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# ※ Derivatives ※ (Chapter no. 2) ①

**Derivative of a function:-** Let $f(x)$ be a real valued function then derivative of $f(x)$ denoted by $f'(x)$ & is defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

**Some standard formulae of differentiation:-** Set ①

① $\frac{d}{dx}(x^n) = n x^{n-1}$

② $\frac{d}{dx}(c) = 0$ where $c$ is any Constt.

③ $\frac{d}{dx}(x) = 1$

④ $\frac{d}{dx}(a^{bx}) = a^{bx} . b . \ln a$

⑤ $\frac{d}{dx}(e^{ax}) = a e^{ax}$

⑥ $\frac{d}{dx}(\ln x) = \frac{1}{x}$

⑦ $\frac{d}{dx}(ax+b)^n = n(ax+b)^{n-1} \cdot a$

⑧ $\frac{d}{dx}(U \pm V) = \frac{dU}{dx} \pm \frac{dV}{dx}$

⑨ $\frac{d}{dx}(U.V) = U.\frac{dV}{dx} + V.\frac{dU}{dx}$

⑩ $\frac{d}{dx}\left(\frac{U}{V}\right) = \frac{V.\frac{dU}{dx} - U.\frac{dV}{dx}}{V^2}$

⑪ $\frac{d}{dx}(cU) = c.\frac{dU}{dx}$

## Set 2

1. $\frac{d}{dx}(\sin x) = \cos x$
2. $\frac{d}{dx}(\cos x) = -\sin x$
3. $\frac{d}{dx}(\tan x) = \sec^2 x$
4. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
5. $\frac{d}{dx}(\sec x) = \sec x . \tan x$
6. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x . \cot x$

## Set 3

1. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
2. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
3. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
4. $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
5. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
6. $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

## Set 4

1. $\frac{d}{dx}(\sinh x) = \cosh x$
2. $\frac{d}{dx}(\cosh x) = \sinh x$
3. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$
4. $\frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$
5. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x . \tanh x$
6. $\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x . \coth x$

Set 5

1. $\dfrac{d}{dx}(\sinh^{-1}x) = \dfrac{1}{\sqrt{1+x^2}}$

2. $\dfrac{d}{dx}(\cosh^{-1}x) = \dfrac{1}{\sqrt{x^2-1}}$

3. $\dfrac{d}{dx}(\tanh^{-1}x) = \dfrac{1}{1-x^2}$ $\qquad |x| < 1$

4. $\dfrac{d}{dx}(\coth^{-1}x) = \dfrac{1}{1-x^2}$ $\qquad |x| > 1$

5. $\dfrac{d}{dx}(\operatorname{sech}^{-1}x) = \dfrac{-1}{x\sqrt{1-x^2}}$

6. $\dfrac{d}{dx}(\operatorname{csch}^{-1}x) = \dfrac{-1}{x\sqrt{1+x^2}}$

# EXERCISE 2.2

**Differentiate** w.r.t. $x$, **(Problems 1 – 14)**

1. $\sqrt{a^2+x^2}$

**Sol. Let** $y = \sqrt{a^2+x^2}$

or $y = (a^2+x^2)^{1/2}$

**Diff. w.r.t. x**

$$\frac{dy}{dx} = \frac{1}{2}(a^2+x^2)^{\frac{1}{2}-1} \cdot \frac{d}{dx}(a^2+x^2).$$

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$$\frac{dy}{dx} = \frac{1}{2}(a^2+x^2)^{-1/2}\cdot 2x$$

$$= \frac{x}{(a^2+x^2)^{1/2}}$$

$$= \frac{x}{\sqrt{a^2+x^2}}$$



**2.** $\sqrt[3]{x^2+x+1}$

**Sol. Let** $y = (x^2+x+1)^{1/3}$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{1}{3}(x^2+x+1)^{1/3-1}\cdot\frac{d}{dx}(x^2+x+1)$$

$$= \frac{1}{3}(x^2+x+1)^{-2/3}\cdot(2x+1)$$

$$= \frac{1}{3}\cdot\frac{1}{(x^2+x+1)^{2/3}}\cdot(2x+1)$$

$$= \frac{2x+1}{3(x^2+x+1)^{2/3}}$$

**3.** $\dfrac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$

**Sol. Let** $y = \dfrac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$

Multiplying numerator & denominator by $\sqrt{a+x}-\sqrt{a-x}$

$$= \frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}} \times \frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}$$

$$= \frac{(\sqrt{a+x}-\sqrt{a-x})^2}{(\sqrt{a+x})^2-(\sqrt{a-x})^2} = \frac{(a+x)+(a-x)-2\sqrt{a+x}\sqrt{a-x}}{(a+x)-(a-x)}$$

$$= \frac{2a-2\sqrt{(a+x)(a-x)}}{a+x-a+x} = \frac{2a-2\sqrt{a^2-x^2}}{2x}$$

$$y = \frac{a-\sqrt{a^2-x^2}}{x}$$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{x\left[0-\frac{1}{2}(a^2-x^2)^{-1/2}(-2x)\right]-\left[a-\sqrt{a^2-x^2}\right]\cdot 1}{x^2}$$

$$= \frac{x\cdot\frac{x}{\sqrt{a^2-x^2}}-a+\sqrt{a^2-x^2}}{x^2}$$

$$= \frac{\frac{x^2}{\sqrt{a^2-x^2}}+\sqrt{a^2-x^2}-a}{x^2} = \frac{\frac{x^2+a^2-x^2}{\sqrt{a^2-x^2}}-a}{x^2}$$

⑥

$$= \frac{\frac{a^2}{\sqrt{a^2-x^2}} - a}{x^2} = \frac{a^2 - a\sqrt{a^2-x^2}}{x^2\sqrt{a^2-x^2}} = \frac{a(a-\sqrt{a^2-x^2})}{x^2\sqrt{a^2-x^2}}$$

4. $y = \dfrac{\sqrt{\sin x}}{\sin\sqrt{x}}$

Sol. Let $y = \dfrac{\sqrt{\sin x}}{\sin\sqrt{x}}$

Diff. w.r.t. x

Sol.

$$\frac{dy}{dx} = \frac{\sin\sqrt{x}\left\{\frac{1}{2\sqrt{\sin x}} \cdot \cos x\right\} - \sqrt{\sin x}\left\{\cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}\right\}}{(\sin\sqrt{x})^2}$$

$$= \frac{\sin\sqrt{x} \cdot \frac{\cos x}{2\sqrt{\sin x}} - \frac{\sqrt{\sin x} \cdot \cos\sqrt{x}}{2\sqrt{x}}}{(\sin\sqrt{x})^2}$$

$$= \frac{\frac{\sqrt{x}\ \sin\sqrt{x}\ \cos x - \sin x\cos\sqrt{x}}{2\sqrt{x}\sqrt{\sin x}}}{(\sin\sqrt{x})^2}$$

$$= \frac{\sqrt{x}\ \sin\sqrt{x}\ \cos x - \sin x\cos\sqrt{x}}{2\sqrt{x}\sqrt{\sin x}\ \sin^2\sqrt{x}}$$

**5.** $\sqrt{\log_{10}(x^2+1)}$

**Sol.** **Let** $y = \sqrt{\log_{10}(x^2+1)}$

or $y = \sqrt{\dfrac{\log_e(x^2+1)}{\log_e 10}} = \sqrt{\dfrac{\ln(x^2+1)}{\ln 10}} = \dfrac{\sqrt{\ln(x^2+1)}}{\sqrt{\ln 10}}$

or $y = \dfrac{1}{\sqrt{\ln 10}} \cdot \sqrt{\ln(x^2+1)}$

Diff. w.r.t. x

$$dy/dx = \frac{1}{\sqrt{\ln 10}} \cdot \frac{1}{2\sqrt{\ln(x^2+1)}} \cdot \frac{1}{x^2+1} \cdot 2x$$

$$\Rightarrow dy/dx = \frac{x}{\sqrt{\ln 10} \cdot \sqrt{\ln(x^2+1)} \cdot (x^2+1)} \quad \text{Ans.}$$

**6.** $y = \tan(\sin x)$

Diff. w.r.t. x

**Sol.** $\dfrac{dy}{dx} = \sec^2(\sin x) \cdot \dfrac{d}{dx}(\sin x)$

$= \sec^2(\sin x) \cdot \cos x$

$= \cos x \cdot \sec^2(\sin x)$.

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<u>Note</u> $\dfrac{\log_a x}{\log_a y} = \log_y x$.

