calculs

Ex 7.1

Asymptote is a straight line 'l' of an infinite branch of a curve 'C' if the distance between 'l' & 'C' tends to zero when distance moved along 'l' tends to infinity.

For Asymptote // to x-axis: Put coeff of highest power of $x = 0$
For Asymptote // to y-axis: Put coeff of highest power of $y = 0$

(1) $Y = \dfrac{(x-2)^2}{x^2}$

$\Rightarrow x^2 y = (x-2)^2$

$\Rightarrow x^2 y = x^2 + 4 - 4x$

$\Rightarrow x^2 y - x^2 - 4 + 4x = 0$

$\Rightarrow x^2(y-1) + 4x - 4 = 0$

Asymptote // to x-axis $\quad y - 1 = 0 \Rightarrow y = 1$

Asymptote // to y-axis $\quad x^2 = 0 \Rightarrow x = 0, 0$ (Two Coincident Asymptotes).

$x = 0,\ y = 1$ are required Asymptotes.

(2) $x^2 y^2 = 12(x-3)$

$x^2 y^2 - 12x + 36 = 0$

Asymptote // to x-axis $\quad y^2 = 0 \Rightarrow y = 0, 0$ (Two Coincident)

Asymptote // to y-axis $\quad x^2 = 0 \Rightarrow x = 0, 0$ (Two Coincident)

$x = 0, y = 0$ are required Asymptotes

Working Rule

Case 1 If $\phi_n'(m) \neq 0$ then use formula
$C\phi_n'(m) + \phi_{n-1}(m) = 0$
'n' = highest power

Case 2 If $\phi_n'(m) = 0$ then
→ If $\phi_{n-1}(m) \neq 0$ then No Asymptote
→ If Also $\phi_{n-1}(m) = 0$ then use $\dfrac{c^2}{2!}\phi_n''(m) + c\phi_{n-1}'(m) + \phi_{n-2} = 0$

(3) $2xy = x^2 + 3$

$x^2 - 2xy + 3 = 0$

Asymptote // to x-axis $\quad$ No Asymptote

Asymptote // to y-axis $\quad -2x = 0 \Rightarrow x = 0$

Inclined Asymptote $\quad y = mx + c$ — (1)

$\phi_2(x, y) = x^2 - 2xy$ $\qquad \phi_2(m) = 1 - 2m = 0 \Rightarrow \boxed{m = \frac{1}{2}}$

$\phi_2(m) = 1 - 2m$

$\phi_1(x, y) = 0$

$\phi_1(m) = 0$

$\phi_2'(m) = -2$ $\qquad \phi_2'(\frac{1}{2}) = -2 \neq 0$ at $m = \frac{1}{2}$

To find 'C' $\quad C\phi_2'(m) + \phi_1(m) = 0$

$\Rightarrow C(-2) + 0 = 0 \quad \Rightarrow \boxed{C = 0}$

Putting values of m & c $\quad Y = \frac{1}{2}x + 0 \Rightarrow Y = \frac{x}{2}$

Hence Asymptotes are $\boxed{Y = \frac{x}{2}}$ & $\boxed{x = 0}$

Q4 $x^2(x-y)^2+a^2(x^2-y^2)=a^2xy$

$x^2(x^2+y^2-2xy)+a^2x^2-a^2y^2-a^2xy=0$

$x^4+x^2y^2-2x^3y+a^2x^2-a^2y^2-a^2xy=0$

$x^4+y^2(x^2-a^2)-2x^3y+a^2x^2-a^2xy=0$

No Asymptote // to x-axis.

Asymptote // to y-axis $\quad x^2-a^2=0$

$\Rightarrow \boxed{x=\pm a}$

Inclined Asymptote $\quad y=mx+c.$

$\phi_4(x,y)=x^4+x^2y^2-2x^3y$

$\phi_4(m)=1+m^2-2m$

$\phi_4(m)=0$

$\Rightarrow m^2-2m+1=0$

$\Rightarrow m=\frac{2\pm\sqrt{4-4}}{2}$

$=\frac{2\pm0}{2}=1,1$

$\phi_3(x,y)=0=\phi_3(m)$

$\phi_4'(m)=2m-2$

At $m=1$ $\phi_4'(m)=2(1)-2=0$

So we can not use $c\,\phi_n'(m)+\phi_{(n-1)}(m)=0$

since $\phi_4'(1)=0$ & $\phi_3(1)=0$ (i.e at $m=1$)

