frac{1}{(x+y)^2}$$

$$\frac{dy}{dx}\left[\frac{1}{(x^2+y)\sqrt{(x^2+y)^2-1}}+\frac{1}{(x+y)^2}\right] = \frac{e^x(x^2+y)(x+y)^2\sqrt{(x^2+y)^2-1}-2x(x+y)^2-(x^2+y)\sqrt{(x^2+y)^2-1}}{(x^2+y)(x+y)^2\sqrt{(x^2+y)^2-1}}$$

$$\frac{dy}{dx}\left[\frac{(x+y)^2+(x^2+y)\sqrt{(x^2+y)^2-1}}{\cancel{(x^2+y)(x+y)^2\sqrt{(x^2+y)^2-1}}}\right] = \frac{e^x(x^2+y)(x+y)^2\sqrt{(x^2+y)^2-1}-2x(x+y)^2-(x^2+y)\sqrt{(x^2+y)^2-1}}{\cancel{(x^2+y)(x+y)^2\sqrt{(x^2+y)^2-1}}}$$

$$\frac{dy}{dx} = \frac{e^x(x^2+y)(x+y)^2\sqrt{(x^2+y)^2-1}-2x(x+y)^2-(x^2+y)\sqrt{(x^2+y)^2-1}}{(x+y)^2+(x^2+y)\sqrt{(x^2+y)^2-1}}$$

53. $x = a(t-\sin t)$, $y = a(1-\cos t)$

$$\frac{dx}{dt} = a(1-\cos t)$$

$$\frac{dy}{dt} = a(0-(-\sin t)) = a\sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = (a\sin t)\frac{1}{a(1-\cos t)} = \frac{\cancel{a}\sin t}{\cancel{a}(1-\cos t)}$$

$$\frac{dy}{dx} = \frac{\cancel{2}\sin(\frac{t}{2})\cos(\frac{t}{2})}{\cancel{2}\sin^2(\frac{t}{2})} = \frac{\cos(t/2)}{\sin(t/2)}$$

$$\frac{dy}{dx} = \cot\left(\frac{t}{2}\right)$$

1. $$x = \frac{3at}{1+t^2}, \quad y = \frac{3at^2}{1+t^2}$$

$$\frac{dx}{dt} = \frac{(1+t^2)(3a) - (3at)(2t)}{(1+t^2)^2}, \quad \frac{dy}{dt} = \frac{(1+t^2)(6at) - 3at^2(2t)}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{3a + 3at^2 - 6at^2}{(1+t^2)^2} \qquad \frac{dy}{dt} = \frac{6at + 6at^3 - 6at^3}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{3a - 3at^2}{(1+t^2)^2} \qquad \frac{dy}{dt} = \frac{6at}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{3a(1-t^2)}{(1+t^2)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{\overset{2}{\cancel{6}}at}{(1+t^2)^2} \times \frac{(1+t^2)^2}{\cancel{3}\cancel{a}(1-t^2)}$$

$$\frac{dy}{dx} = \frac{2t}{1-t^2}$$

55. **Diff w.r.t 'x' (55-60)**

$$y = \sqrt[3]{\frac{x(x^2+1)}{(x-1)^2}} = \left(\frac{x(x^2+1)}{(x-1)^2}\right)^{1/3}$$

Taking natural log

$$\ln y = \ln\left(\frac{x(x^2+1)}{(x-1)^2}\right)^{1/3} = \frac{1}{3}\left[\ln\left(\frac{x(x^2+1)}{(x-1)^2}\right)\right]$$

$$\ln y = \frac{1}{3}\left[\ln x + \ln(x^2+1) - \ln(x-1)^2\right]$$

$$\ln y = \frac{1}{3}\left[\ln x + \ln(x^2+1) - 2\ln(x-1)\right]$$

differentiating both sides;