**7.** $y = \arctan\left(\dfrac{x\sin a}{1 - x\cos a}\right)$

Diff. w.r.t. x

**Sol.** $\dfrac{dy}{dx} = \dfrac{1}{1+\left(\dfrac{x\sin a}{1-x\cos a}\right)^2}\cdot\dfrac{d}{dx}\left(\dfrac{x\sin a}{1-x\cos a}\right)$

$= \dfrac{1}{1+\dfrac{(x\sin a)^2}{(1-x\cos a)^2}}\cdot\dfrac{(1-x\cos a).\sin a - x\sin a.(-\cos a)}{(1-x\cos a)^2}$

$= \dfrac{1}{\dfrac{(1-x\cos a)^2+(x\sin a)^2}{(1-x\cos a)^2}}\cdot\dfrac{\sin a - \cancel{x\sin a\cos a} + \cancel{x\sin a\cos a}}{(1-x\cos a)^2}$

$=$

$= \dfrac{\sin a}{1-2x\cos a + x^2\cos^2 a + x^2\sin^2 a} = \dfrac{\sin a}{1-2x\cos a + x^2(\cos^2 a+\sin^2 a)}$

$= \dfrac{\sin a}{1-2x\cos a + x^2}$

**8.** $y = \ln\dfrac{x^2+x+1}{x^2-x+1}$

**Sol.** $y = \ln(x^2+x+1) - \ln(x^2-x+1)$

Diff. w.r.t. x

$\dfrac{dy}{dx} = \dfrac{1}{(x^2+x+1)}\cdot\dfrac{d}{dx}(x^2+x+1) - \dfrac{1}{(x^2-x+1)}\cdot\dfrac{d}{dx}(x^2-x+1)$

$= \dfrac{2x+1}{x^2+x+1} - \dfrac{2x-1}{x^2-x+1}$

$= \dfrac{(2x+1)(x^2-x+1)-(2x-1)(x^2+x+1)}{(x^2+x+1)(x^2-x+1)}$

$= \dfrac{(2x^3-2x^2+2x+x^2-x+1)-(2x^3+2x^2+2x-x^2-x-1)}{(x^2+x+1)(x^2-x+1)}$

$= \dfrac{2x^3-x^2+x+1-2x^3-x^2-x+1}{(x^2+x+1)(x^2-x+1)} = \dfrac{-2x^2+2}{(x^2+x+1)(x^2-x+1)}$

$= \dfrac{2(1-x^2)}{(x^2+x+1)(x^2-x+1)}$

**9.** $y = x^{x^2}$

**Sol.** Let $y = x^{x^2}$

taking log on both sides

$\ln y = \ln x^{x^2}$

$\ln y = x^2\ln x$

Diff. w.r.t. x

$\dfrac{1}{y}\cdot\dfrac{dy}{dx} = x^2\cdot\dfrac{1}{x} + \ln x\cdot 2x$

$$= x + 2x\ln x$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = x\,(1 + 2\ln x)$$

$$\Rightarrow \quad \frac{dy}{dx} = y\,.\,x\,(2\ln x + 1)$$

$$= x^{x^2}\cdot \quad x\,(2\ln x + 1)$$

$$= x^{x^2+1}\cdot \quad (2\ln x + 1)$$

**10.** $\ln(x^2 + x)$

**Sol.** Let $y = \ln(x^2 + x)$

Differentiating w.r.t $x$, we have

$$\frac{dy}{dx} = \frac{1}{(x^2 + x)}\cdot\frac{d}{dx}(x^2 + x)$$

$$= \frac{1}{x^2 + x}\cdot(2x+1)$$

$$= \frac{2x+1}{x(x+1)}$$

**11.** $y = (\text{arc}\sin x)^{x^{\frac{1}{x}}}$

**Sol.** Taking logarithm of both sides

$$\ln y = \ln(\sin^{-1}x)^{x^{1/x}}$$

$$\ln y = x^{1/x}\cdot\ln(\sin^{-1}x)$$

Diff. w.r.t. $x$

$$\frac{1}{y}\frac{dy}{dx} = x^{1/x}\cdot\frac{1}{\sin^{-1}x}\cdot\frac{1}{\sqrt{1-x^2}} + \ln\sin^{-1}x\cdot\frac{d}{dx}\left(x^{\frac{1}{x}}\right) \quad \text{---} (1)$$

Now, let $u = x^{1/x}$

$$\ln u = \ln x^{1/x}$$

or $\ln u = \frac{1}{x}\cdot\ln x$

Diff. w.r.t. $x$

$$\frac{1}{u}\frac{du}{dx} = \frac{1}{x}\cdot\frac{1}{x} + \ln x\left(-\frac{1}{x^2}\right)$$

$$= \frac{1}{x^2} + \ln x\left(-\frac{1}{x^2}\right)$$

$$\frac{du}{dx} = u\left(\frac{1}{x^2} - \frac{1}{x^2}\ln x\right)$$

$$= x^{1/x}\frac{1}{x^2}(1 - \ln x)$$

$$\frac{du}{dx} = x^{\frac{1}{x}-2}(1 - \ln x)$$

$$\text{or } \frac{d}{dx}(x^{1/x}) = x^{\frac{1}{x}-2}(1-\ln x)$$

**Putting in (1)**

$$\frac{1}{y}\frac{dy}{dx} = x^{\frac{1}{x}} \cdot \frac{1}{\sin^{-1}x \cdot \sqrt{1-x^2}} + \ln\sin^{-1}x \cdot x^{\frac{1}{x}-2}(1-\ln x)$$

$$\Rightarrow \frac{dy}{dx} = y \cdot \left[x^{\frac{1}{x}} \cdot \frac{1}{\sin^{-1}x \cdot \sqrt{1-x^2}} + x^{\frac{1}{x}-2} \cdot \ln\sin^{-1}x \cdot \ln(1-x)\right]$$

$$= (\sin^{-1}x)^{x^{\frac{1}{x}}}\left[x^{\frac{1}{x}} \cdot \frac{1}{\sin^{-1}x \cdot \sqrt{1-x^2}} + x^{\frac{1}{x}-2} \cdot \ln\sin^{-1}x \cdot \ln(1-x)\right]$$

12. $y = |x^2 - 9|$

Sol. Given $y = |x^2 - 9|$

Here
$$\left.\begin{aligned} y &= x^2 - 9, & \text{if } |x| \geq 3 \\ &= -(x^2 - 9), & \text{if } |x| < 3 \end{aligned}\right]$$

Note Let $x \in R$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Diff. w.r.t. $x$

$$\left.\begin{aligned} \frac{dy}{dx} &= 2x & \text{if } |x| \geq 3 \\ &= -2x & \text{if } |x| < 3 \end{aligned}\right] \text{— Ans.}$$

13. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

**Sol.** Let $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

Diff. w.r.t. $x$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{d}{dx}\left(x + \sqrt{x+\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \left(1 + \frac{1}{2\sqrt{x+\sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}}\right)\right)$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \left(1 + \frac{1}{2\sqrt{x+\sqrt{x}}}\right) \cdot \left(1 + \frac{1}{2\sqrt{x}}\right) \text{— Ans.}$$

**14.** $y = (x + |x|)^{1/2}$

**Sol.** Here, $y = (x+x)^{1/2}$, if $x \geq 0$

$= (x-x)^{1/2}$ if $x < 0$

or $y = (2x)^{1/2}$ if $x \geq 0$

$= 0$ if $x < 0$

Diff. w.r.t. $x$

$\frac{dy}{dx} = \frac{1}{2}(2x)^{-1/2}\cdot 2 = \frac{1}{\sqrt{2x}}$ if $x \geq 0$

$= 0$ if $x < 0$

**15.** Differentiate

$$\arctan \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \quad \text{w.r.t.} \quad \arccos x^2$$

Sol. Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right)$ & $u = \cos^{-1}x^2$ then $\frac{dy}{du} = ?$

as $u = \cos^{-1}x^2 \Rightarrow x^2 = \cos u$. Put in eq.

$$y = \tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{1+\cos u} - \sqrt{1-\cos u}}{\sqrt{1+\cos u} + \sqrt{1-\cos u}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\frac{u}{2}} - \sqrt{2\sin^2\frac{u}{2}}}{\sqrt{2\cos^2\frac{u}{2}} + \sqrt{2\sin^2\frac{u}{2}}}\right)$$

$$= \tan^{-1}\left[\frac{\sqrt{2}\cdot\cos u/2 - \sqrt{2}\sin u/2}{\sqrt{2}\cos u/2 + \sqrt{2}\sin u/2}\right]$$

$$= \tan^{-1}\left[\frac{\cos u/2 - \sin u/2}{\cos u/2 + \sin u/2}\right]$$

Dividing numerator & denominator by $\cos u/2$.

$$= \tan^{-1}\left(\frac{1-\tan u/2}{1+\tan u/2}\right)$$

$$y = \tan^{-1}\left[\frac{\tan\pi/4 - \tan u/2}{1+\tan\pi/4\cdot\tan u/2}\right] = \tan^{-1}.\tan(\pi/4 - u/2)$$

So $y = \pi/4 - u/2$

Diff. w.r.t. $u$

$\frac{dy}{du} = -1/2$ —— Ans.

**16.** $y = x^{\sin y}$

**Sol.** $y = x^{\sin y}$

Taking logarithm of both sides, we have

$\ln y = \ln x^{\sin y}$

or $\ln y = \sin y . \ln x$

Diff. w.r.t. $x$

$\frac{1}{y} . \frac{dy}{dx} = \sin y . \frac{1}{x} + \ln x . \cos y . \frac{dy}{dx}$

Multiplying both sides by $xy$.

