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# CHAPTER # 7

## PLANE CURVES II

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### Asymptotes.

**Def.** An asymptote is a straight line for a given curve C. If the distance between $l$ and C tends to 0 as infinite distance is moved along $l$.

e.g.

$$C: \; xy = 1$$
$$y = \frac{1}{x}$$

As $x \longrightarrow \infty$, $y \longrightarrow 0$

$y = 0$ is an asymptote for curve C.

Like wise for $x = \frac{1}{y}$

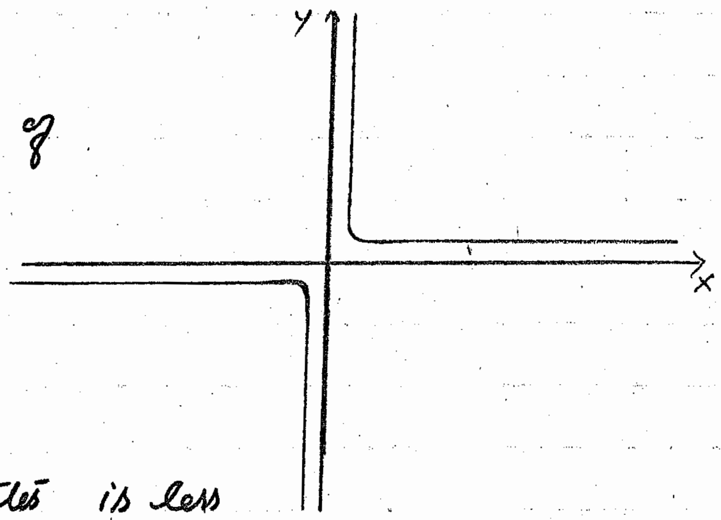
As $y \longrightarrow \infty$, $x \longrightarrow 0$

$x = 0$ is also an asymptote.

The graph of the asymptote is as shown.

$$y = \frac{1}{x}$$

| $x$ | 1 | 2 | 3 | ..... | 1/2 | 1/3 | 1/4 | ...... |
|---|---|---|---|---|---|---|---|---|
| $y$ | 1 | 1/2 | 1/3 | ..... | 2 | 3 | 4 | ..... |




### Types of Asymptotes

There are three types of asymptotes.

1) Horizontal asymptote
2) Vertical asymptote
3) Inclined asymptote

### No. of Asymptotes.

The number of asymptotes is less than or equal to the degree of the given equation.

## How to Find the Asymptote.

Arrange the given equation in desscending powers of $x$ and $y$ like,

$$a_n x^n + a_{n-1} x^{n-1} y + a_{n-2} x^{n-2} y^2 + \cdots + y^n + \cdots + x^2 + y^2 + x + y + 1 = 0$$

### For Horizontal asymptote.

Equate the Coefficient of heighest power of $x$ to zero if any.

### For Vertical Asymptote.

Equate the Co-efficient of heighest power of $y$ to zero if any.

### Inclined Asymptote.

Equation of inclined asymptote is

$$y = mx + C$$

put $x=1$, $y=m$ in the heighest degree terms and equate to zero.

$$\text{i.e} \quad \phi_n(m) = 0$$

$$\Rightarrow \quad m = m_1, m_2, m_3 \text{ ----- } m_n$$

### Value of C.

$$C = -\frac{\phi_{n-1}(m)}{\phi_n'(m)}$$

### Value of C in the pressence of two equal Values of m.

By Putting the values of $m$ in the below formula we will get the values of $C$.

$$\frac{c^2}{2!}\phi_n''(m) + \frac{c}{1!}\phi_{n-1}'(m) + \phi_{n-2}(m) = 0$$

### For three equal values of m.

We will use this formula for three equal values of $m$ and get the values of $C$.

$$\frac{c^3}{3!}\phi_n'''(m) + \frac{c^2}{2!}\phi_{n-1}''(m) + \frac{c}{1!}\phi_{n-2}'(m) + \phi_{n-3}(m) = 0$$

## Asymptotes of Polar Curves:

Let $r = f(\theta)$ be curve.

Put $r = \infty$ in the equation of the curve and find the value of $\theta$

say $\theta = \alpha, \beta, \gamma, \ldots\ldots$

Then by using the formula

$$p = r\sin(\alpha - \theta)$$

we can find the equation of the asymptote where

$$p = \lim_{\theta \to \alpha, \beta, \gamma \ldots} r^2 \frac{d\theta}{dr}$$

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# Exercise 7.1

Find equations of the asymptotes of the following curves.

**Q.1.** $y = \frac{(x-2)^2}{x^2}$

The equation can be written as

$$x^2 y = x^2 + 4 - 4x$$

$$x^2(y-1) + 4x - 4 = 0 \quad \text{——— I}$$

H.A. Coefficient of highest power of $x = y-1$

Put $y-1=0$

is an asymptote // to x-axis.

V.A Co-efficient of heighest power of $y = x^2$

Put $x^2 = 0$

$\Rightarrow x=0, x=0$

i.e y-axis plays the role of asymptote.

Hence the required asymptotes are

$y-1=0, x=0, x=0$

Note Number of asymptotes is at the most three which has been acheived. There is no need to look for an inclined asymptote.

**Q.2.** $x^2y^2 = 12(x-3)$

$$x^2y^2 = 12(x-3)$$

$$x^2y^2 - 12x + 36 = 0$$

H.A: Co-efficient of heighest power of $x = y^2$

Put $y^2 = 0$

$\Rightarrow y=0, y=0$

i.e X-axis plays the role of an asymptote.

V.A. Co-efficient of the highest power of $y = x^2$

Put $x^2 = 0$

$\Rightarrow x=0, x=0$

i.e y-axis plays the role of an asymptote.

Hence the required asymptotes are

$y=0, y=0, x=0$ and $x=0$

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**Q.3.** $2xy = x^2 + 3 \Rightarrow x^2 + 3 - 2xy = 0$

**H.A.** $x^2 - 2xy + 3 = 0$ ——— (1)

Co-efficient of highest power of $x = 1$

$\Rightarrow$ There is no asymptote parallel to X-axis.

**V.A.**

Co-efficient of highest power of $y = -2x$

Put $-2x = 0$

$\Rightarrow x = 0$

$\Rightarrow$ y-axis plays the role of asymptote.

**I.A.** Arrange the given eq in descending power of $x$

i.e. $x^2 - 2xy + 3 = 0$

Put $x = 1$ and $y = m$, we have

$\phi_2(m) = 1 - 2m$

Put $\phi_2(m) = 0$

$\Rightarrow 1 - 2m = 0$

$-2m = -1$

$2m = 1$

$m = 1/2$

Now for value of $c$

$\frac{c}{1!}\phi_2'(m) + \phi_1(m) = 0$ ——— I

$\phi_2'(m) = -2$

$\phi_1(m) = 0$

Put these values in (I) we have

$c(-2) + 0 = 0$

$-2c = 0$

$c = 0$

So eq. of Inclined asymptote is

$y = mx + c$

$y = \frac{1}{2}x + 0$

$y = \frac{1}{2}x$

Hence the required asymptotes are

$x = 0$ and $y = \frac{1}{2}x$.