So Use this formula. $\frac{c^2}{2}\phi_4''(m)+c\phi_3'(m)+\phi_2(m)=0$

$\therefore \phi_4''(m)=2$

$\phi_2(m)=-a^2m^2+a^2-a^2m$

$\phi_3'(m)=0$

$\frac{c^2}{2}(2)+c(0)+a^2-a^2m^2-a^2m=0$

$c^2=-a^2+a^2m^2+a^2m$

$c^2=-a^2+a^2\cdot1+a^2\cdot1$ for $m=1$

$c^2=a^2$

$c=\pm a.$

Therefore when $m=1, c=a$ $\quad y=mx+c \Rightarrow \boxed{y=x+a}$

when $m=1, c=-a$ $\quad y=mx+c \Rightarrow \boxed{y=x-a}$

Thus inclined asymptotes are $y=x\pm a$.

x———————x

(5) $(x-y)^2(x^2+y^2)-10(x-y)x^2+12y^2+2x+y=0$

$(x^2+y^2-2xy)(x^2+y^2)-10x^3+10yx^2+12y^2+2x+y=0$

$x^4+x^2y^2+y^2x^2+y^4-2x^3y-2xy^3-10x^3+10yx^2+12y^2+2x+y=0$

Asymptote // to x-axis No

Asymptote // to y-axis No

Inclined Asymptote $\boxed{y=mx+c}$ —(1) Find values of m & c and Put in (1) to get Inclined Asympt

$\phi_4(x,y)=x^4+2x^2y^2-2x^3y-2xy^3+y^4$

$\phi_4(m)=1+2m^2-2m-2m^3+m^4$

$\phi_4'(m)=4m-2-6m^2+4m^3$

$\phi_4'(1)=4-2-6+4=0$

$\phi_3(m)=-10+10m$

$\phi_3(1)=-10+10=0$

$\phi_4(m)=0$

$\Rightarrow m^4-2m^3+2m^2-2m+1=0$

	1	-2	2	-2	1
1	$\downarrow$	1	-1	1	-1
	1	-1	1	-1	$\underline{0}$
1	$\downarrow$	1	0	1	
	1	0	1	$\underline{0}$	