$x \, dy/dx = y\sin y + xy\ln x . \cos y . dy/dx$

$x \, dy/dx - xy\cos y . \ln x . dy/dx = y\sin y$

$\frac{dy}{dx}(x - xy\cos y . \ln x) = y\sin y$

$\Rightarrow \frac{dy}{dx} = \frac{y\sin y}{x - xy\cos y . \ln x}$

**17.** $x^y = e^{x-y}$

**Sol.** Taking logarithm of both sides, we have

$\ln x^y = \ln e^{x-y}$

$y\ln x = (x-y).\ln e$

or $y\ln x = x - y$

$y\ln x + y = x$

$y(\ln x + 1) = x$

$\Rightarrow y = \frac{x}{1+\ln x}$

Diff. w.r.t. $x$

$$\frac{dy}{dx} = \frac{(1+\ln x).1 - x . \frac{1}{x}}{(1+\ln x)^2}$$

$$= \frac{1 + \ln x - 1}{(1+\ln x)^2}$$

$$\frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$$

18.

$y^x + x^y = c$

Sol.- Given $y^x + x^y = c$

**Sol.** Let $u = y^x$ and $v = x^y$

Taking logarithm of both sides of the first equation

$$\ln u = \ln y^x$$

$$\ln u = x \ln y$$

Differentiating w.r.t. $x$, we have

$$\frac{1}{u}\frac{du}{dx} = x \cdot \frac{1}{y}\frac{dy}{dx} + \ln y$$

$$\frac{du}{dx} = u\left(\frac{x}{y}\cdot\frac{dy}{dx} + \ln y\right)$$

$$= y^x\left(\frac{x}{y}\frac{dy}{dx} + \ln y\right)$$

Now from $v = x^y$, taking logarithm, we get

$$\ln v = \ln x^y$$

or $\ln v = y \ln x$

Differentiating w.r.t. $x$, we obtain

$$\frac{1}{v}\frac{dv}{dx} = \frac{y}{x} + \ln x \frac{dy}{dx}$$

$$\frac{dv}{dx} = v\left[\frac{y}{x} + \ln x \cdot \frac{dy}{dx}\right]$$

$$= x^y\left[\frac{y}{x} + \ln x \cdot \frac{dy}{dx}\right]$$

The given equation is

$$u + v = c$$

Differentiating w.r.t. $x$, we get

$$\frac{du}{dx} + \frac{dv}{dx} = 0$$

Putting the values of $\frac{du}{dx}$ and $\frac{dv}{dx}$ from (1) and (2) into (3), we have

$$y^x\left[\frac{x}{y}\frac{dy}{dx} + \ln y\right] + x^y\left[\frac{y}{x} + \ln x \cdot \frac{dy}{dx}\right] = 0$$

$$x y^{x-1} \, dy/dx + y^x \ln y + x^y \cdot y/x + x^y \cdot \ln x \cdot dy/dx = 0$$

or $$(x y^{x-1} + x^y \ln x)\frac{dy}{dx} + \left[y^x \ln y + x^y \cdot \frac{y}{x}\right] = 0$$

$$(x y^{x-1} + x^y \cdot \ln x) \cdot \frac{dy}{dx} = -(y^x \cdot \ln y + y \cdot x^{y-1})$$

or $\dfrac{dy}{dx} = -\dfrac{y^x \ln y + y x^{y-1}}{xy^{x-1} + y^x \ln x}$

**19.** $\dfrac{x+y}{x-y} = x^2 + y^2$

**Sol.** **Differentiating** w.r.t. $x$, we have

$$\frac{(x-y)(1+y') - (x+y)(1-y')}{(x-y)^2} = 2x + 2yy'$$

or $$\frac{(x-y) + (x-y)y' - (x+y) + y'(x+y)}{(x-y)^2} = 2x + 2yy'$$

$$(\cancel{x}-y-\cancel{x}-y) + y'(x\cancel{-y}+x\cancel{+y}) = 2(x+yy')(x-y)^2$$

$$-2y + y'(2x) = 2(x+yy')(x-y)^2$$

$$-y + xy' = (x+yy')(x-y)^2$$

$$xy' - y = x(x-y)^2 + yy'(x-y)^2$$

$$xy' - yy'(x-y)^2 = x(x-y)^2 + y$$

$$y'(x-y(x-y)^2) = y + x(x-y)^2$$

$$\frac{dy}{dx} = \frac{y + x(x-y)^2}{x - y(x-y)^2}$$

**20.** $x + \text{arc}\sin y = xy$

**Sol.** $x + \sin^{-1} y = xy$

**Differentiating** (1) w.r.t. $x$, we have

$$1 + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = x\frac{dy}{dx} + y.1$$

$$\frac{1}{\sqrt{1-y^2}}\cdot\frac{dy}{dx} - x\frac{dy}{dx} = y - 1$$

or $$\frac{dy}{dx}\left(\frac{1}{\sqrt{1-y^2}} - x\right) = y - 1$$

or $$\frac{dy}{dx}\left(\frac{1 - x\sqrt{1-y^2}}{\sqrt{1-y^2}}\right) = y - 1$$

**Therefore,**

$$\frac{dy}{dx} = \frac{(y-1)\sqrt{1-y^2}}{1 - x\sqrt{1-y^2}}$$

In Problems 21-30, find $f'(x)$:

**21.** $f(x) = \ln\left(x+\sqrt{x^2-1}\right)$

Sol. Here $f(x) = \ln\left(x+\sqrt{x^2-1}\right)$

Diff. w.r.t. $x$

$$f'(x) = \frac{1}{\left(x+\sqrt{x^2+1}\right)} \cdot \frac{d}{dx}\left(x+\sqrt{x^2-1}\right).$$

$$= \frac{1}{\left(x+\sqrt{x^2-1}\right)} \cdot \left(1+\frac{1}{2\sqrt{x^2-1}} \cdot 2x\right)$$

$$= \frac{1}{\left(x+\sqrt{x^2-1}\right)} \cdot \left(1+\frac{x}{\sqrt{x^2-1}}\right)$$

$$= \frac{1}{\left(x+\sqrt{x^2-1}\right)} \cdot \left[\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}\right]$$

$$= \frac{1}{\sqrt{x^2-1}}$$

**22.** $f(x) = \ln \dfrac{e^x}{1+e^x}$

**Sol.** $f(x) = \ln \dfrac{e^x}{1+e^x}$

$$= \ln e^x - \ln(1+e^x)$$

$$f(x) = x - \ln(1+e^x)$$

Diff. w.r.t. $x$

$$f'(x) = 1 - \frac{1}{1+e^x} \cdot e^x = 1 - \frac{e^x}{1+e^x}$$

$$= \frac{1+e^x-e^x}{1+e^x} = \frac{1}{1+e^x}$$

**23.** $f(x) = x^{\ln x}$

**Sol.** Taking ln of both sides, we get

$$\ln(f(x)) = \ln x^{\ln x}$$

$$\text{or } \ln f(x) = \ln x \cdot \ln x$$

Diff. w.r.t. $x$

$$\frac{1}{f(x)} \cdot f'(x) = \ln x \cdot \frac{1}{x} + \ln x \cdot \frac{1}{x}$$

$$f'(x) = f(x)\left[\frac{\ln x}{x} + \frac{\ln x}{x}\right]$$

$$= x^{\ln x}\left[\frac{2\ln x}{x}\right]$$

$$= x^{\ln x - 1} \cdot (2\ln x)$$

**24.** $f(x) = \ln \dfrac{1+\sqrt{x}}{1-\sqrt{x}}$

**Sol.** $f(x) = \ln\left(1+\sqrt{x}\right) - \ln\left(1-\sqrt{x}\right)$

Diff. w.r.t. x

$$f'(x) = \frac{1}{1+\sqrt{x}}\cdot\left(0+\frac{1}{2\sqrt{x}}\right) - \frac{1}{1-\sqrt{x}}\cdot\left(0-\frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{x}(1+\sqrt{x})} + \frac{1}{2\sqrt{x}(1-\sqrt{x})}$$

$$= \frac{1}{2\sqrt{x}}\left[\frac{1}{1+\sqrt{x}} + \frac{1}{1-\sqrt{x}}\right]$$

$$= \frac{1}{2\sqrt{x}}\left[\frac{1-\sqrt{x}+1+\sqrt{x}}{(1+\sqrt{x})(1-\sqrt{x})}\right]$$

$$= \frac{1}{2\sqrt{x}}\cdot\frac{2}{(1-x)}$$

$$= \frac{1}{\sqrt{x}(1-x)}$$

**25.** $f(x) = e^{ax} \cos\,(b \operatorname{arc} \tan x)$

**Sol.** $f(x) = e^{ax} \cos\,(b\tan^{-1} x)$

Diff. w.r.t. x

$$f'(x) = e^{ax}\cdot -\sin(b\tan^{-1}x)\cdot b\cdot\frac{1}{1+x^2}) + \cos(b\tan^{-1}x)\cdot e^{ax}\cdot a$$

$$= e^{ax}\left[a\cos(b\tan^{-1}x) - \frac{b}{1+x^2}\cdot\sin(b\tan^{-1}x)\right]$$

$$= e^{ax}\left[\frac{a(1+x^2)\cdot\cos(b\tan^{-1}x) - b\sin(b\tan^{-1}x)}{(1+x^2)}\right]$$