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{3}\left[\frac{1}{x} + \frac{1}{x^2+1}\frac{d}{dx}(x^2+1) - 2\cdot\frac{1}{x-1}\right]$$

$$\frac{dy}{dx} = \frac{1}{3}y\left[\frac{1}{x} + \frac{2x}{x^2+1} - \frac{2}{x-1}\right]$$

$$\frac{dy}{dx} = \frac{1}{3}y\left[\frac{(x^2+1)(x-1) + 2x.x(x-1) - 2x(x^2+1)}{x(x^2+1)(x-1)}\right]$$

$$\frac{dy}{dx} = \frac{1}{3}y\left[\frac{x^3+x-x^2-1+\cancel{2x^3}-2x^2-\cancel{2x^3}-2x}{x(x^2+1)(x-1)}\right]$$

$$\frac{dy}{dx} = \frac{1}{3}y\left[\frac{x^3-3x^2-x-1}{x(x^2+1)(x-1)}\right] = \frac{1}{3}\left[\frac{x(x^2+1)}{(x-1)^2}\right]^{1/3}\left[\frac{x^3-3x^2-x-1}{x(x^2+1)(x-1)}\right]$$

$$\frac{dy}{dx} = \frac{1}{3}\,\frac{x^{1/3}(x^2+1)^{1/3}}{(x-1)^{2/3}}\,\frac{(x^3-3x^2-x-1)}{x(x^2+1)(x-1)}$$

$$\frac{dy}{dx} = \frac{x^3-3x^2-x-1}{3x^{2/3}(x+1)^{2/3}(x-1)^{5/3}}$$

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56. $\dfrac{\sqrt{x}\,(1-2x)^{2/3}}{(2-3x)^{3/4}(3-4x)^{4/3}}$

$$y = \frac{\sqrt{x}\,(1-2x)^{2/3}}{(2-3x)^{3/4}(3-4x)^{4/3}}$$

$$\ln y = \ln\left(\frac{x^{1/2}(1-2x)^{2/3}}{(2-3x)^{3/4}(3-4x)^{4/3}}\right)$$

$$\ln y = \ln x^{1/2} + \ln(1-2x)^{2/3} - \ln(2-3x)^{3/4} - \ln(3-4x)^{4/3}$$

$$\ln y = \frac{1}{2}\ln x + \frac{2}{3}\ln(1-2x) - \frac{3}{4}\ln(2-3x) - \frac{4}{3}\ln(3-4x)$$

differentiating both sides w.r.t 'x'

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2x} + \frac{2}{3}\cdot\frac{1}{1-2x}(0-2) - \frac{3}{4}\cdot\frac{1}{(2-3x)}(0-3) - \frac{4}{3}\cdot\frac{1}{(3-4x)}(0-4)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2x} + \frac{2(-2)}{3(1-2x)} - \frac{3(-3)}{4(2-3x)} - \frac{4(-4)}{3-4x}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2x} - \frac{4}{3(1-2x)} + \frac{9}{4(2-3x)} + \frac{16}{3(3-4x)}$$

$$\frac{1}{y}\frac{dy}{dx} = \left[\frac{1}{2x} + \frac{9}{4(2-3x)}\right] + \left[\frac{16}{3(3-4x)} - \frac{4}{3(1-2x)}\right]$$

$$= \left[\frac{2(2-3x)+9x}{4x(2-3x)}\right] + \left[\frac{16(1-2x) - 4(3-4x)}{3(3-4x)(1-2x)}\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \left[\frac{4-6x+9x}{4x(2-3x)}\right] + \left[\frac{16-32x-12+16x}{3(3-4x)(1-2x)}\right]$$

$$\frac{dy}{dx} = y\left[\frac{4+3x}{4x(2-3x)} + \frac{4-16x}{3(3-4x)(1-2x)}\right]$$

$$\frac{dy}{dx} = \frac{\sqrt{x}\,(1-2x)^{2/3}}{(2-3x)^{3/4}(3-4x)^{4/3}}\left[\frac{4+3x}{4x(2-3x)} + \frac{4(1-4x)}{3(3-4x)(1-2x)}\right]$$

57. $y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$

Let $u = (\tan x)^{\cot x}$ $\qquad$ $v = (\cot x)^{\tan x}$

then $y = u + v$

differentiating wrt 'x'