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**Q.4** $x^2(x-y)^2 + a^2(x^2-y^2) = a^2xy$.

$x^2(x^2-2xy+y^2) + a^2x^2 - a^2y^2 = a^2xy$

$x^4 - 2x^3y + x^2y^2 + a^2x^2 - a^2y^2 - a^2xy = 0$ ——— I

**H.A:** Co-efficient of highest power of $x = 1$

$\Rightarrow$ There is no horizontal asymptote.

**V.A.** Co-efficient of highest power of $y = x^2 - a^2$

Put $x^2 - a^2 = 0$

$x^2 = a^2$

$x = \pm a$

$\Rightarrow$ $x = a$, $x = -a$

are the asymptotes // to y-axis.

**I.A** Put $x=1$ and $y=m$ in the highest degree terms of (I)

i.e $\phi_4(m) = 1 - 2m + m^2$

$\phi_4(m) = 1 + m^2 - 2m$

equal to zero

$1 + m^2 - 2m = 0$

$(1-m)^2 = 0$

$\Rightarrow$ $1-m = 0$, $1-m=0$

$\Rightarrow$ $m = 1$, $m = 1$

Now

$$\frac{c^2}{2!}\phi_4''(m) + \frac{c}{1!}\phi_3'(m) + \phi_2(m) = 0 \quad \text{——— II}$$

So $\phi_4(m) = 1 + m^2 - 2m$

$\phi_4'(m) = 2m - 2$

$\phi_4''(m) = 2$

and $\phi_3(m) = 0$

$\phi_3'(m) = 0$

Also $\phi_2(m) = a^2 - a^2m^2 - a^2m$

$\phi_2(m) = a^2(1 - m^2 - m)$

Put these values in II, we have

$\frac{c^2}{2}(2) + c(0) + a^2(1-m^2-m) = 0$

$c^2 + a^2(1-m^2-m) = 0$

Put $m = 1$

$$c^2 + a^2(1-1-1) = 0$$
$$c^2 + a^2(-1) = 0$$
$$c^2 = a^2$$
$$c = \pm a$$
$$\Rightarrow \quad c = a \quad \text{and} \quad c = -a$$

Hence the inclined asymptotes are

$$y = x + a \quad \text{and} \quad y = x - a$$

Hence the required asymptotes are

$$x = a, \; x = -a, \; y = x + a \quad \text{and} \quad y = x - a.$$

**Q.5.** $(x-y)^2(x^2+y^2) - 10(x-y)x^2 + 12y^2 + 2x + y = 0$

$$\Rightarrow (x^2+y^2-2xy)(x^2+y^2) - 10(x-y)x^2 + 12y^2 + 2x + y = 0$$
$$x^4 + 2x^2y^2 + y^4 - 2x^3y - 2xy^3 - 10x^3 + 10x^2y + 12y^2 + 2x + y = 0$$

**H.A.** Co-efficient of highest power of $x = 1$

$\Rightarrow$ There is no horizontal asymptote.

**V.A.** Co-efficient of highest power of $y = 1$

$\Rightarrow$ There is no vertical asymptote.

**I.A.** Put $x = 1$ and $y = m$ in the highest powers of $x$ and $y$

$$\Rightarrow \quad \phi_4(m) = 1 + 2m^2 + m^4 - 2m - 2m^3$$

Put $\phi_4(m) = 0$

$$\Rightarrow \quad 1 + 2m^2 + m^4 - 2m - 2m^3 = 0$$
$$(m^2+1)^2 - 2m(m^2+1) = 0$$
$$(m^2+1)(m^2+1) - 2m(m^2+1) = 0$$
$$(m^2+1-2m)(m^2+1) = 0$$
$$(m-1)^2(m^2+1) = 0$$
$$\Rightarrow \quad (m-1)^2 = 0 \quad \text{and} \quad (m^2+1) = 0$$
$$\Rightarrow (m-1)(m-1) = 0 \quad \text{and} \quad m^2 = -1$$
$$\Rightarrow \quad m = 1, 1 \qquad \text{Imaginary, not included.}$$

Now $$\frac{c^2}{2!}\phi_4''(m) + \frac{c}{1!}\phi_3'(m) + \phi_2(m) = 0 \quad \text{——— I}$$

So $$\phi_4(m) = 1 + 2m^2 + m^4 - 2m - 2m^3$$

$$\phi'(m) = 1 + 4m + 4m^3 - 2 - 6m^2$$

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$$\phi_4''(m) = 4 + 12m^2 - 12m$$

Now $\phi_3(m) = -10 + 10m$

$$\phi_3'(m) = 10$$

and $\phi_2(m) = 12m^2$

Put in I

$$\Rightarrow \frac{c^2}{2}(4+12m^2-12m) + c(10) + 12m^2 = 0$$

$$\Rightarrow \frac{c^2}{2}.2(2+6m^2-6m) + 10c + 12m^2 = 0$$

$$\Rightarrow 2c^2 + 6c^2m^2 - 6c^2m + 10c + 12m^2 = 0$$

$$c^2(2+6m^2-6m) + 10c + 12m^2 = 0$$

Putting $m = 1$, the above eq. is

$$c^2(2+6-6) + 10c + 12 = 0$$

$$2c^2 + 10c + 12 = 0$$

$$c^2 + 5c + 6 = 0$$

$$(c+2)(c+3) = 0$$

$$\Rightarrow c+2 = 0 \quad \text{or} \quad c+3 = 0$$

$$\Rightarrow c = -2 \quad \text{or} \quad c = -3$$

So the Eqs of Inclined asymptotes are

$$y = x-2 \quad \text{and} \quad y = x-3$$

Hence the required asymptotes are

$$y = x-2, \quad y = x-3$$

**Q.6.** $x^2y + xy^2 + xy + y^2 + 3x = 0$

Here $x^2y + xy^2 + y^2 + xy + 3x = 0$ ——— (1)

H.A. Co-efficient of highest power of $x = y$

Put $y = 0$ is an asymptote

i.e x-axis plays the role of asymptote.

V.A. Co-efficient of highest power of $y = x+1$

Put $x+1 = 0$

$$\Rightarrow x = -1$$

is an asymptote.

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Inclined Asymptote:

put $x=1$ and $y=m$ in the highest degree terms of I, we have

$$\phi_3(m) = m+m^2$$

put $\phi_3(m)=0$

$\Rightarrow m+m^2=0$

$\Rightarrow m(1+m)=0$

$\Rightarrow m=0$ and $1+m=0$

$\Rightarrow m=0$ and $m=-1$

Value of C for $m=0$.

For Value of C we use the formula

i.e

$$C = -\frac{\phi_{n-1}(m)}{\phi_n'(m)} \quad \text{——— II}$$

$\Rightarrow \phi_3(m) = m+m^2 \Rightarrow \phi_3'(m) = 1+2m$

and $\phi_2(m) = m^2+m$

put these values in (2)

$$C = -\frac{m^2+m}{1+2m}$$

Put $m=0$, we have

$\Rightarrow C = 0$

So the equation of the asymptote for $m=0$, and $c=0$ is $y=0$

which we have already found.