**26.** $f(x) = \dfrac{1}{\sqrt{b^2-a^2}} \ln \dfrac{\sqrt{b+a}+\sqrt{b-a}\tan\left(\frac{x}{2}\right)}{\sqrt{b+a}-\sqrt{b-a}\tan\left(\frac{x}{2}\right)}$

**Sol.**

Here $f(x) = \dfrac{1}{\sqrt{b^2-a^2}}\left[\ln\left(\sqrt{b+a}+\sqrt{b-a}\tan\frac{x}{2}\right) - \ln\left(\sqrt{b+a}-\sqrt{b-a}\tan\frac{x}{2}\right)\right]$

Diff. w.r.t. x

$$f'(x) = \frac{1}{\sqrt{b^2-a^2}}\left[\frac{1}{\sqrt{b+a}+\sqrt{b-a}\tan x/2}\cdot\sqrt{b-a}\cdot\sec^2\frac{x}{2}\cdot\frac{1}{2} - \frac{-\sqrt{b-a}\cdot\sec^2\frac{x}{2}\cdot\frac{1}{2}}{\sqrt{b+a}-\sqrt{b-a}\tan x/2}\right]$$

$$f'(x) = \frac{1}{\sqrt{b^2-a^2}}\left[\frac{\frac{1}{2}\sqrt{b-a}\cdot\sec^2\frac{x}{2}}{\sqrt{b+a}+\sqrt{b-a}\tan\frac{x}{2}} + \frac{\frac{1}{2}\sqrt{b-a}\cdot\sec^2\frac{x}{2}}{\sqrt{b+a}-\sqrt{b-a}\tan x/2}\right]$$ 16

$$= \frac{\sqrt{b-a}\sec^2\frac{x}{2}}{2\sqrt{(b-a)(b+a)}}\left[\frac{1}{\sqrt{b+a}+\sqrt{b-a}\tan\frac{x}{2}} + \frac{1}{\sqrt{b+a}-\sqrt{b-a}\tan\frac{x}{2}}\right]$$

$$= \frac{\cancel{\sqrt{b-a}}\sec^2\frac{x}{2}}{2\cancel{\sqrt{b-a}}\sqrt{b+a}}\cdot\left[\frac{\sqrt{b+a}-\sqrt{b-a}\cancel{\tan x/2}+\sqrt{b+a}+\sqrt{b-a}\cancel{\tan x/2}}{(\sqrt{b+a}+\sqrt{b-a}\tan x/2)(\sqrt{b+a}-\sqrt{b-a}\tan x/2)}\right]$$

$$= \frac{\sec^2\frac{x}{2}}{2\cancel{\sqrt{b+a}}}\cdot\frac{2\cancel{\sqrt{b+a}}}{(b+a)-(b-a)\cdot\frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}}$$

$$= \frac{\sec^2\frac{x}{2}}{\frac{(b+a)\cos^2\frac{x}{2}-(b-a)\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}} =$$

$$= \frac{\sec^2\frac{x}{2}\cdot\cos^2\frac{x}{2}}{b\cos^2\frac{x}{2}+a\cos^2\frac{x}{2}-b\sin^2\frac{x}{2}+a\sin^2\frac{x}{2}}$$

$$= \frac{1}{a\left(\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)+b\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}$$

$$= \frac{1}{a+b\cos x}$$

**27.** $f(x) = xa^x \sinh x$

Sol: $f(x) = x\cdot a^x\cdot\sinh x$

Diff. w.r.t. $x$

$$f'(x) = x\cdot a^x\cdot\cosh x + x\sinh x\cdot a^x\ln a + 1\cdot a^x\cdot\sinh x$$
$$= a^x[x\cosh x + x\sinh x\cdot\ln a + \sinh x]$$

**28.** $f(x) = -\dfrac{\cos x}{2\sin^2 x} + \dfrac{1}{2}\ \ln\tan\dfrac{x}{2}$

Sol:- Here $f(x) = -\frac{1}{2}\cdot\frac{\cos x}{\sin^2 x} + \frac{1}{2}\ln\tan\frac{x}{2}$

Diff. w.r.t. $x$

$$f'(x) = -\frac{1}{2}\left[\frac{\sin^2 x\cdot(-\sin x)-\cos x\cdot 2\sin x\cos x}{\sin^4 x}\right]+\frac{1}{2}\cdot\frac{1}{\tan\frac{x}{2}}\cdot\sec^2\frac{x}{2}\cdot\frac{1}{2}$$

$$= -\frac{1}{2}\left[\frac{-\sin^3 x-2\sin x\cos^2 x}{\sin^4 x}\right]+\frac{1}{4}\cdot\frac{\sec^2\frac{x}{2}}{\tan\frac{x}{2}}$$

$$= -\frac{1}{2}\left[\frac{-\sin^2 x-2\cos^2 x}{\sin^3 x}\right]+\frac{1}{4}\ \frac{\frac{1}{\cos^2 x/2}}{\frac{\sin\ x/2}{\cos\ x/2}}$$

$$= -\frac{1}{2}\left[\frac{-\sin^2 x - 2\cos^2 x}{\sin^3 x}\right] + \frac{1}{(4\sin\frac{x}{2}\cos\frac{x}{2})}$$ 17

$$= -\frac{1}{2}\left[\frac{-\sin^2 x - 2\cos^2 x}{\sin^3 x}\right] + \frac{1}{2(2\sin\frac{x}{2}\cos\frac{x}{2})}$$

$$= \frac{\sin^2 x + 2\cos^2 x}{2\sin^3 x} + \frac{1}{2\sin x}$$

$$= \frac{\sin^2 x + 2\cos^2 x + \sin^2 x}{2\sin^3 x} = \frac{2\sin^2 x + 2\cos^2 x}{2\sin^3 x}$$

$$= \frac{2(\sin^2 x + \cos^2 x)}{2\sin^3 x} = \frac{1}{\sin^3 x} = \operatorname{Cosec}^3 x$$

**29.** $f(x) = \operatorname{arcsec}\left(\csc x + \sqrt{x}\right)$

Sol:- Here $f(x) = \sec^{-1}(\operatorname{Cosec} x + \sqrt{x})$

Diff. w.r.t. x

$$f'(x) = \frac{1}{(\operatorname{Cosec} x + \sqrt{x})\sqrt{(\operatorname{Cosec} x + \sqrt{x})^2 - 1}} \cdot \frac{d}{dx}(\operatorname{Cosec} x + \sqrt{x})$$

$$= \frac{1}{(\operatorname{Cosec} x + \sqrt{x})\sqrt{(\operatorname{Cosec} x + \sqrt{x})^2 - 1}} \times \left(-\operatorname{Cosec} x . \operatorname{Cot} x + \frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{(\operatorname{Cosec} x + \sqrt{x})\sqrt{\operatorname{Cosec}^2 x + x + 2\sqrt{x}\operatorname{Cosec} x - 1}} \cdot \left[\frac{-2\sqrt{x}\operatorname{Cosec} x \operatorname{Cot} x + 1}{2\sqrt{x}}\right]$$

$$= \frac{1 - 2\sqrt{x}\csc x \cot x}{2\sqrt{x}\left(\operatorname{Cosec} x + \sqrt{x}\right)\sqrt{\operatorname{Cosec}^2 x + x + 2\sqrt{x}.\operatorname{Cosec} x - 1}}$$

**30.** $f(x) = \left(1 + \frac{1}{x}\right)^{x^2}$

**Sol.** Taking ln of both sides, we have

$$\ln f(x) = \ln\left(1 + \frac{1}{x}\right)^{x^2}$$

$$\ln f(x) = x^2 \ln\left(\frac{x+1}{x}\right).$$

Diff. w.r.t. x

$$\frac{1}{f(x)} \cdot f'(x) = x^2 \cdot \frac{1}{\frac{x+1}{x}} \cdot \left[\frac{x . 1 - (x+1) . 1}{x^2}\right] + \ln\left(\frac{x+1}{x}\right) . 2x$$

$$\frac{1}{f(x)} . f'(x) = \frac{x}{x+1}\left[\frac{x - x - 1}{x^2}\right] + 2x\ln\left(\frac{x+1}{x}\right)$$

$$= \frac{-1}{x(x+1)} + 2x\ln\frac{(x+1)}{x}$$

$$\Rightarrow f'(x) = f(x)\left[2x\ln\left(\frac{x+1}{x}\right) - \frac{1}{x(x+1)}\right]$$

$$= \left(1+\frac{1}{x}\right)^{x^2}\left[2x\ln\left(\frac{x+1}{x}\right) - \frac{1}{x(x+1)}\right]$$