$m^2+1=0$

$m^2=-1 \Rightarrow m=\pm i$ So leave it, being imaginary

$m=1,1$ only

So we can not use $c\phi_n'(m)+\phi_{(n-1)}(m)=0$

We use $\frac{c^2}{\underline{2}}\phi_4''(m)+c\phi_3'(m)+\phi_2(m)=0$

$\phi_3'(m)=10$

$\phi_2(m)=12m^2$

$\phi_4''(m)=12m^2-12m+4$

$\frac{c^2}{\underline{2}}(12m^2-12m+4)+c(10)+12m^2=0$

$\therefore \frac{c^2}{\underline{2}}(12m^2-12m+4)+c(10)+12m^2=0$

Put $m=1$ $\frac{c^2}{2}(\cancel{12}-\cancel{12}+4)+10c+12=0$

$\frac{4c^2}{2}+10c+12=0$

$2c^2+10c+12=0 \Rightarrow c=\frac{-10\pm\sqrt{100-4\cdot2\cdot12}}{4}$

$=\frac{-10\pm\sqrt{100-96}}{4}$

$c=\frac{-10\pm2}{4}=-2,-3$

$m=1, c=-2$ $y=mx+c \Rightarrow \boxed{y=x-2}$

$m=1$ $c=-3$ $y=mx+c \Rightarrow \boxed{y=x-3}$

We leave imaginary values of m. Hence two asymptotes

Q6 $x^2y + xy^2 + xy + y^2 + 3x = 0$

$y^2(x+1) + x^2y + xy + 3x = 0$

Asymptote // to x-axis $\boxed{y=0}$

Asymptote // to y-axis $\boxed{x+1=0}$

Inclined Asymptote $y = mx + c$

$\phi_3(m) = m + m^2$

$\phi_3(m) = 0$

$\Rightarrow m + m^2 = 0$

$m(1+m) = 0$

$m = 0, -1$

$\phi_3'(m) = 1 + 2m$

$\phi_3'(0) = 1 + 0 \neq 0$

So Use formula $c\phi_3'(m) + \phi_2(m) = 0$

$\phi_2(m) = m + m^2$; $\Rightarrow c(1+2m) + m + m^2 = 0$

when $m = 0$ $\Rightarrow c(1+0) + 0 + 0 = 0$

$\therefore$ when $m = 0$ $\Rightarrow \boxed{c=0}$

$\phi_3'(-1) = 1 + 2(-1) \neq$ so use $c\phi_3'(m) + \phi_2(m) = 0$

$\Rightarrow c\,(1+2m) + m + m^2 = 0$

when $m = -1$ $\Rightarrow c(1-2) - 1 + 1 = 0$

when $m = -1$ $\Rightarrow \boxed{c=0}$

$\therefore m = 0, c = 0 \Rightarrow y = mx + c \Rightarrow y = 0 \cdot x + 0 \Rightarrow \boxed{y=0}$ as already seen.

$m = -1, c = 0 \Rightarrow y = mx + c \Rightarrow y = -x + 0$

$\Rightarrow \boxed{y = -x}$

Hence $y = 0$

$x + 1 = 0$

$y = -x$ are the required asymptotes.

Q7. $(x-y+1)(x-y-2)(x+y) = 8x-1$

$(x^2 - xy - 2x - xy + y^2 + 2y + x - y - 2)x+y = 8x-1$

$x^3 - \cancel{x^2y} - 2x^2 - x^2y + \cancel{xy^2} + \cancel{2yx} + x^2 - \cancel{xy} - 2x + \cancel{x^2y} - \cancel{xy^2} - \cancel{2xy} - xy^2 + y^3 + 2y^2$
$+ \cancel{xy} - y^2 - 2y - 8x + 1 = 0$

$x^3 + y^3 - 2x^2 + x^2 - x^2y - xy^2 - 10x + 2y^2 - y^2 - 2y + 1 = 0$

$x^3 + y^3 - x^2 - x^2y - xy^2 - 10x + y^2 - 2y + 1 = 0$

Asymptote // to x-axis No Asymptote

Asymptote // to y-axis No Asymptote.

Inclined Asymptote $y = mx + c$

$\phi_3(m) = 1 + m^3 - m - m^2$ $\phi_3(m) = 0$

$\phi_3'(m) = 3m^2 - 1 - 2m$ $\Rightarrow m^3 - m^2 - m + 1 = 0$

$\Rightarrow$
1	1	−1	−1	1
	↓	1	0	−1
	1	0	−1	$\boxed{0}$

$\phi_3'(1) = 3(1) - 1 - 2(1) = 0$

$\phi_3'(-1) = 3(1) - 1 - 2(-1) \neq 0$ $m^2 - 1 = 0$

$\phi_2(m) = -1 + m^2$ $m^2 = 1 \Rightarrow \boxed{m = \pm 1}$

$\phi_2(1) = -1 + 1 = 0$

since $\phi_3'(-1) \neq 0$ so use $c\,\phi_3'(m) + \phi_2(m) = 0$

$c[3m^2 - 1 - 2m] + m^2 - 1 = 0$ when $m = -1$

$c[3(-1)^2 - 1 - 2(-1)] + (-1)^2 - 1 = 0$

$c(4) = 0 \Rightarrow \boxed{c = 0}$ when $m = -1$.

$\therefore$ when $m = -1$, $c = 0$ $y = mx + c \Rightarrow y = -x + 0 \Rightarrow \boxed{y = -x}$

Since $\phi_3'(1) = 0 = \phi_2(1)$ So Use $\frac{c^2}{\lfloor 2}\phi_3''(m) + c\,\phi_2'(m) + \phi_1(m) = 0$