Value of C for $m=-1$.

From above

$$C = -\frac{m^2+m}{1+2m}$$

Put $m=-1$

$$\Rightarrow C = -\frac{(-1)^2+(-1)}{1+2(-1)}$$

$$\Rightarrow C = -\frac{1-1}{1-2}$$

$$\Rightarrow C = 0$$

So eq. of asymptote is

$$y = -x$$

Hence required asymptote are

$$y=0 \;,\quad x+1=0 \quad \text{and} \quad y=-x$$

**Q.7.** $(x-y+1)(x-y-2)(x+y) = 8x-1$

$$(x^2-xy-2x-xy+y^2+2y+x-y-2)(x+y) = 8x-1$$
$$(x^2+y^2-2xy-2x+2y+x-y-2)(x+y) = 8x-1$$
$$(x^2+y^2-2xy-x+y-2)(x+y) = 8x-1$$
$$(x^3+xy^2-2x^2y-x^2+xy-2x+x^2y+y^3-2xy^2-xy+y^2-2y)=8x-1$$
$$x^3+y^3-x^2y-xy^2-x^2+y^2-2x-2y-8x+1=0$$
$$x^3+y^3-x^2y-xy^2-x^2+y^2-10x-2y+1=0 \quad \text{—— I}$$

__Horizontal Asymptote.__

Co-efficient of highest power of $x = 1$

$\Rightarrow$ No Horizontal asymptote.

__Vertical Asymptote.__

Co-efficient of highest power of $y = 1$

$\Rightarrow$ No vertical asymptote.

__Inclined Asymptote.__

Put $x=1$ and $y=m$ in the highest degree terms of $x$ and $y$.

$$\Rightarrow \quad \phi_3(m) = 1+m^3-m-m^2$$
$$\Rightarrow \quad 1-m^2-m+m^3=0$$
$$\Rightarrow \quad 1-m^2-m(1-m^2)=0$$
$$\Rightarrow \quad (1-m^2)(1-m)=0$$
$$\Rightarrow \quad 1-m^2=0 \;,\quad 1-m=0$$
$$\Rightarrow \quad m^2=1 \;,\quad m=1$$
$$\Rightarrow \quad m=1,-1 \;,\quad m=1$$

__Value of $c$ For $m=1$.__

$$\frac{c^2}{2!}\phi_3''(m) + \frac{c}{1!}\phi_{3-1}'(m) + \phi_{3-2}(m) = 0 \quad \text{—— I}$$

$$\Rightarrow \quad \phi_3(m) = m^3-m^2-m+1$$
$$\Rightarrow \quad \phi_3'(m) = 3m^2-2m-1$$
$$\phi''(m) = 6m-2$$

$$\phi_2(m) = -1 + m^2$$

$$\phi_2'(m) = 2m$$

and $$\phi_1(m) = -10 - 2m$$

Put these values in I, we have

$$\frac{c^2}{2} \cdot 2(3m-1) + c \cdot 2m - 10 - 2m = 0$$

$$c^2(3m-1) + 2cm - 2m - 10 = 0$$

Put $m = 1$

$$c^2(3-1) + 2c - 2 - 10 = 0$$

$$2c^2 + 2c - 12 = 0$$

$$c^2 + c - 6 = 0$$

$$(c+3)(c-2) = 0$$

$$\Rightarrow \quad c = -3 \quad , \quad c = 2$$

So the eq. of asymptote when $m = 1$ are

$$y = x - 3 \quad , \quad y = x + 2$$

<u>Value of c For $m = -1$</u>

we know that

$$c = -\frac{\phi_2(m)}{\phi_3'(m)}$$

$$\Rightarrow \quad c = -\frac{m^2 - 1}{3m^2 - 2m - 1}$$

Put $m = -1$

$$\Rightarrow \quad c = -\frac{(-1)^2 - 1}{3(-1)^2 - 2(-1) - 1}$$

$$\Rightarrow \quad c = 0$$

So eq. of the asymptote when $m = -1$ is

$$y = -x$$

Hence the required asymptotes are

$$y = x - 3 \,, \quad y = x + 2 \quad \text{and} \quad y = -x$$

Q.8. $y^3 + x^2y + 2xy^2 - y + 1 = 0$

Horizontal asymptote.

Co-efficient of highest power of $x = y$

Put $y = 0$

is an asymptote.

Vertical Asymptote.

Coefficient of highest power of $y = 1$

$\Rightarrow$ There is no vertical Asymptote.

Inclined Asymptote.

put $x=1$ and $y=m$ in the highest degree terms of $x$ and $y$.

$$\Rightarrow \quad \phi_3(m) = m^3 + m + 2m^2$$

Put $\phi_3(m) = 0$

$$m^3 + 2m^2 + m = 0$$

$$m(m^2 + 2m + 1) = 0$$

$$\Rightarrow \quad m = 0 \quad , \quad m^2 + 2m + 1 = 0$$

$$m = 0 \quad , \quad (m+1)^2 = 0$$

$$m = 0 \quad , \quad (m+1)(m+1) = 0$$

$$m = 0 \quad , \quad m = -1, -1$$

Now Value of $c$ For $m = 0$

$$c = -\frac{\phi_2(m)}{\phi_3'(m)}$$

$$c = -\frac{0}{\phi_3'(m)}$$

$$\Rightarrow \quad c = 0$$

so eq. of the asymptote is

$$y = 0$$

Now Value of $C$ For $m = -1$

$$\frac{c^2}{2!}\phi_3''(m) + \frac{c}{1!}\phi_2'(m) + \phi_1(m) = 0 \quad \text{——— I}$$

Now $\phi_3(m) = m^3 + 2m^2 + m$

$\phi_3'(m) = 3m^2 + 4m$

$\phi_3''(m) = 6m + 4$

and $\qquad \phi_2(m) = 0$

$\Rightarrow \phi_2'(m) = 0$

and $\qquad \phi_1(m) = -m$

Put these values in I

$$\frac{c^2}{2} \cdot 2(3m+2) + c \cdot 0 - m = 0$$

Put $m = -1$

$$\Rightarrow c^2(3(-1)+2) - (-1) = 0$$

$$c^2(-1) + 1 = 0$$

$$1 - c^2 = 0$$

$$c^2 = 1$$

$$c = \pm 1$$

$$\Rightarrow c = 1, \quad c = -1$$

So eqs. of the asymptotes are

$$y = -x + 1 \quad , \quad y = -x - 1$$

Hence the required asymptotes are

$$y = 0 \ , \ y = -x - 1 \text{ and } y = -x + 1$$

**Q.9.** $\qquad y(x-y)^2 = x + y$

$$y(x^2 + y^2 - 2xy) = x + y$$

$$x^2y + y^3 - 2xy^2 = x + y$$

$$y^3 + x^2y - 2xy^2 - x - y = 0$$

<u>Horizontal Asymptote</u>:

Co-efficient of highest power of $x = y$

$\Rightarrow y = 0$ is an asymptote.