---

Differentiate with respect to $x$ each of the following (Problems 31-42)

**31.** $\arctan\left(\dfrac{1+2x}{2-x}\right)$

**Sol.** Let $y = \tan^{-1}\left(\dfrac{1+2x}{2-x}\right)$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{1}{1+\left(\frac{1+2x}{2-x}\right)^2}\cdot\frac{d}{dx}\left(\frac{1+2x}{2-x}\right)$$

$$= \frac{1}{1+\frac{(1+2x)^2}{(2-x)^2}}\cdot\frac{(2-x)\cdot 2-(1+2x)\cdot(-1)}{(2-x)^2}$$

$$= \frac{1}{\frac{(2-x)^2+(1+2x)^2}{(2-x)^2}}\cdot\frac{4-2x+1+2x}{(2-x)^2}$$

$$= \frac{\cancel{(2-x)^2}}{4-4x+x^2+1+4x+4x^2}\cdot\frac{5}{\cancel{(2-x)^2}} = \frac{5}{5+5x^2} = \frac{5}{5(1+x^2)} = \frac{1}{1+x^2}$$

**32.** $\ln(\arcsin e^x) + yx^2 = 1$

**Sol.** Given $\ln(\sin^{-1}e^x) + yx^2 = 1$

Diff. w.r.t. x

$$\frac{1}{\sin^{-1}(e^x)}\cdot\frac{d}{dx}(\sin^{-1}e^x) + \frac{d}{dx}(yx^2) = 0$$

$$\frac{1}{\sin^{-1}(e^x)}\cdot\frac{1}{\sqrt{1-(e^x)^2}}\cdot e^x + y\cdot 2x + x^2\frac{dy}{dx} = 0$$

$$\frac{1}{\sin^{-1}(e^x)}\cdot\frac{e^x}{\sqrt{1-e^{2x}}} + 2xy + x^2\frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} = -2xy - \frac{e^x}{\sin^{-1}(e^x)\sqrt{1-e^{2x}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x^2}\left[-2xy - \frac{e^x}{\sin^{-1}(e^x)\sqrt{1-e^{2x}}}\right] = -\frac{2y}{x} - \frac{e^x}{x^2\sin^{-1}(e^x)\sqrt{1-e^{2x}}}$$

**33.** $y = (\arcsin x^2)^{\pi}$

**Sol.** Let $y = (\sin^{-1} x^2)^{\pi}$

Diff. w.r.t. $x$

$$\frac{dy}{dx} = \pi (\sin^{-1}x^2)^{\pi-1} \cdot \frac{d}{dx}(\sin^{-1}x^2)$$

$$= \pi (\sin^{-1}x^2)^{\pi-1} \cdot \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$$

$$= \pi \cdot (\sin^{-1}x^2)^{\pi-1} \cdot \frac{2x}{\sqrt{1-x^4}}$$

$$= \frac{2x\pi(\sin^{-1}x^2)^{\pi-1}}{\sqrt{1-x^4}}$$

**34.** $\arctan\left(\dfrac{y}{x}\right) + yx^2 = 1$

**Sol.** Given $\tan^{-1}(y/x) + yx^2 = 1$

Diff. w.r.t. $x$

$$\frac{1}{1+(y/x)^2} \cdot \left(\frac{x\, dy/dx - y \cdot 1}{x^2}\right) + y \cdot 2x + x^2 \frac{dy}{dx} = 0$$

$$\frac{1}{1+y^2/x^2} \cdot \left(\frac{x\, dy/dx - y}{x^2}\right) + 2xy + x^2\frac{dy}{dx} = 0$$

$$\frac{1}{\frac{x^2+y^2}{x^2}} \cdot \left(\frac{x\frac{dy}{dx} - y}{x^2}\right) + 2xy + x^2\frac{dy}{dx} = 0$$

$$\frac{x^2}{x^2+y^2} \cdot \frac{x\frac{dy}{dx} - y}{x^2} + 2xy + x^2\frac{dy}{dx} = 0$$

$$\frac{x\, dy/dx - y}{x^2+y^2} + x^2\frac{dy}{dx} = -2xy$$

or $x\dfrac{dy}{dx} - y + x^2(x^2+y^2)\dfrac{dy}{dx} = -2xy(x^2+y^2)$

$dy/dx\,(x + x^4 + x^2y^2) = y - 2xy(x^2+y^2)$

$\Rightarrow dy/dx = \dfrac{y(1-2x(x^2+y^2))}{x(1+x^3+xy^2)}$ — Ans.

**35.** $y = \dfrac{1-\cosh x}{1+\cosh x}$

**Sol.**

Given $y = \dfrac{1-\cosh x}{1+\cosh x}$

Diff. w.r.t. $x$

$$\frac{dy}{dx} = \frac{(1+\cosh x)\cdot\frac{d}{dx}(1-\cosh x) - (1-\cosh x)\cdot\frac{d}{dx}(1+\cosh x)}{(1+\cosh x)^2}$$

$$= \frac{(1+\cosh x)(-\sinh x) - (1-\cosh x)\cdot\sinh x}{(1+\cosh x)^2}$$

$$= \frac{-\sinh x - \cosh x \cdot \sinh x - \sinh x + \cosh x \cdot \sinh x}{(1+\cosh x)^2} = \frac{-2\sinh x}{(1+\cosh x)^2}$$

$$= \frac{-2\cdot 2\sinh\frac{x}{2}\cosh\frac{x}{2}}{(2\cosh^2\frac{x}{2})^2} = \frac{-4\sinh\frac{x}{2}\cosh\frac{x}{2}}{4\cosh^4\frac{x}{2}} = -\frac{\sinh\frac{x}{2}}{\cosh\frac{x}{2}} \cdot \frac{1}{\cosh^2\frac{x}{2}} = -\tanh\frac{x}{2}\,\mathrm{sech}^2\frac{x}{2}$$

**36.** $y = \ln(\tanh 2x)$

Sol.- $y = \ln(\tanh 2x)$

Diff. w.r.t. $x$

$$\frac{dy}{dx} = \frac{1}{\tanh 2x} \cdot \frac{d}{dx}(\tanh 2x)$$

$$= \frac{1}{\tanh 2x} \cdot \operatorname{sech}^2 2x \cdot 2$$

$$= \coth 2x \cdot \operatorname{sech}^2 2x \cdot 2$$

$$= \frac{\cosh 2x}{\sinh 2x} \cdot \frac{1}{\cosh^2 2x} \cdot 2$$

$$= \frac{2}{\sinh 2x \cdot \cosh 2x} = \frac{4}{2\sinh 2x \cdot \cosh 2x}$$

$$= \frac{4}{\sinh 4x} = 4\operatorname{cosech} 4x$$

**37.** $\log_{10}\left(\frac{x+1}{x}\right)$

**Sol.** $y = \log_{10}\left(\frac{x+1}{x}\right)$

$$y = \frac{\log_e\left(\frac{x+1}{x}\right)}{\log_e 10} = \frac{\ln\left(\frac{x+1}{x}\right)}{\ln 10}$$

Diff. w.r.t. $x$

$$\frac{dy}{dx} = \frac{1}{\ln 10} \cdot \frac{1}{\frac{x+1}{x}} \cdot \left[\frac{x \cdot 1 - (x+1)\cdot 1}{x^2}\right]$$

$$= \frac{x}{(x+1)\cdot \ln 10} \cdot \left(\frac{x - x - 1}{x^2}\right) = \frac{-x}{(x+1)\cdot \ln 10 \cdot x^2}$$

$$= \frac{-1}{(x+1)\cdot \ln 10 \cdot x} \text{ — Ans.}$$

**38.** $\arccos \sqrt{1-x^2}$

**Sol.** Let $y = \cos^{-1}\sqrt{1-x^2}$

Diff. w.r.t. $x$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(\sqrt{1-x^2})^2}} \cdot \frac{d}{dx}\left(\sqrt{1-x^2}\right)$$

$$= \frac{-1}{\sqrt{1-(1-x^2)}} \cdot \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) = \frac{x}{\sqrt{1-1+x^2} \cdot \sqrt{1-x^2}}$$

$$= \frac{x}{\sqrt{x^2}} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{x}{|x|} \cdot \frac{1}{\sqrt{1-x^2}}$$