$\phi_3''(m) = 6m - 2$ $\frac{c^2}{2}(6m-2) + c(2m) + (-10 - 2m) = 0$

$\phi_2'(m) = 2m$ Put $m=1$ $\frac{c^2}{2}(6-2) + c(2) + (-10 - 2) = 0$

$\phi_1(m) = -10 - 2m$ $2c^2 + 2c - 12 = 0$

$c^2 + c - 6 = 0 \Rightarrow c = \frac{-1 \pm \sqrt{1+24}}{2}$
$= 2, -3$

Therefore when $m = 1$, $c = 2$, $y = mx + c \Rightarrow \boxed{y = x + 2}$

when $m = 1$, $c = -3$, $y = mx + c \Rightarrow \boxed{y = x - 3}$

(8) $y^3 + x^2y + 2xy^2 - y + 1 = 0$

Asymptote // to x-axis $\boxed{y=0}$

Asymptote // to y-axis No Asymptote

Inclined Asymptote $y = mx + c$

$\phi_3(m) = m^3 + 2m^2 + m$

$\phi_3'(m) = 3m^2 + 4m + 1$

$\phi_3'(0) = 3(0) + 4(0) + 1 \neq 0$

$\phi_3(m) = 0$

$\Rightarrow m^3 + 2m^2 + m = 0$

$$\begin{array}{r|rrr} -1 & 1 & 2 & 1 \\ & \downarrow & -1 & -1 \\ \hline & 1 & 1 & \boxed{0} \end{array}$$

$m^2 + m = 0$

$m(m+1) = 0$

$m = 0, -1$

Since $\phi_3'(0) \neq 0$

So use $c\phi_3'(m) + \phi_2(m) = 0$

$\phi_2(m) = 0, \Rightarrow c(3m^2 + 4m + 1) + 0 = 0$

$\Rightarrow c.(3(0) + 4(0) + 1) = 0$ when $m = 0$

$\Rightarrow c = 0$

$\therefore y = mx + c$

$y = 0 + 0$

$\boxed{y = 0}$ as already seen.

$\phi_3'(-1) = 3(-1)^2 + 4(-1) + 1 = 0$

$\phi_2(-1) = 0$

Since $\phi_3'(-1) = 0 = \phi_2(-1)$ So use $\frac{c^2}{\underline{|2}}\phi_3''(m) + c\phi_2'(m) + \phi_1(m) = 0$

$\phi_3''(m) = 6m + 4$

$\phi_2'(m) = 0$

$\phi_1(m) = -m$

$\Rightarrow \frac{c^2}{\underline{|2}}(6m + 4) + 0 - m = 0$

$\Rightarrow c^2(3m + 2) - m = 0$

$\Rightarrow c^2(3(-1) + 2) - (-1) = 0$ when $m = -1$

$-c^2 + 1 = 0 \Rightarrow c^2 = 1$

$\boxed{c = \pm 1}$

$\therefore$ when $m = -1, c = 1$ $y = mx + c \Rightarrow \boxed{y = -x + 1}$

when $m = -1, c = -1$ $y = mx + c \Rightarrow \boxed{y = -x - 1}$

Hence $y = -x + 1$

$y = -x - 1$

$y = 0$ are required Asymptotes.

(9) $y(x-y)^2 = x+y$

$y(x^2+y^2-2xy)-x-y=0$

$yx^2+y^3-2xy^2-x-y=0$

Asymptote // to x-axis $\boxed{y=0}$

Asymptote // to y-axis No Asymptote

Inclined Asymptote $y=mx+c$

$\phi_3(m) = m+m^3-2m^2$

$\phi_3'(m) = 3m^2-4m+1$

$\phi_3'(0) = 3(0)-4(0)+1 \neq 0$

Since $\phi_3'(0) \neq 0$

$\phi_3(m)=0$

$\Rightarrow m^3-2m^2+m=0$

$\Rightarrow m(m^2-2m+1)=0$

$\Rightarrow m(m-1)^2=0$

$\Rightarrow m(m-1)(m-1)=0$

$\Rightarrow m=0,1,1$

So use $c\,\phi_3'(m)+\phi_2(m)=0$

$\phi_2(m)=0$ $\quad c(3m^2-4m+1)+0=0$

$c(3(0)-4(0)+1)=0$ when $m=0$

$\boxed{c=0}$ when $m=0$

$\therefore$ when $m=0, c=0$ $\quad y=mx+c \Rightarrow \boxed{y=0}$

$\phi_3'(1)=3(1)-4(1)+1=0$

$\phi_2(1)=0$

Since $\phi_3'(1)=0=\phi_2(1)$ $\quad$ So use $\frac{c^2}{\underline{|2}}\phi_3''(m)+c\,\phi_2'(m)+\phi_1(m)=0$