<u>Vertical Asymptote</u>:

Co-efficient of highest power of $y = 1$

$\Rightarrow$ There is no horizontal asymptote.

<u>Oblique Asymptote</u>.

Put $x = 1$ and $y = m$ in highest degree terms of $x$ and $y$.

$$\Rightarrow \phi_3(m) = m^3 + m - 2m^2$$

$$\Rightarrow m^3 + m - 2m^2 = 0$$

$$m^3 - 2m^2 + m = 0$$

$$m(m^2 - 2m + 1) = 0$$

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$m=0 \quad , \quad (m-1)^2=0$

$m=0 \quad (m-1)(m-1)=0$

$m=0 \quad m=1,1$

<u>Value of C For $m=1$</u>.

$$\frac{c^2}{2!}\phi_3''(m)+\frac{c}{1!}\phi_2'(m)+\phi_1(m)=0 \quad \text{———} \quad I$$

Now $\phi_3(m) = m^3-2m^2+m$

$\phi_3'(m) = 3m^2-4m+1$

$\phi_3''(m) = 6m-4$

and $\phi_2(m) = 0$

$\phi_2'(m) = 0$

$\phi_1(m) = -1-m$

Put these values in I

$$\frac{c^2}{2}(6m-4)+c(0)-1-m=0$$

Put $m=1$

$$\frac{c^2}{2}(6-4)-1-1=0$$

$$\frac{c^2}{2}.2 \; -2=0$$

$$c^2-2=0$$

$$c^2=2$$

$$c=\pm\sqrt{2}$$

So eqs. of asymptotes are

$$y=x+\sqrt{2} \; , \; y=x-\sqrt{2}$$

Now <u>Value of C For $m=0$</u>.

We know that

$$c=-\frac{\phi_2(m)}{\phi_3'(m)}$$

$$c=-\frac{0}{\phi_3'(m)}$$

$$\Rightarrow c=0$$

So eq. of asymptote is $y=0$

Hence the required asymptotes are

$y=0$, $y=x+\sqrt{2}$ and $y=x-\sqrt{2}$

**Q.10.** $x^2y^2(x^2-y^2)^2 = (x^2+y^2)^3$

$x^2y^2(x^4+y^4-2x^2y^2) - (x^2+y^2)^3 = 0$

$x^6y^2 + x^2y^6 - 2x^4y^4 - (x^6+y^6+3x^2y^4+3x^4y^2) = 0$

$x^6y^2 - 2x^4y^4 + x^2y^6 - x^6 - y^6 - 3x^4y^2 - 3x^2y^4 = 0$

$x^6y^2 - 2x^4y^4 + x^2y^6 - x^6 - y^6 - 3x^4y^2 - 3x^2y^4 = 0$

$(a+b)^3 = a^3+b^3+3a^2b+3ab^2$

<u>Horizontal Asymptote</u>

Co-efficients of highest power of $x = y^2-1$

Put $y^2-1=0$

$\Rightarrow y^2 = 1$

$\Rightarrow y = \pm 1$ is an asymptote.

<u>Vertical Asymptote.</u>

Co-efficient of highest power of $y = x^2-1$

Put $x^2-1=0$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$ is an asymptote.

<u>Inclined Asymptote.</u>

Put $x=1$ and $y=m$ in the highest degree terms.

i.e. $\phi_8(m) = m^2 - 2m^4 + m^6$

Put $\phi_8(m) = 0$

$\Rightarrow m^6 - 2m^4 + m^2 = 0$

$\Rightarrow m^2(m^4 - 2m^2 + 1) = 0$

$\Rightarrow m^2 = 0 \quad , \quad (m^2-1)^2 = 0$

$\Rightarrow m = 0,0 \quad , \quad (m^2-1)(m^2-1) = 0$

$\Rightarrow m^2-1=0 \quad , \quad m^2-1=0$

$\Rightarrow m^2 = 1 \quad , \quad m^2 = 1$

$\Rightarrow m = \pm 1 \quad , \quad m = \pm 1$

$\Rightarrow m = 1, 1, -1, -1$

$\because$ $m = 0,0$ was already used

$\therefore$ No need to look for it.

<u>Value of C For $m = 1,1$.</u>

<u>Formula for C</u>

$$\frac{c^2}{2!}\phi_8''(m) + \frac{c}{1!}\phi_7'(m) + \phi_6(m) = 0 \quad \text{——— I}$$

Now $\phi_8(m) = m^6 - 2m^4 + m^2$

$\phi_8'(m) = 6m^5 - 8m^3 + 2m$

$\phi_8''(m) = 30m^4 - 24m^2 + 2$

and $\phi_7(m) = 0$

$\phi_7'(m) = 0$

and $\phi_6(m) = -1 - m^6 - 3m^2 - 3m^4$

Put these values in I

$$\frac{c^2}{2!}(30m^4 - 24m^2 + 2) + C(0) - 1 - m^6 - 3m^2 - 3m^4 = 0 \quad \text{—— II}$$

Put $m = 1$, we have

$$\frac{c^2}{2}(30 - 24 + 2) - 1 - 1 - 3 - 3 = 0$$

$$\frac{c^2}{2}(8) - 8 = 0$$

$$4c^2 - 8 = 0$$

$$c^2 - 2 = 0$$

$$c^2 = 2$$

$$\Rightarrow \quad c = \sqrt{2}, -\sqrt{2}$$

So the inclined asymptotes when $m = 1, 1$ and $c = \sqrt{2}, -\sqrt{2}$ are $y = x + \sqrt{2}$ and $y = x - \sqrt{2}$

<u>Value of C for $m = -1, -1$.</u>

Put $m = -1$ in II, we have

$$\frac{c^2}{2}(30 - 24 + 2) - 1 - 1 - 3 - 3 = 0$$

$$\Rightarrow \quad c = \sqrt{2}, -\sqrt{2}$$

So eqs. of asymptotes when $m = -1, -1$ and $= \sqrt{2}, -\sqrt{2}$ are

$$y = -x + \sqrt{2}, \quad y = -x - \sqrt{2}$$

Hence the required asymptotes are

$$y = \pm 1, \quad x = \pm 1, \quad y = \pm x + \sqrt{2}, \quad y = \pm x - \sqrt{2}$$

<u>Q.12.</u> $xy^2 = (x+y)^2$

$$xy^2 = x^2 + y^2 + 2xy$$

$$x^2 + 2xy - xy^2 + y^2 = 0 \Rightarrow xy^2 - x^2 - 2xy - y^2 = 0$$

<u>Horizontal asymptote.</u>

Co-efficient of highest power of $x = -1$

$\Rightarrow$ No horizontal asymptote.

<u>Vertical Asymptote.</u>

Co-efficient of highest power of $y = x - 1$

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$\Rightarrow \quad x-1=0$

$\Rightarrow \quad x=1$ is an asymptote.

Inclined Asymptote.