39. $\text{arc Sec}(\sinh x)$

Sol:- Let $y = \sec^{-1}(\sinh x)$

Diff. w.r.t. $x$

$$\frac{dy}{dx} = \frac{1}{\sinh x\sqrt{\sinh^2 x - 1}} \cdot \frac{d}{dx}(\sinh x)$$

$$= \frac{1}{\sinh x\sqrt{\sinh^2 x - 1}} \cdot \cosh x$$

$$= \frac{\coth x}{\sqrt{\sinh^2 x - 1}}$$

40. $\text{arcSin}(\text{arcCot} \ln x)$

Sol:- Let $y = \sin^{-1}(\cot^{-1} \ln x)$

Diff. w.r.t. $x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\cot^{-1}\ln x)^2}} \cdot \frac{d}{dx}(\cot^{-1}\ln x)$$

$$= \frac{1}{\sqrt{1-(\cot^{-1}\ln x)^2}} \cdot \frac{-1}{1+(\ln x)^2} \cdot \frac{1}{x}$$

$$= \frac{-1}{x(1+\ln^2 x)\sqrt{1-(\cot^{-1}\ln x)^2}}$$

41. $\cosh^{-1}(1+x^2)$

Sol:- Let $y = \cosh^{-1}(1+x^2)$

Diff. w.r.t. $x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{(1+x^2)^2 - 1}} \cdot \frac{d}{dx}(1+x^2)$$

$$= \frac{1}{\sqrt{\cancel{1}+2x^2+x^4-\cancel{1}}} \cdot 2x = \frac{2x}{\sqrt{2x^2+x^4}} = \frac{2x}{|x|\sqrt{2+x^2}}$$

42. $\sinh^{-1}(\tanh x)$

Sol:- Let $y = \sinh^{-1}(\tanh x)$

Diff. w.r.t. $x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+\tanh^2 x}} \cdot \frac{d}{dx}(\tanh x) = \frac{\operatorname{sech}^2 x}{\sqrt{1+\tanh^2 x}}$$ —— Ans.

Differentiate (logarithmically) with respect to $x$ **(Problems 43-47)**

**43.** $y = \sqrt[3]{\dfrac{x\,(x^2+1)}{(x-1)^2}}$

**Sol.**
Here $y = \left[\dfrac{x(x^2+1)}{(x-1)^2}\right]^{1/3}$

taking ln on both sides

$$\ln y = \ln\left[\frac{x(x^2+1)}{(x-1)^2}\right]^{1/3}$$
$$= \frac{1}{3}\ln\left[\frac{x(x^2+1)}{(x-1)^2}\right]$$
$$= \frac{1}{3}\left[\ln x + \ln(x^2+1) - \ln(x-1)^2\right]$$
$$\ln y = \frac{1}{3}\left[\ln x + \ln(x^2+1) - 2\ln(x-1)\right]$$

Diff. w.r.t. $x$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{3}\left[\frac{1}{x} + \frac{1}{x^2+1}\cdot 2x - 2\cdot\frac{1}{x-1}\cdot 1\right] = \frac{1}{3}\left[\frac{1}{x} + \frac{2x}{x^2+1} - \frac{2}{x-1}\right]$$
$$= \frac{1}{3}\left[\frac{(x^2+1)(x-1) + 2x\cdot x(x-1) - 2x(x^2+1)}{x(x^2+1)(x-1)}\right] = \frac{1}{3}\left[\frac{x^3 - x^2 + x - 1 + \cancel{2x^3} - 2x^2 - \cancel{2x^3} - 2x}{x(x^2+1)(x-1)}\right]$$
$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{3}\left[\frac{x^3 - 3x^2 - x - 1}{x(x^2+1)(x-1)}\right]$$
$$\Rightarrow \frac{dy}{dx} = \frac{y}{3}\left[\frac{x^3-3x^2-x-1}{x(x^2+1)(x-1)}\right] = \frac{x^{1/3}\cdot(x^2+1)^{1/3}}{3(x-1)^{2/3}}\cdot\frac{x^3-3x^2-x-1}{x(x^2+1)(x-1)}$$
$$= \frac{x^3-3x^2-x-1}{3x^{1-1/3}\cdot(x-1)^{2/3+1}\cdot(x^2+1)^{1-1/3}} = \frac{x^3-3x^2-x-1}{3x^{2/3}\cdot(x-1)^{5/3}\cdot(x^2+1)^{2/3}} \text{ — Ans.}$$

**44.** $y = \dfrac{\sqrt{x}\,(1-2x)^{2/3}}{(2-3x)^{3/4}\,(3-4x)^{4/3}}$

**Sol.** Taking logarithm of both sides, we get

$$\ln y = \ln\left[\frac{\sqrt{x}\,(1-2x)^{2/3}}{(2-3x)^{3/4}\cdot(3-4x)^{4/3}}\right]$$
$$= \ln\left(\sqrt{x}\,(1-2x)^{2/3}\right) - \ln\,(2-3x)^{3/4}\cdot(3-4x)^{4/3}$$
$$= \left[\ln\sqrt{x} + \ln(1-2x)^{2/3}\right] - \left[\ln(2-3x)^{3/4} + \ln(3-4x)^{4/3}\right]$$
$$= \ln x^{1/2} + \ln(1-2x)^{2/3} - \ln(2-3x)^{3/4} - \ln(3-4x)^{4/3}$$
$$\ln y = \frac{1}{2}\ln x + \frac{2}{3}\ln(1-2x) - \frac{3}{4}\ln(2-3x) - \frac{4}{3}\ln(3-4x)$$

Diff. w.r.t. x

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{x} + \frac{2}{3}\cdot\frac{1}{1-2x}\cdot(-2) - \frac{3}{4}\cdot\frac{1}{2-3x}(-3) - \frac{4}{3}\cdot\frac{1}{3-4x}(-4)$$

$$= \frac{1}{2x} - \frac{4}{3(1-2x)} + \frac{9}{4(2-3x)} + \frac{16}{3(3-4x)}$$

$$= \frac{1}{2x} + \frac{9}{4(2-3x)} + \frac{-4}{3(1-2x)} + \frac{16}{3(3-4x)}$$

$$= \frac{2(2-3x)+9x}{4x(2-3x)} + \frac{-4(3-4x)+16(1-2x)}{3(1-2x)(3-4x)}$$

$$= \frac{4-6x+9x}{4x(2-3x)} + \frac{-12+16x+16-32x}{3(1-2x)(3-4x)}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{4+3x}{4x(2-3x)} + \frac{4-16x}{3(1-2x)(3-4x)}$$

$$\frac{dy}{dx} = y\left[\frac{4+3x}{4x(2-3x)} + \frac{4-16x}{3(1-2x)(3-4x)}\right]$$

$$= \frac{\sqrt{x}\,(1-2x)^{4/3}}{(2-3x)^{3/4}(3-4x)^{4/3}}\left[\frac{4+3x}{4x(2-3x)} + \frac{4(1-4x)}{3(1-2x)(3-4x)}\right] \text{ — Ans.}$$

**45.** $y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$ ——— (1)

**Sol.** Let $u = (\tan x)^{\cot x}$

taking ln on both sides

$\ln u = \ln(\tan x)^{\cot x}$

$\ln u = \cot x \ln(\tan x)$

**Differentiating,** w.r.t. $x$, we have

$$\frac{1}{u}\cdot\frac{du}{dx} = \cot x\cdot\frac{1}{\tan x}\cdot\sec^2 x + \ln(\tan x)\cdot -\operatorname{cosec}^2 x$$

$$\frac{du}{dx} = u\left[\cot x\cdot\cot x\cdot\sec^2 x - \ln(\tan x)\cdot\operatorname{cosec}^2 x\right]$$

$$= (\tan x)^{\cot x}\left[\cot^2 x\cdot\sec^2 x - \ln(\tan x)\cdot\operatorname{cosec}^2 x\right]$$

$$= (\tan x)^{\cot x}\left[\frac{\cos^2 x}{\sin^2 x}\cdot\frac{1}{\cos^2 x} - \ln(\tan x)\cdot\operatorname{cosec}^2 x\right]$$

$$\frac{du}{dx} = (\tan x)^{\cot x}\left[\operatorname{cosec}^2 x - \ln(\tan x)\cdot\operatorname{cosec}^2 x\right]$$

& $v = (\cot x)^{\tan x}$

$\ln v = \ln(\cot x)^{\tan x}$

$\ln v = \tan x \ln(\cot x)$

Diff. w.r.t. x

$$\frac{1}{v}\cdot\frac{dv}{dx} = \tan x\cdot\frac{1}{\cot x}\cdot -\operatorname{cosec}^2 x + \ln(\cot x)\cdot\sec^2 x$$

$$\frac{dv}{dx} = v\left[-\tan x\cdot\tan x\cdot\operatorname{cosec}^2 x + \ln(\cot x)\cdot\sec^2 x\right]$$

$$= v\left[-\tan^2 x\cdot\operatorname{cosec}^2 x + \ln(\cot x)\cdot\sec^2 x\right]$$

$$= (\cot x)^{\tan x}\left[-\frac{\sin^2 x}{\cos^2 x}\cdot\frac{1}{\sin^2 x} + \ln(\cot x)\cdot\sec^2 x\right]$$

$$dv/dx = (\cot x)^{\tan x}\left[-\sec^2 x + \ln(\cot x)\cdot\sec^2 x\right]$$

Now Eq. (1) is

$y = U + V$

Diff. w.r.t. $x$

$\frac{dy}{dx} = \frac{du}{dx} + dV/dx$

Putting values

$$\frac{dy}{dx} = (\tan x)^{\cot x}\left[\operatorname{cosec}^2 x - \ln(\tan x).\operatorname{cosec}^2 x\right] + (\cot x)^{\tan x}\left[-\sec^2 x + \ln(\cot x).\sec^2 x\right]$$ — Ans