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \longrightarrow (1)$$

$u = (\tan x)^{\cot x}$ $\qquad$ $v = (\cot x)^{\tan x}$

$\ln u = \ln(\tan x)^{\cot x}$ $\qquad$ $\ln v = \ln \cot x^{\tan x}$

$\ln u = \cot x \ln \tan x$ $\qquad$ $\ln v = \tan x \ln \cot x$

differentiating w.r.t 'x'

$$\frac{d\ln u}{dx} = \frac{d}{dx}\left[\cot x \ln\tan x\right]$$

$$\frac{1}{u}\frac{du}{dx} = \cot x \frac{d}{dx}(\ln\tan x) + \ln\tan x \frac{d}{dx}(\cot x)$$

$$\frac{1}{u}\frac{du}{dx} = \cot x \cdot \frac{1}{\tan x} \cdot \frac{d}{dx}(\tan x) + \ln\tan x(-\operatorname{cosec}^2 x)$$

$$\frac{1}{u}\frac{du}{dx} = \frac{\cot x}{\tan x} \cdot \sec^2 x + \ln\tan x(-\operatorname{cosec}^2 x)$$

$$= \frac{\frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x}} \times \frac{1}{\cos^2 x} + \ln\tan x(-\operatorname{cosec}^2 x)$$

$$\frac{1}{u}\frac{du}{dx} = \frac{\cos^2 x}{\sin^2 x} \times \frac{1}{\cos^2 x} - \operatorname{cosec}^2 x \ln\tan x$$

$$\frac{1}{u}\frac{du}{dx} = \frac{1}{\sin^2 x}(1) - \operatorname{cosec}^2 x \ln\tan x$$

$$\frac{du}{dx} = u\left[\operatorname{cosec}^2 x \ln e - \operatorname{cosec}^2 x \ln\tan x\right]$$

$$\frac{du}{dx} = (\tan x)^{\cot x}\left[\operatorname{cosec}^2 x(\ln e - \ln\tan x)\right]$$

$$\frac{du}{dx} = (\tan x)^{\cot x}\left[\operatorname{cosec}^2 x \ln\left(\frac{e}{\tan x}\right)\right]$$

differentiating w.r.t "x"

$$\frac{d}{dx}(\ln v) = \frac{d}{dx}\left[\tan x \ln\cot x\right]$$

$$\frac{1}{v}\frac{dv}{dx} = \tan x \frac{d}{dx}(\ln\cot x) + \ln\cot x \frac{d}{dx}\tan x$$

$$\frac{1}{v}\frac{dv}{dx} = \frac{\tan x}{\cot x}\frac{d}{dx}(\cot x) + \ln\cot x(\sec^2 x)$$

$$\frac{1}{v}\frac{dv}{dx} = \frac{\tan x}{\cot x}(-\operatorname{cosec}^2 x) + \sec^2 x \ln\cot x$$

$$\frac{1}{v}\frac{dv}{dx} = \frac{\frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}} \times \left(-\frac{1}{\sin^2 x}\right) + \sec^2 x \ln\cot x$$

$$\frac{1}{v}\frac{dv}{dx} = \frac{\sin^2 x}{\cos^2 x}\left(-\frac{1}{\sin^2 x}\right) + \sec^2 x \ln\cot x$$

$$\frac{1}{v}\frac{dv}{dx} = -\frac{1}{\cos^2 x}(1) + \sec^2 x \ln\cot x$$

$$\frac{dv}{dx} = v\left[-\sec^2 x \ln e + \sec^2 x \ln\cot x\right]$$

$$\frac{dv}{dx} = (\cot x)^{\tan x}\left[\sec^2 x(-\ln e + \ln\cot x)\right]$$

$$= (\cot x)^{\tan x}\left[-\sec^2 x(\ln e - \ln\cot x)\right]$$

$$= -(\cot x)^{\tan x}\left[\sec^2 x \ln\frac{e}{\cot x}\right]$$

$$\frac{dv}{dx} = -(\cot x)^{\tan x}\sec^2 x \ln(e\tan x)$$

putting values in (1)

$$\frac{dy}{dx} = (\tan x)^{\cot x}\operatorname{cosec}^2 x \ln\left(\frac{e}{\tan x}\right) - (\cot x)^{\tan x}\sec^2 x \ln(e\tan x)$$

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58. $$y = x^x \cdot e^x \cdot \sin x \cdot (\ln x)$$

$$\ln y = \ln(x^x e^x \sin x(\ln x))$$

$$\ln y = \ln x^x + \ln e^x + \ln\sin x + \ln(\ln x)$$

$$\ln y = x\ln x + x\ln e + \ln\sin x + \ln(\ln x)$$

$$\ln y = x\ln x + x + \ln\sin x + \ln(\ln x)$$

differentiating w.r.t 'x'