Put $x=1$ and $y=m$ in the highest degree terms

i.e. $\phi_3(m) = m^2$

Put $\phi_3(m)=0 \quad \Rightarrow \quad m^2=0$

$\Rightarrow \quad m=0;\ 0$

$\because m=0,0$

$\therefore$ There is no oblique asymptote.

Hence the required asymptotes are

$x=1$

Q.12 $\quad xy^2-x^2y-3x^2-2xy+y^2+x-2y+1=0$

Horizontal Asymptote.

Co-efficients of highest powers of $x = -y-3$

Put $\quad -y-3=0$

$-y=3$

$y=-3$ is an asymptote.

Vertical Asymptote.

Co-efficients of highest powers of $y = x+1$

put $\quad x+1=0$

$\Rightarrow \quad x=-1$ is an asymptote.

Oblique Asymptote.

Put $x=1$ and $y=m$ in the highest degree terms.

$\Rightarrow \quad \phi_3(m) = m^2-m$ ~~[illegible]~~

Put $\phi_3(m)=0 \quad \Rightarrow \quad m^2-m=0$

$m(m-1)=0$

$\Rightarrow \quad m=0\ ,\quad m-1=0$

$m=0\ ,\quad m=1$

$\because$ when $m=0$ there is no inclined asymptote

$\therefore$ We look for $m=1$

Value of C for m=1.

$$c = -\frac{\phi_2(m)}{\phi_3'(m)} \qquad \text{—— I}$$

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Now $\phi_3(m) = m^2 - m$

$\phi_3'(m) = 2m - 1$

and $\phi_2(m) = -3 - 2m + m^2$

Put these values in I, we have

$$C = -\frac{-3-2m+m^2}{2m-1}$$

Put $m = 1$

$$\Rightarrow C = -\frac{-3-2+1}{2-1} = -\frac{-4}{1} = 4$$

So eq. of inclined asymptote when $m=1$ and $c=4$ is

$$y = x+4$$

Hence the required asymptotes are

$$y+3=0, \quad x+1=0 \text{ and } y = x+4$$

**Q.13** $\quad r = \dfrac{a}{\theta}$ ——— 2

Put $r = \infty \Rightarrow \cancel{\alpha} = \dfrac{a}{\theta}$

$1) \Rightarrow \quad \theta = 0$

i.e $\alpha = 0$

Diff. w.r.t. $\theta$.

$$\frac{dr}{d\theta} = -\frac{a}{\theta^2}$$

$$\frac{d\theta}{dr} = -\frac{\theta^2}{a}$$

x by $r^2$

$$r^2\frac{d\theta}{dr} = \frac{a^2}{\theta^2} \cdot -\frac{\theta^2}{a} = -a$$

Now $p = \lim\limits_{\theta \to \alpha} r^2 \dfrac{d\theta}{dr}$

$$p = \lim_{\theta \to 0} -a$$

$$p = -a$$

Hence the asymptote is

$$p = r\sin(\alpha - \theta)$$

$$-a = r\sin(0-\theta)$$

$$-a = r\sin(-\theta)$$

$$-a = -r\sin\theta$$

Now $\phi_3(m) = m^2 - m$

$\phi_3'(m) = 2m - 1$

and $\phi_2(m) = -3 - 2m + m^2$

Put these values in I, we have

$$c = -\frac{-3-2m+m^2}{2m-1}$$

Put $m=1$

$$\Rightarrow c = -\frac{-3-2+1}{2-1} = -\frac{-4}{1} = 4$$

So eq. of inclined asymptote when $m=1$ and $c=4$ is

$$y = x+4.$$

Hence the required asymptotes are

$$y+3=0, \quad x+1=0 \text{ and } y = x+4$$

**Q.13** $\quad r = \dfrac{a}{\theta}$ ——— I

Put $r=\infty \Rightarrow \infty = \dfrac{a}{\theta}$

$\text{I} \Rightarrow \quad \theta = 0$

i.e $\alpha = 0$

Diff. w.r.t. $\theta$.

$$\frac{dr}{d\theta} = -\frac{a}{\theta^2}$$

$$\frac{d\theta}{dr} = -\frac{\theta^2}{a}$$

x by $r^2$

$$r^2\frac{d\theta}{dr} = \frac{a^2}{\theta^2} \cdot -\frac{\theta^2}{a} = -a$$

Now $p = \lim\limits_{\theta \to \alpha} r^2 \dfrac{d\theta}{dr}$

$$p = \lim_{\theta \to 0} -a$$

$$p = -a$$

Hence the asymptote is

$$p = r\sin(\alpha - \theta)$$

$$-a = r\sin(0-\theta)$$

$$-a = r\sin(-\theta)$$

$$-a = -r\sin\theta$$

$a = r\sin\theta$ is the required asymptote.

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**Q.14.** $r = \dfrac{a}{\sqrt{\theta}}$ ——— I

Put $r = \infty$

$\Rightarrow \infty = \dfrac{a}{\sqrt{\theta}} \Rightarrow \sqrt{\theta} = \dfrac{a}{\infty} \Rightarrow \sqrt{\theta} = 0 \Rightarrow \theta = 0$

Diff. I w.r.t. $\theta$. i.e $\alpha = 0$

$$\frac{dr}{d\theta} = -\frac{1}{2} a \theta^{-3/2}$$

$$\frac{d\theta}{dr} = -\frac{2}{a\theta^{-3/2}}$$

$$\frac{d\theta}{dr} = -\frac{2}{a}\theta^{3/2}$$

$\times$ by $r^2$

$$\Rightarrow r^2\frac{d\theta}{dr} = \frac{a^2}{\theta} \cdot -\frac{2}{a}\theta^{3/2}$$

$$= -2a\sqrt{\theta}$$

Now $p = \lim\limits_{\theta \to 0} r^2 \dfrac{d\theta}{dr}$

$p = \lim\limits_{\theta \to 0} -2a\sqrt{\theta}$

$p = 0$

So the equation of the asymptote is

$$p = r\sin(\alpha - \theta)$$

$$0 = r\sin(0 - \theta)$$

$$r\sin(-\theta) = 0$$

$$-r\sin\theta = 0$$

$$r\sin\theta = 0$$

$$\sin\theta = 0$$

$$\theta = 0$$

**Q.15.** $r = a\,\mathrm{cosec}\,\theta + b$ ——— (A)

$$r = \frac{a}{\sin\theta} + b$$

$r = \dfrac{a + b\sin\theta}{\sin\theta}$ ——— I

Put $r = \infty$

$\Rightarrow \sin\theta = 0$

$\theta = 0, \pi$

i.e $\alpha = 0,\ \beta = \pi$

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Now Diff (A) w.r.t. $\theta$.

$$\frac{dr}{d\theta} = -a \operatorname{Cosec}\theta \cot\theta$$

$$\frac{d\theta}{dr} = -\frac{1}{a \operatorname{Cosec}\theta \cot\theta}$$

$\times$ by $r^2$

$$\Rightarrow r^2 \frac{d\theta}{dr} = -\frac{(a+b\sin\theta)^2}{\sin^2\theta} \cdot \frac{\sin\theta \cdot \sin\theta}{a \cos\theta}$$

$$= -\frac{(a+b\sin\theta)^2}{a\cos\theta}.$$

Value of $P$ when $\alpha = 0$.

$$P = -\lim_{\theta \to 0} \frac{(a+b\sin\theta)^2}{a\cos\theta}$$

$$= -\frac{(a+b(0))^2}{a(1)} = -\frac{a^2}{a} = -a$$

So eq of the asymptote when $\alpha = 0$ is

$$p = r\sin(\alpha - \theta)$$

$$-a = r\sin(0-\theta)$$

$$-a = -r\sin\theta$$

$a = r\sin\theta$ is the asymptote.