**46.** $y = x^x . e^x \sin x . \ln x$

**Sol.** Taking logarithm of both sides, we get

$\ln y = \ln(x^x . e^x . \sin x . \ln x)$

$= \ln x^x + \ln e^x + \ln \sin x + \ln(\ln x)$

$= x\ln x + x\ln e + \ln\sin x + \ln(\ln x)$

$\ln y = x.\ln x + x + \ln\sin x + \ln(\ln x)$

Diff. w.r.t. $x$

$\frac{1}{y}.\frac{dy}{dx} = x.\frac{1}{x} + \ln x.1 + 1 + \frac{1}{\sin x}.\cos x + \frac{1}{\ln x}.\frac{1}{x}$

$\frac{dy}{dx} = y\left[1 + \ln x + 1 + \cot x + \frac{1}{x\ln x}\right]$

$= (x^x . e^x . \sin x . \ln x)\left[2 + \ln x + \cot x + \frac{1}{x\ln x}\right]$ — Ans

In Problems 48 – 60, find $\frac{dy}{dx}$ :

**47.** $y = \frac{(x+2)^2}{(x+1)(x^2+3)^3}$

**Sol.** Taking ln of both sides, we have

$\ln y = \ln\left[\frac{(x+2)^2}{(x+1)(x^2+3)^3}\right] = \ln(x+2)^2 - \ln(x+1) - \ln(x^2+3)^3$

$\ln y = 2\ln(x+2) - \ln(x+1) - 3\ln(x^2+3)$

Diff. w.r.t. $x$

$\frac{1}{y}.\frac{dy}{dx} = 2.\frac{1}{x+2} - \frac{1}{x+1} - 3.\frac{1}{x^2+3}.2x$

$\frac{dy}{dx} = y\left[\frac{2}{x+2} - \frac{1}{x+1} - \frac{6x}{x^2+3}\right] = y\left[\frac{2(x+1)(x^2+3) - (x+2)(x^2+3) - 6x(x+1)(x+2)}{(x+1)(x+2)(x^2+3)}\right]$

$= y\left[\frac{2x^3+6x+2x^2+6 - x^3 - 3x - 2x^2 - 6 - 6x(x^2+3x+2)}{(x+1)(x+2)(x^2+3)}\right]$

$= y\left[\frac{x^3+3x - 6x^3 - 18x^2 - 12x}{(x+1)(x+2)(x^2+3)}\right] = \frac{(x+2)^2}{(x+1)(x^2+3)^3}\left[\frac{-5x^3-18x^2-9x}{(x+1)(x+2)(x^2+3)}\right]$

**48.** $\sqrt{x} + \sqrt{y} = \sqrt{a}$

**Sol.** Differentiating, both sides w.r.t. $x$, we get

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}.\frac{dy}{dx} = 0$

$\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}}.\frac{dy}{dx} = 0$

$\frac{1}{\sqrt{y}}.\frac{dy}{dx} = -\frac{1}{\sqrt{x}}$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

or $$\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$$

**49.** $x^3 + y^3 - 3axy = 0$

**Sol.** Differentiating w.r.t. $x$, we get

$$3x^2 + 3y^2\frac{dy}{dx} - 3a\left[x\frac{dy}{dx} + y \cdot 1\right] = 0$$

or $$x^2 + y^2\frac{dy}{dx} - a\left[x\frac{dy}{dx} + y \cdot 1\right] = 0$$

$$x^2 + y^2\,dy/dx - ax \cdot dy/dx - ay = 0$$

$$\frac{dy}{dx}(y^2 - ax) = ay - x^2$$

or $$\frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$

**50.** $y - \cos(x+y) = 0$

**Sol.** It can be rewritten as

$$y = \cos(x+y)$$

Differentiating both sides w.r.t. $x$, we get

$$\frac{dy}{dx} = -\sin(x+y)\left[1 + \frac{dy}{dx}\right]$$

$$\frac{dy}{dx} = -\sin(x+y) - \sin(x+y) \cdot dy/dx$$

$$\frac{dy}{dx} + \sin(x+y) \cdot \frac{dy}{dx} = -\sin(x+y).$$

$$\frac{dy}{dx}(1 + \sin(x+y)) = -\sin(x+y).$$

or $$\frac{dy}{dx} = -\frac{\sin(x+y)}{1 + \sin(x+y)}$$

**51.** $\arctan(x+y) = \arcsin(e^y + x)$

Sol. $$\tan^{-1}(x+y) = \sin^{-1}(e^y + x)$$

Diff. w.r.t. x

$$\frac{1}{1+(x+y)^2} \cdot \left(1 + dy/dx\right) = \frac{1}{\sqrt{1-(e^y+x)^2}} \cdot \left(e^y \cdot \frac{dy}{dx} + 1\right)$$

$$\frac{1}{1+(x+y)^2} + \frac{1}{1+(x+y)^2} \cdot \frac{dy}{dx} = \frac{1}{\sqrt{1-(e^y+x)^2}} \cdot e^y\frac{dy}{dx} + \frac{1}{\sqrt{1-(e^y+x)^2}}$$

$$\frac{1}{1+(x+y)^2} \cdot \frac{dy}{dx} - \frac{e^y}{\sqrt{1-(e^y+x)^2}} \cdot \frac{dy}{dx} = \frac{1}{\sqrt{1-(e^y+x)^2}} - \frac{1}{1+(x+y)^2}$$

$$\frac{dy}{dx}\left[\frac{1}{1+(x+y)^2} - \frac{e^y}{\sqrt{1-(e^y+x)^2}}\right] = \frac{1+(x+y)^2 - \sqrt{1-(e^y+x)^2}}{\sqrt{1-(e^y+x)^2}\,(1+(x+y)^2)}$$

$$\frac{dy}{dx}\left[\frac{\sqrt{1-(e^y+x)^2}-e^y\left[1+(x+y)^2\right]}{\left[1+(x+y)^2\right]\sqrt{1-(e^y+x)^2}}\right]=\frac{\left[1+(x+y)^2-\sqrt{1-(e^y+x)^2}\right]}{\sqrt{1-(e^y+x)^2}\left[1+(x+y)^2\right]}$$

$$\frac{dy}{dx}\left[\sqrt{1-(e^y+x)^2}-e^y\left(1+(x+y)^2\right)\right]=1+(x+y)^2-\sqrt{1-(e^y+x)^2}$$

or $$\frac{dy}{dx}=\frac{1+(x+y)^2-\sqrt{1-(e^y+x)^2}}{\sqrt{1-(e^y+x)^2}-e^y(1+(x+y)^2)}.$$

52. $x = a(t-\sin t),\ y = a(1-\cos t)$

**Sol.** Differentiating w.r.t. $t$, we get

$$\frac{dx}{dt}=a(1-\cos t)$$

and $$\frac{dy}{dt}=a(\sin t)=a\sin t$$

Now $$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$$

$$=\frac{a\sin t}{a(1-\cos t)}$$

$$=\frac{2\sin\frac{t}{2}\cos\frac{t}{2}}{2\sin^2\frac{t}{2}}=\frac{\cos\frac{t}{2}}{\sin\frac{t}{2}}=\cot\frac{t}{2}.$$

53. $x = a\cos^3 t,\ y = b\sin^3 t$

**Sol.** $x = a\cos^3 t$ & $y = b\sin^3 t$

Diff. w.r.t. $t$

$$\frac{dx}{dt}=a.3\cos^2 t.(-\sin t)$$
$$=-3a\cos^2 t.\sin t$$

& as $y = b\sin^3 t$

Diff. w.r.t. $t$

$$\frac{dy}{dt}=b(3\sin^2 t.\cos t)$$
$$=3b\sin^2 t\cos t$$

Now $$\frac{dy}{dx}=\frac{dy}{dt}.\frac{dt}{dx}$$
$$=\frac{3b\sin^2 t\cos t}{-3a\cos^2 t\sin t}$$
$$=-\frac{b}{a}.\frac{\sin t}{\cos t}=-\frac{b}{a}\tan t$$