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln(x) \cdot (1) + (1) + \frac{1}{\sin x}\frac{d}{dx}(\sin x) + \frac{1}{\ln x}\frac{d}{dx}(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = 1 + \ln x + 1 + \frac{\cos x}{\sin x} + \frac{1}{x\ln x}$$

$$\frac{dy}{dx} = y\left[2 + \ln x + \cot x + \frac{1}{x\ln x}\right]$$

$$\frac{dy}{dx} = x^x e^x \sin x \cdot \ln x\left[2 + \ln x + \cot x + \frac{1}{x\ln x}\right]$$

$$y = \frac{(x+2)^2}{(x+1)(x^2+3)^3}$$

$$\ln y = \ln \frac{(x+2)^2}{(x+1)(x^2+3)^3}$$

$$\ln y = \ln(x+2)^2 - \ln(x+1) - \ln(x^2+3)^3$$

$$\ln y = 2\ln(x+2) - \ln(x+1) - 3\ln(x^2+3)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{(x+2)}(1) - \frac{1}{x+1} - \frac{3}{x^2+3}(2x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{(x+2)} - \frac{1}{x+1} - \frac{6x}{x^2+3}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2(x+1)(x^2+3) - 1(x+2)(x^2+3) - 6x(x+2)(x+1)}{(x+2)(x+1)(x^2+3)}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2(x^3+3x+x^2+3) - (x^3+3x+2x^2+6) - 6x(x^2+3x+2)}{(x+2)(x+1)(x^2+3)}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x^3+6x+\cancel{2x^2}+6 - x^3 - 3x - \cancel{2x^2} - 6 - 6x^3 - 18x^2 - 12x}{(x+1)(x+2)(x^2+3)}$$

$$\frac{dy}{dx} = y\left[\frac{-5x^3 - 18x^2 - 9x}{(x+1)(x+2)(x^2+3)}\right]$$

$$\frac{dy}{dx} = -\frac{(x+2)^{\cancel{2}}}{(x+1)(x^2+3)^3}\left[\frac{5x^3+18x^2+9x}{(x+1)\cancel{(x+2)}(x^2+3)}\right]$$

$$\frac{dy}{dx} = -\frac{(x+2)(5x^3+18x^2+9x)}{(x+1)^2(x^2+3)^4}$$

60.

$$y = \exp\left(\text{arc Cosec}\left(\frac{1}{x}\right)\right) = e^{\text{Cosec}^{-1}\left(\frac{1}{x}\right)}$$

$$y = e^{\text{Cosec}^{-1}\left(\frac{1}{x}\right)}$$

diff. both sides.

$$\frac{dy}{dx} = e^{\text{Cosec}^{-1}\left(\frac{1}{x}\right)} \frac{d}{dx}\left(\text{Cosec}^{-1}\left(\frac{1}{x}\right)\right)$$

$$\frac{dy}{dx} = e^{\text{Cosec}^{-1}\left(\frac{1}{x}\right)} \cdot \frac{-1}{\frac{1}{x}\sqrt{\frac{1}{x^2}-1}} \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = e^{\text{Cosec}^{-1}\left(\frac{1}{x}\right)} \frac{-x}{\sqrt{\frac{1-x^2}{x^2}}} \cdot \left(-\frac{1}{x^2}\right)$$

$$\frac{dy}{dx} = e^{\text{Cosec}^{-1}\left(\frac{1}{x}\right)} \frac{-x}{\frac{\sqrt{1-x^2}}{\sqrt{x^2}}} \cdot \left(\frac{-1}{x^2}\right) = e^{\text{Cosec}^{-1}\left(\frac{1}{x}\right)} \frac{-x}{\frac{\sqrt{1-x^2}}{x}} \left(-\frac{1}{x^2}\right)$$

$$\frac{dy}{dx} = e^{\text{Cosec}^{-1}\left(\frac{1}{x}\right)} \frac{\cancel{x^2}}{\sqrt{1-x^2}} \cdot \left(\frac{1}{\cancel{x^2}}\right)$$

$$\frac{dy}{dx} = \frac{e^{\text{Cosec}^{-1}\left(\frac{1}{x}\right)}}{\sqrt{1-x^2}}$$