Value of $P$ when $\beta = \pi$.

$$P = -\lim_{\theta \to \pi} \frac{(a+b\sin\theta)^2}{a\cos\theta}$$

$$= -\frac{(a+b(0))^2}{a(-1)}$$

$$= -\frac{a^2}{-a} = a$$

So eq. of the asymptote when $\beta = \pi$ is

$$p = r\sin(\beta - \theta)$$

$$a = r\sin(\pi - \theta)$$

$$a = r\sin\theta$$

$\Rightarrow r\sin\theta = a$ is the asymptote.

Q.16.
$$r = 2a\sin\theta\tan\theta$$
$$r = 2a\sin\theta\,\frac{\sin\theta}{\cos\theta}$$
$$r = \frac{2a\sin^2\theta}{\cos\theta} \quad \text{———} \; I$$

Put $r = \infty$

$$\Rightarrow \quad \cos\theta = 0$$
$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$
$$\text{i.e } \alpha = \frac{\pi}{2}, \quad \beta = \frac{3\pi}{2}$$

Diff. I w.r.t. $\theta$
$$\frac{dr}{d\theta} = 2a\left[\frac{\cos\theta\cdot 2\sin\theta\cos\theta - \sin^2\theta(-\sin\theta)}{\cos^2\theta}\right]$$
$$= 2a\left[\frac{2\sin\theta\cos^2\theta + \sin^3\theta}{\cos^2\theta}\right]$$

$\times$ by $1/r^2$
$$\frac{1}{r^2}\frac{dr}{d\theta} = \frac{\cos^2\theta}{4a^2\sin^4\theta}\cdot 2a\left[\frac{2\sin\theta\cos^2\theta + \sin^3\theta}{\cos^2\theta}\right]$$
$$= \frac{2a}{4a^2\sin^4\theta}\cdot\sin\theta\left[\frac{2\cos^2\theta + \sin^2\theta}{1}\right]$$
$$= \frac{2\cos^2\theta + \sin^2\theta}{2a\sin^3\theta}$$
$$\Rightarrow r^2\frac{d\theta}{dr} = \frac{2a\sin^3\theta}{2\cos^2\theta + \sin^2\theta}$$

Now
$$p = \lim_{\theta\to\frac{\pi}{2}} r^2\frac{d\theta}{dr}$$
$$p = \lim_{\theta\to\pi/2}\frac{2a\sin^3\theta}{2\cos^2\theta + \sin^2\theta}$$
$$p = \frac{2a(1)^3}{0 + (1)^2} = 2a$$

So eq. of the asymptote when $\alpha = \pi/2$ is
$$2a = r\sin(\pi/2 - \theta)$$
$$2a = r\cos\theta \text{ is the asymptote}$$

Now
$$p = \lim_{\theta\to 3\pi/2}\frac{2a\sin^3\theta}{2\cos^2\theta + \sin^2\theta}$$
$$= \frac{2a(-1)^3}{0 + (-1)^2} = -2a$$

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So Eq. of the asymptote when $\beta = 3\pi/2$ and $p = -2a$ is

$$-2a = r\sin(3\pi/2 - \theta)$$

$$-2a = -r\cos\theta$$

$$\Rightarrow \quad 2a = r\cos\theta \text{ is an asymptote.}$$

Hence the required asymptote is

$$2a = r\cos\theta.$$

Q.N

$$r\sin 2\theta = a\cos 3\theta$$

$$r = \frac{a\cos 3\theta}{\sin 2\theta} \quad \text{—— I}$$

put $r = \infty$ in I

$$\Rightarrow \quad \infty = \frac{a\cos 3\theta}{\sin 2\theta}$$

$$\Rightarrow \quad \sin 2\theta = 0$$

$$2\theta = 0, \pi$$

$$\theta = 0, \pi/2 \quad \text{i.e} \quad \alpha = 0, \ \beta = \pi/2$$

Diff. (1) w.r.t. $\theta$

$$\frac{dr}{d\theta} = a\left[\frac{\sin 2\theta \cdot (-3\sin 3\theta) - \cos 3\theta \, 2\cos 2\theta}{\sin^2 2\theta}\right]$$

$$\frac{dr}{d\theta} = a\left[\frac{-3\sin 2\theta\sin 3\theta - 2\cos 3\theta\cos 2\theta}{\sin^2 2\theta}\right]$$

$\times$ by $\frac{1}{r^2}$

$$\Rightarrow \quad \frac{1}{r^2}\frac{dr}{d\theta} = \frac{\sin^2 2\theta}{a^2\cos^2 3\theta} \cdot a\left[\frac{-3\sin 2\theta\sin 3\theta - 2\cos 3\theta\cos 2\theta}{\sin^2 2\theta}\right]$$

$$\frac{1}{r^2}\frac{dr}{d\theta} = \frac{-3\sin 2\theta\sin 3\theta - 2\cos 3\theta\cos 2\theta}{a\cos^2 3\theta}$$

$$\Rightarrow \quad r^2\frac{d\theta}{dr} = -\frac{a\cos^2 3\theta}{3\sin 2\theta\sin 3\theta + 2\cos 3\theta\cos 2\theta}$$

Value of $P$ when $\alpha = 0$.

$$P = \lim_{\theta \to \alpha} r^2\frac{d\theta}{dr}$$

$$P = -\lim_{\theta \to 0} \frac{a\cos^2 3\theta}{3\sin 2\theta\sin 3\theta + 2\cos 3\theta\cos 2\theta}$$

$$P = -\frac{a(1)^2}{3(0)(0) + 2(1)(1)}$$

$$P = -\frac{a}{2}$$

So eq. of the asymptote is

$$p = r\sin(\alpha - \theta)$$

$$-\frac{a}{2} = r\sin(0-\theta)$$

$$-\frac{a}{2} = -r\sin\theta$$

$2r\sin\theta = a$ is an asymptote.