54. $x = \dfrac{3at}{1+t^2}$, $y = \dfrac{3at^2}{1+t^2}$

**Sol.** Diff. w.r.t. t

$$\frac{dx}{dt} = 3a\left[\frac{(1+t^2)\cdot 1 - t\cdot(2t)}{(1+t^2)^2}\right]$$

$$= 3a\left[\frac{1+t^2-2t^2}{(1+t^2)^2}\right]$$

$$\frac{dx}{dt} = \frac{3a(1-t^2)}{(1+t^2)^2}$$

Now $\dfrac{dy}{dt} = 3a\left[\dfrac{(1+t^2)\cdot 2t - t^2(2t)}{(1+t^2)^2}\right]$

$$= 3a\left[\frac{2t+2t^3-2t^3}{(1+t^2)^2}\right]$$

$$= \frac{3a(2t)}{(1+t^2)^2}$$

$$\frac{dy}{dt} = \frac{6at}{(1+t^2)^2}$$

Now $\dfrac{dy}{dx} = \dfrac{dy}{dt}\cdot\dfrac{dt}{dx} = \dfrac{6at}{(1+t^2)^2}\cdot\dfrac{(1+t^2)^2}{3a(1-t^2)}$

$$= \frac{2t}{(1-t^2)} \text{ —— Ans.}$$

55. $y = (\sec x^3 + \operatorname{arcsec} x)^2$

**Sol.** $y = (\sec x^3 + \sec^{-1} x)^2$

Diff. w.r.t. x

$$\frac{dy}{dx} = 2(\sec x^3 + \sec^{-1}x)\cdot\frac{d}{dx}(\sec x^3 + \sec^{-1}x)$$

$$= 2(\sec x^3 + \sec^{-1}x)\cdot\left[\sec x^3\cdot\tan x^3\cdot 3x^2 + \frac{1}{x\sqrt{x^2-1}}\right]$$

$$= 2(\sec x^3 + \sec^{-1}x)\left[3\sec x^3\cdot\tan x^3\cdot x^2 + \frac{1}{x\sqrt{x^2-1}}\right] \text{ —— Ans.}$$

56. $y = \exp\left(\operatorname{arccsc}\left(\dfrac{1}{x}\right)\right)$

**Sol.** $y = e^{\csc^{-1}(1/x)}$

Diff. w.r.t. x

$$\frac{dy}{dx} = e^{\csc^{-1}(1/x)}\cdot\frac{-1}{\frac{1}{x}\sqrt{\left(\frac{1}{x}\right)^2-1}}\cdot\frac{-1}{x^2}$$

$$= e^{\csc^{-1}(1/x)}\,\frac{1}{x\sqrt{\frac{1}{x^2}-1}} = e^{\csc^{-1}(\frac{1}{x})}\cdot\frac{1}{x\sqrt{\frac{1-x^2}{x^2}}}$$

$$\frac{dy}{dx} = e^{\operatorname{Cosec}^{-1}(1/x)} \cdot \frac{1}{\frac{x\sqrt{1-x^2}}{x}}$$

$$\frac{dy}{dx} = e^{\operatorname{Cosec}^{-1}\frac{1}{x}} \cdot \frac{1}{\sqrt{1-x^2}} \text{ —— Ans.}$$

**57.** $y = \arcsin(\ln x) - \ln(\arctan x)$

Sol. $y = \sin^{-1}(\ln x) - \ln(\tan^{-1}x)$

Diff. w.r.t. $x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\ln x)^2}} \cdot \frac{1}{x} - \frac{1}{\tan^{-1}x} \cdot \frac{1}{1+x^2}$$

$$= \frac{1}{x\sqrt{1-(\ln x)^2}} - \frac{1}{(1+x^2)\tan^{-1}x}$$

**58.** $y \arcsin x - x \arctan y = 1$

Sol. $y\sin^{-1}x - x\tan^{-1}y = 1$

Diff. w.r.t. $x$

$$y \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x \cdot \frac{dy}{dx} - x \cdot \frac{1}{1+y^2} \cdot \frac{dy}{dx} - \tan^{-1}y = 0$$

$$\frac{dy}{dx}\left(\sin^{-1}x - \frac{x}{1+y^2}\right) + \frac{y}{\sqrt{1-x^2}} - \tan^{-1}y = 0$$

$$\frac{dy}{dx} \cdot \left(\frac{(1+y^2)\sin^{-1}x - x}{(1+y^2)}\right) = -\frac{y}{\sqrt{1-x^2}} + \tan^{-1}y$$

$$\frac{dy}{dx} \cdot \left(\frac{(1+y^2)\sin^{-1}x - x}{1+y^2}\right) = \frac{-y + \sqrt{1-x^2}\tan^{-1}y}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{1+y^2}{(1+y^2)\sin^{-1}x - x} \times \frac{\left[-y + \sqrt{1-x^2} \cdot \tan^{-1}y\right]}{\sqrt{1-x^2}}$$

$$= \frac{(1+y^2)\left(\sqrt{1-x^2}\tan^{-1}y - y\right)}{\sqrt{1-x^2}\left[(1+y^2)\sin^{-1}x - x\right]} \text{ —— Ans.}$$

**59.** $\arcsin(\ln xy) = x + y^2$

Sol. $\sin^{-1}(\ln xy) = x + y^2$

Diff. w.r.t. $x$

$$\frac{1}{\sqrt{1-(\ln xy)^2}} \cdot \frac{d}{dx}(\ln xy) = 1 + 2y \cdot \frac{dy}{dx}$$

$$\frac{1}{\sqrt{1-(\ln xy)^2}} \cdot \frac{1}{xy} \cdot \left[x\frac{dy}{dx} + y \cdot 1\right] = 1 + 2y \cdot \frac{dy}{dx}$$

$$x\frac{dy}{dx} + y = xy\sqrt{1-(\ln xy)^2}\left[1 + 2y \cdot \frac{dy}{dx}\right]$$

$$x\frac{dy}{dx} + y = xy\sqrt{1-(\ln xy)^2} + 2xy^2\sqrt{1-(\ln xy)^2} \cdot \frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2xy^2\sqrt{1-(\ln xy)^2} \cdot \frac{dy}{dx} = xy\sqrt{1-(\ln xy)^2} - y$$

$$x\frac{dy}{dx}\left(1-2y^2\sqrt{1-(\ln xy)^2}\right) = y\left(x\sqrt{1-(\ln xy)^2}-1\right)$$ (29)

$$\frac{dy}{dx} = \frac{y\left(x\sqrt{1-(\ln xy)^2}-1\right)}{x\left(1-2y^2\sqrt{1-(\ln xy)^2}\right)} = \frac{y\left(x\sqrt{1-\ln^2 xy}-1\right)}{x\left(1-2y^2\sqrt{1-\ln^2 xy}\right)}$$

**60.** $\operatorname{arcsec}(x^2+y) - e^x = \dfrac{1}{x+y}$

**Sol.** $\sec^{-1}(x^2+y) - e^x = \dfrac{1}{x+y}$

Diff. w.r.t. $x$

$$\frac{1}{(x^2+y)\sqrt{(x^2+y)^2-1}}\cdot\frac{d}{dx}(x^2+y) - e^x = \frac{d}{dx}(x+y)^{-1}$$

$$\frac{1}{(x^2+y)\sqrt{(x^2+y)^2-1}}\cdot\left(2x+\frac{dy}{dx}\right) - e^x = -(x+y)^{-2}\cdot\left(1+\frac{dy}{dx}\right)$$

$$\frac{2x+\frac{dy}{dx}}{(x^2+y)\sqrt{(x^2+y)^2-1}} - e^x = \frac{-1-\frac{dy}{dx}}{(x+y)^2}$$

$$\frac{2x+\frac{dy}{dx} - e^x(x^2+y)\sqrt{(x^2+y)^2-1}}{(x^2+y)\sqrt{(x^2+y)^2-1}} = \frac{-1-\frac{dy}{dx}}{(x+y)^2}$$

$$(x+y)^2\left(2x+\frac{dy}{dx}\right) - (x+y)^2\cdot e^x(x^2+y)\sqrt{(x^2+y)^2-1} = \left(-1-\frac{dy}{dx}\right)(x^2+y)\sqrt{(x^2+y)^2-1}$$

$$2x(x+y)^2 + (x+y)^2\cdot\frac{dy}{dx} - (x+y)^2\cdot e^x\cdot(x^2+y)\sqrt{(x^2+y)^2-1} = -(x^2+y)\sqrt{(x^2+y)^2-1} - \frac{dy}{dx}(x^2+y)\sqrt{(x^2+y)^2-1}$$

$$(x+y)^2\cdot\frac{dy}{dx} + \frac{dy}{dx}(x^2+y)\sqrt{(x^2+y)^2-1} = (x+y)^2\cdot e^x(x^2+y)\sqrt{(x^2+y)^2-1} - (x^2+y)\sqrt{(x^2+y)^2-1} - 2x(x+y)^2$$

$$\frac{dy}{dx}\left((x+y)^2+(x^2+y)\sqrt{(x^2+y)^2-1}\right) = e^x(x+y)^2(x^2+y)\sqrt{(x^2+y)^2-1} - (x^2+y)\sqrt{(x^2+y)^2-1} - 2x(x+y)^2$$

$$\frac{dy}{dx} = \frac{e^x(x+y)^2(x^2+y)\sqrt{(x^2+y)^2-1} - (x^2+y)\sqrt{(x^2+y)^2-1} - 2x(x+y)^2}{(x+y)^2+(x^2+y)\sqrt{(x^2+y)^2-1}}$$ — Ans