$\phi_3''(m)=6m-4$

$\phi_2'(m)=0$

$\phi_1(m)=-1-m$

$\Rightarrow \frac{c^2}{\underline{|2}}(6m-4)+c.(0)+(-1-m)=0$

$\Rightarrow c^2(3m-2)-1-m=0$

$\Rightarrow c^2(3-2)-1-1=0$ when $m=1$

$\Rightarrow c^2=2$

$\Rightarrow c=\pm\sqrt{2}$

when $m=1, c=\sqrt{2}$ $\quad y=mx+c \Rightarrow \boxed{y=x+\sqrt{2}}$

when $m=1, c=-\sqrt{2}$ $\quad y=mx+c \Rightarrow \boxed{y=x-\sqrt{2}}$

Hence $y=0$; $y=x+\sqrt{2}$, $y=x-\sqrt{2}$ are required Asymptotes.

(10) $x^2y^2(x^2-y^2)^2=(x^2+y^2)^3$

$x^2y^2(x^4+y^4-2x^2y^2)=x^6+y^6+3(x^2y^2)(x^2+y^2)$ $\qquad (a+b)^3=a^3+b^3+3ab(a+b)$

$x^6y^2+x^2y^6-2x^4y^4 \Rightarrow x^6-y^6-3x^4y^2-3x^2y^4=0$

$x^6(y^2-1)+y^6(x^2-1)-2x^4y^4-3x^4y^2-3x^2y^4=0$

Asymptote // to x-axis $\quad y^2-1=0$

$\Rightarrow \boxed{y=\pm 1}$

Asymptote // to y-axis $\quad x^2-1=0$

$\boxed{x=\pm 1}$

Inclined Asymptote $\qquad y=mx+c$

$\phi_8(m)=m^2+m^6-2m^4$

$\phi_8'(m)=2m+6m^5-8m^3$

$\phi_8'(0)=2(0)+6(0)-8(0)=0$

$\phi_7(m)=0$

Since $\phi_8'(0)=0=\phi_7(0)$

$\phi_8(m)=0$

$\Rightarrow m^6-2m^4+m^2=0$

$\Rightarrow m^2(m^4-2m^2+1)=0$

$\Rightarrow m^2(m^2-1)^2=0$

$\Rightarrow m^2(m^2-1)(m^2-1)=0$

$\Rightarrow m=0,0,-1,1,1,-1$

So use formula $\frac{c^2}{\lfloor 2}\phi_8''(m)+c\,\phi_7'(m)+\phi_6(m)=0$

$\phi_8''(m)=2+30m^4-24m^2$

$\phi_7'(m)=0$

$\phi_6(m)=-1-m^6-3m^2-3m^4$

$\Rightarrow \frac{c^2}{\lfloor 2}(2+30m^4-24m^2)+c\cdot 0-1-m^6-3m^2-3m^4=0$

$\Rightarrow \frac{c^2}{\cancel{2}}\cancel{2}(1+15m^4-12m^2)-1-m^6-3m^2-3m^4=0$

$\Rightarrow c^2(1+0-0)-1-0-0-0=0$

$\Rightarrow c^2=1 \qquad \Rightarrow c=\pm 1$

when $m=0$, $c=1$ $\qquad y=mx+c \Rightarrow \boxed{y=1}$

when $m=0$, $c=-1$ $\qquad y=mx+c \Rightarrow \boxed{y=-1}$ as already seen.