Value of $p$ when $\beta = \pi/2$.

$$p = \lim_{\theta \to \beta} r^2 \frac{d\theta}{dr}$$

$$p = -\lim_{\theta \to \pi/2} \frac{a\cos^2 3\theta}{3\sin 2\theta \sin 3\theta + 2\cos 3\theta \cos 2\theta}$$

$$p = -\frac{a\cos^2(3\pi/2)}{3\sin(2\pi/2)\sin 3\pi/2 + 2\cos 3\pi/2 \cos 2\pi/2}$$

$$p = -\frac{a(0)}{\sim}$$

$$p = 0$$

So eq. of the asymptote is

$$p = r\sin(\beta - \theta)$$

$$0 = r\sin(\pi/2 - \theta)$$

$$0 = r\cos\theta$$

$$\Rightarrow \quad r\cos\theta = 0$$

$$\cos\theta = 0$$

$$\theta = \pi/2$$

Hence the required asymptotes are

$2r\sin\theta = a$ and $\theta = \pi/2$

**Q.28.** $r = \dfrac{a}{1-\cos\theta}$ ——— I

Put $r = \infty$

$$\Rightarrow \quad 1 - \cos\theta = 0$$

$$\cos\theta = 1$$

$$\theta = 0 \quad \text{i.e } \alpha = 0$$

Diff. I w.r.t. '$\theta$'

$$\frac{dr}{d\theta} = -a(1-\cos\theta)^{-2}(0 + \sin\theta)$$

$$= -\frac{a}{(1-\cos\theta)^2}(+\sin\theta) = -\frac{a\sin\theta}{(1-\cos\theta)^2}$$

$\times$ by $1/r^2$

$$\frac{1}{r^2}\frac{dr}{d\theta} = \frac{(1-\cos\theta)^2}{a^2} ; -\frac{a\sin\theta}{(1-\cos\theta)^2}$$

$$= -\frac{1}{a}\sin\theta$$

$$r^2\frac{d\theta}{dr} = -\frac{a}{\sin\theta}$$

<u>Value of $p$ when $\alpha = 0$.</u>

$$p = \lim_{\theta\to\alpha} r^2\frac{d\theta}{dr}$$

$$p = -\lim_{\theta\to 0}\frac{a}{\sin\theta}$$

$$p = -\infty$$

$\because$ $p$ tends to $-\infty$

$\therefore$ there is no asymptote for the given curve.

**Q.19.** $r\sin n\theta = a$

$$r = \frac{a}{\sin n\theta} \quad \text{———} \quad I$$

Put $r = \infty$

$$\Rightarrow \sin n\theta = 0$$

$$n\theta = K\pi \qquad \text{where } K = 0, 1, 2, 3\ldots.$$

$$\theta = \frac{K}{n}\pi \quad \text{i.e } \alpha = \frac{K}{n}\pi$$

Diff. w.r.t. $\theta$.

$$\frac{dr}{d\theta} = -\frac{a}{\sin^2 n\theta}\, n\cos n\theta$$

$$\frac{dr}{d\theta} = -\frac{an\cos n\theta}{\sin^2 n\theta}$$

$\times$ by $1/r^2$

$$\frac{1}{r^2}\frac{dr}{d\theta} = \frac{\sin^2 n\theta}{a^2} \cdot -\frac{an\cos n\theta}{\sin^2 n\theta}$$

$$= -\frac{n\cos n\theta}{a}$$

$$\Rightarrow r^2\frac{d\theta}{dr} = -\frac{a}{n\cos n\theta}$$

Applying $\lim_{\theta\to K\pi/n}$

$$\lim_{\theta\to K\pi/n} r^2\frac{d\theta}{dr} = -\lim_{\theta\to K\pi/n}\frac{a}{n\cos n\theta}$$

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$$\Rightarrow \quad p = -\frac{a}{n\cos n \cdot \frac{k\pi}{n}}$$

$$= -\frac{a}{n\cos k\pi}$$

$$= -\frac{a}{n}\sec k\pi$$

So eq. of the Asymptote is

$$p = r\sin(\alpha - \theta)$$

$$-\frac{a}{n}\sec k\pi = r\sin\left(\frac{k}{n}\pi - \theta\right)$$

$$\frac{a}{n}\sec k\pi = -r\sin\left(\frac{k}{n}\pi - \theta\right)$$

$$\frac{a}{n}\sec k\pi = r\sin\left(-\frac{k}{n}\pi + \theta\right)$$

$$\frac{a}{n}\sec k\pi = r\sin\left(\theta - \frac{k}{n}\pi\right)$$

is the required asymptote.

**Q.20.** $r(e^\theta - 1) = a(e^\theta + 1)$

$$\Rightarrow \quad r = \frac{a(e^\theta + 1)}{e^\theta - 1} \quad \text{———} \quad I$$

Put $r = \infty$

$$\Rightarrow \quad e^\theta - 1 = 0$$

$$\Rightarrow \quad e^\theta = 1$$

$$\Rightarrow \quad e^\theta = e^0 \qquad \because e^0 = 1$$

$$\Rightarrow \quad \theta = 0 \quad \text{i.e. } \alpha = 0$$

Diff. (1) w.r.t. $\theta$.

$$\frac{dr}{d\theta} = a\left[\frac{(e^\theta - 1)(e^\theta) - (e^\theta + 1)e^\theta}{(e^\theta - 1)^2}\right]$$

$$= a\left[\frac{e^{2\theta} - e^\theta - e^{2\theta} - e^\theta}{(e^\theta - 1)^2}\right]$$

$$= \frac{-2ae^\theta}{(e^\theta - 1)^2}$$

$\times$ by $1/r^2$

$$\Rightarrow \quad \frac{1}{r^2}\frac{dr}{d\theta} = \frac{(e^\theta - 1)^2}{a^2(e^\theta + 1)^2} \cdot -\frac{2ae^\theta}{(e^\theta - 1)^2}$$

$$= -\frac{2e^\theta}{a(e^\theta + 1)^2}$$

$$\Rightarrow \quad r^2\frac{d\theta}{dr} = -\frac{a(e^\theta + 1)^2}{2e^\theta}$$

Now $\quad p = \lim_{\theta \to 0} r^2 \frac{d\theta}{dr}$

$$p = -\lim_{\theta \to 0} \frac{a(e^{\theta}+1)^2}{2e^{\theta}}$$

$$p = -\frac{a(e^{0}+1)^2}{2e^{0}} = -\frac{a(1+1)^2}{2(1)} = -\frac{4a}{2} = -2a$$

So eq. of the asymptote is

$$p = r(\sin(\alpha - \theta))$$

$$-2a = r\sin(0-\theta)$$

$$2a = r\sin\theta$$

is the required asymptote.

**Q.2.**

$$r^n \sin n\theta = a^n$$

$$\Rightarrow r^n = \frac{a^n}{\sin n\theta} \quad \text{——— I}$$

Put $r = \infty$

$$\Rightarrow \sin n\theta = 0$$

$$n\theta = k\pi \qquad \text{where } k = 0, 1, 2, 3 \ldots$$

$$\theta = \frac{k}{n}\pi$$

Diff. (1) w.r.t. $\theta$.

$$\cancel{n}\, r^{n-1} \frac{dr}{d\theta} = -\frac{a^n}{\sin^2 n\theta} \cancel{n}\cos n\theta$$

$$\frac{dr}{d\theta} = -\frac{a^n \cos n\theta}{\sin^2 n\theta \; r^{n-1}}$$

$$\frac{dr}{d\theta} = -\frac{a^n \cos n\theta}{\sin^2 n\theta \; r^n \cdot r^{-1}}$$

$$r^{-1} \frac{dr}{d\theta} = -\frac{a^n \cos n\theta}{\sin^2 n\theta \; r^n}$$

$$\frac{1}{r}\frac{dr}{d\theta} = -\frac{\cancel{a^n} \cos n\theta}{\sin^2 n\theta \; \frac{\cancel{a^n}}{\sin n\theta}}$$

$$\frac{1}{r}\frac{dr}{d\theta} = -\frac{\cos n\theta}{\sin n\theta}$$

$\times$ by $r$

$$r\frac{d\theta}{dr} = -\frac{\sin n\theta}{\cos n\theta}$$

$$r^2 \frac{d\theta}{dr} = -\frac{\sin n\theta}{\cos n\theta} \cdot \frac{a}{\sin^{1/n} n\theta}$$

$$\because r^n = \frac{a^n}{\sin n\theta}$$

$$(r^n)^{1/n} = \frac{(a^n)^{1/n}}{(\sin n\theta)^{1/n}}$$

$$r = \frac{a}{(\sin n\theta)^{1/n}}$$

$$r^2 \frac{d\theta}{dr} = -\frac{a \sin n\theta \cdot \sin^{-1/n} n\theta}{\cos n\theta}$$

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$$r^2 \frac{d\theta}{dr} = -\frac{a(\sin n\theta)^{1-\frac{1}{n}}}{\cos n\theta}$$

$$r^2 \frac{d\theta}{dr} = -\frac{a(\sin n\theta)^{\frac{n-1}{n}}}{\cos n\theta}$$

Now $\quad p = \lim\limits_{\theta \to \frac{k\pi}{n}} r^2 \frac{d\theta}{dr}$

$$= -\lim_{\theta \to \frac{k\pi}{n}} \frac{a(\sin n\theta)^{\frac{n-1}{n}}}{\cos n\theta}$$

$$= -\lim_{\theta \to \frac{k\pi}{n}} \frac{a(\sin n\theta)^{\frac{n-1}{n}}}{\cos n\theta}$$

$$= -\frac{a(\sin n(\frac{k\pi}{n}))^{\frac{n-1}{n}}}{\cos n(\frac{\pi k}{n})}$$

$$= -\frac{a(\sin k\pi)^{\frac{n-1}{n}}}{\cos(k\pi)}$$

$$= -\,0$$

$$= 0$$

نوٹ.
چونکہ $K = 0,1,2\ldots$ ہے لہذا اگر
$\sin \pi$ لیا جائے تو جواب صفر آئے گا
اسی طرح اگر $K=2$ لیا جائے تو
$\sin 2\pi$ بھی صفر آئے گا اسی طرح
اگر $K=3$ لیا جائے تو بھی جواب صفر ہی
آئے لہذا ہم نے $\sin K\pi$ کو صفر
دکھایا ہے۔

So eq. of asymptote is

$$p = r\sin(\alpha - \theta)$$

$$0 = r\sin\left(\frac{k\pi}{n} - \theta\right)$$

$$\Rightarrow \sin\left(\frac{k\pi}{n} - \theta\right) = 0$$

$$\frac{k\pi}{n} - \theta = 0$$

$$\theta = \frac{k\pi}{n}$$

is an asymptote.

**Q.22.** $\quad r^2 \sin\theta = a^2 \cos 2\theta$

$$r^2 = \frac{a^2 \cos 2\theta}{\sin\theta} \quad \text{———} \quad (I)$$

Put $r = \infty$

$$\Rightarrow \sin\theta = 0$$

$$\theta = 0, \pi \qquad \text{i.e. } \alpha = 0, \beta = \pi$$

Diff. (1) w.r.t. '$\theta$'

$$2r\frac{dr}{d\theta} = a^2\left[\frac{-\sin\theta\, 2\sin 2\theta - \cos 2\theta \cos\theta}{\sin^2\theta}\right]$$

$$\frac{dr}{d\theta} = -\frac{a^2}{2r}\left[\frac{2\sin\theta\sin 2\theta + \cos\theta\cos 2\theta}{\sin^2\theta}\right]$$

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$\times$ by $\frac{1}{r^2}$

$$\Rightarrow \frac{1}{r^2}\frac{dr}{d\theta} = -\frac{\sin\theta}{a^2\cos 2\theta}\cdot\frac{a^2}{2r}\left[\frac{2\sin\theta\sin 2\theta + \cos\theta\cos 2\theta}{\sin^2\theta}\right]$$

$$\frac{1}{r^2}\frac{dr}{d\theta} = -\frac{1}{2r\cos 2\theta}\cdot\left[\frac{2\sin\theta\sin 2\theta + \cos\theta\cos 2\theta}{\sin\theta}\right]$$

$$\frac{1}{r^2}\frac{dr}{d\theta} = -\frac{1}{2\,\frac{a\sqrt{\cos 2\theta}}{\sqrt{\sin\theta}}.\cos 2\theta}\cdot\left[\frac{2\sin\theta\sin 2\theta + \cos\theta\cos 2\theta}{\sin\theta}\right]$$

$$= -\frac{\sqrt{\sin\theta}}{2a(\cos 2\theta)^{3/2}}\cdot\left[\frac{2\sin\theta\sin 2\theta + \cos\theta\cos 2\theta}{\sqrt{\sin\theta}}\right]$$

$$r^2\frac{d\theta}{dr} = -\frac{2a\cos^{3/2}2\theta\,\sqrt{\sin\theta}}{2\sin\theta\sin 2\theta + \cos\theta\cos 2\theta}$$

Now <u>Value of $p$ when $\alpha = 0$.</u>

$$p = \lim_{\theta\to\alpha} r^2\frac{d\theta}{dr}$$

$$p = -\lim_{\theta\to 0}\frac{2a\cos^{3/2}2\theta\,\sqrt{\sin\theta}}{2\sin\theta\sin 2\theta + \cos\theta\cos 2\theta}$$

$$\Rightarrow p = 0$$

So eq. of asymptote is

$$p = r\sin(\alpha - \theta)$$
$$0 = r\sin(0-\theta)$$
$$0 = -r\sin\theta$$
$$\Rightarrow r\sin\theta = 0$$
$$\sin\theta = 0$$
$$\theta = 0 \text{ is an asymptote.}$$

Now <u>Value of $p$ when $\beta = \pi$.</u>

$$p = \lim_{\theta\to\beta} r^2\frac{d\theta}{dr}$$

$$p = -\lim_{\theta\to\pi}\frac{2a\cos^{3/2}2\theta\,\sqrt{\sin\theta}}{2\sin\theta\sin 2\theta + \cos\theta\cos 2\theta}$$

$$\Rightarrow p = 0$$

So eq. of asymptote is

$$0 = r\sin(\pi - \theta)$$
$$0 = +r\sin\theta \quad\rightarrow\quad \theta = 0 \text{ is an asymptote}$$

Hence the required asymptote is $\theta = 0$.