Again $\phi_8'(1)=2(1)+6(1)-8=0$

$\phi_7(1)=0$

Since $\phi_8'(1)=0=\phi_7(1)$ $\quad \therefore \frac{c^2}{\lfloor 2}\phi_8''(m)+c\,\phi_7'(m)+\phi_6(m)=0$

$\Rightarrow \frac{c^2}{2}(2+30m^4-24m^2)+0-1-m^6-3m^2-3m^4=0$

(when $m=1$) $\Rightarrow \frac{c^2}{2}(2+30-24)+0-1-1-3-3=0$

$\Rightarrow 4c^2-8=0 \qquad \Rightarrow c^2=\frac{8}{4}=2$

$c=\pm\sqrt{2}$

when $m=1$ $c=\sqrt{2}$ $\qquad y=mx+c \Rightarrow \boxed{y=x+\sqrt{2}}$

when $m=1$ $c=-\sqrt{2}$ $\qquad y=mx+c \Rightarrow \boxed{y=x-\sqrt{2}}$

$\phi_8'(-1) = 2(-1)+6(-1)^5-8(-1)^3$
$= -2-6+8 = 0$
$\phi_7(-1) = 0$

$\because \phi_8'(-1) = 0 = \phi_7(-1)$ So use $\frac{c^2}{\underline{2}}\phi_8''(m) + c\phi_7'(m) + \phi_6(m) = 0$

$\Rightarrow \frac{c^2}{2}(2+30m^4-24m^2)+0.-1-m^6-3m^2-3m^4=0$

$\Rightarrow c^2(1+15m^4-12m^2)-1-m^6-3m^2-3m^4=0$

when $m=-1 \Rightarrow c^2(1+15-12)-1-(-1)^6-3(-1)^2-3(-1)^4=0$

$\Rightarrow 4c^2-1-1-3-3=0$

$\Rightarrow c^2 = \frac{8}{4} = 2$

$\Rightarrow c = \pm\sqrt{2}$

when $m=-1$ $c=\sqrt{2}$ $y=mx+c \Rightarrow \boxed{y=-x+\sqrt{2}}$
$m=-1$ $c=-\sqrt{2}$ $y=mx+c \Rightarrow \boxed{y=-x-\sqrt{2}}$

x———————————x

Q11 $xy^2=(x+y)^2$
$xy^2-x^2-y^2-2xy=0$
$y^2(x-1)-x^2-2xy=0$
Asymptote // to x-axis No Asymptote
Asymptote // to y-axis $x-1=0 \Rightarrow \boxed{x=1}$
Inclined Asymptote $y=mx+c$.

$\phi_3(m) = m^2$ $\qquad \phi_3(m)=0$
$m^2=0 \Rightarrow m=0,0$

$\phi_3'(m) = 2m$
$\phi_3'(0) = 2(0) = 0$
$\phi_2(m) = -1-m^2-2m$
$\phi_2(0) = -1$

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Since $\phi_3'(0)=0$ So No Asymptote
and $\phi_2(0)=-1\neq 0$ (See working Rule Case 2)

So only one Asymptote $\boxed{x=1}$

x———————————x

(12) $xy^2 - x^2y - 3x^2 - 2xy + y^2 + x - 2y + 1 =$

$y^2(x+1) - x^2(y+3) - 2xy + x - 2y + 1 = 0$

Asymptote // to x-axis $\boxed{y+3=0}$

Asymptote // to y-axis $\boxed{x+1=0}$

Inclined Asymptote $y = mx + c$

$\phi_3(m) = m^2 - m$

$\phi_3'(m) = 2m - 1$

$\phi_3'(0) = 0 - 1 \neq 0$

$\phi_3(m) = 0$

$\Rightarrow m^2 - m = 0$

$\Rightarrow m(m-1) = 0$

$\Rightarrow m = 0, 1$

Since $\phi_3'(0) \neq 0$ so use $c\phi_3'(m) + \phi_2(m) = 0$

$\phi_2(m) = -2m - 3 + m^2$

$c(2m-1) + (-2m - 3 + m^2) = 0$

$\Rightarrow c(0-1) + (0 - 3 + 0) = 0$

$\Rightarrow -c - 3 = 0 \quad \Rightarrow \boxed{c=-3}$

when $m = 0, c = -3$ $\quad y = mx + c \Rightarrow \boxed{y = -3}$ as already seen

$\phi_3'(m) = 2m - 1$

$\phi_3'(1) = 2(1) - 1 \neq 0$

Since $\phi_3'(1) \neq 0$ so use $c\phi_3'(m) + \phi_2(m) = 0$

$\Rightarrow c(2m-1) + (-2m - 3 + m^2) = 0$

when $m=1$ $\Rightarrow c(2-1) + (-2\cdot1 - 3 + 1) = 0$

$\Rightarrow c + (-4) = 0$

$\Rightarrow c = 4$ when $m = 1$

when $m = 1$ $c = 4$ $\quad y = mx + c \Rightarrow \boxed{y = x + 4}$

Hence required Asymptotes are

$y + 3 = 0$

$x + 1 = 0$

$y = x + 4$

Asymptotes of the Polar Curves.

Working Rule:- $f(r,\theta)=0$ ——— ①

(i) Put $r=\infty$ in ① and find values of θ.

Denote these values of θ by α ∴ α is that value of θ for which $r=\infty$.

(ii) find $r^2\dfrac{d\theta}{dr}$ from ① ——— ⅱ

(iii) find $P=\lim\limits_{\theta\to\alpha} r^2\dfrac{d\theta}{dr}$ ——— ⅲ

(iv) Asymptote in Polar form. $P=r\sin(\alpha\pm\theta)$ ——— ⅳ

Q13 $r=\dfrac{a}{\theta}$ ——— ①

$r=\infty$

$\infty=\dfrac{a}{\theta} \Rightarrow \dfrac{1}{0}=\dfrac{a}{\theta} \Rightarrow \theta=0=\alpha$ (alpha)

Differentiate ① w.r.t 'r' $r=a\theta^{-1}$

$$1=-\frac{a}{\theta^2}\frac{d\theta}{dr}$$

$$\frac{d\theta}{dr}=-\frac{\theta^2}{a} \qquad r^2\frac{d\theta}{dr}=r^2\left(\frac{-\theta^2}{a}\right)$$

$$r^2\frac{d\theta}{dr}=\frac{a^2}{\theta^2}\left(\frac{-\theta^2}{a}\right)=-a \text{ ——— ⅱ}$$

(iii) $P=\lim\limits_{\theta\to\alpha} r^2\dfrac{d\theta}{dr}=\lim\limits_{\theta\to 0}(-a)=-a$

Eq of Asymptote $P = r\,\mathrm{Sin}(\alpha - \theta)$

$p = -a \quad \alpha = 0, \quad -a = r\,\mathrm{Sin}(0-\theta)$

$-a = -r\,\mathrm{Sin}\,\theta$

$a = r\,\mathrm{Sin}\,\theta$ Ans.

$r = \dfrac{a}{\sqrt{\theta}}$ ——— (I)

$\infty = \dfrac{a}{\sqrt{\theta}} \Rightarrow \dfrac{1}{0} = \dfrac{a}{\sqrt{\theta}} \Rightarrow \sqrt{\theta} = 0$

$\Rightarrow \theta = 0 = \alpha$

Diff (I) w.r.t 'r' $\quad r = a(\theta)^{-\frac{1}{2}}$

$1 = a\left(-\dfrac{1}{2}\right)\dfrac{1}{\theta^{3/2}}\dfrac{d\theta}{dr}$

$1 = -\dfrac{a}{2\theta^{3/2}}\dfrac{d\theta}{dr} \Rightarrow \dfrac{d\theta}{dr} = \dfrac{-2\theta^{3/2}}{a}$

$r^2\dfrac{d\theta}{dr} = \dfrac{a^2}{\theta}\left(\dfrac{-2}{a}\right)\theta^{3/2} = -2a\theta^{\frac{1}{2}}$ ——— (II)

$P = \lim_{\theta \to \alpha} r^2\dfrac{d\theta}{dr} \Rightarrow P = \lim_{\theta \to 0}\left(-2a\theta^{\frac{1}{2}}\right)$ using (II)

$\Rightarrow P = -2a(0)^{\frac{1}{2}} = 0$ ——— (III)

Eq of Asymptote is $p = r\,\mathrm{Sin}(\alpha - \theta)$

$P = 0, \alpha = 0 \qquad 0 = r\,\mathrm{Sin}(0-\theta)$ $\qquad \mathrm{Sin}(-\theta) = -\mathrm{Sin}\,\theta$

$0 = -r\,\mathrm{Sin}\,\theta$

$0 = \mathrm{Sin}\,\theta \Rightarrow \boxed{\theta = 0}$

(15) $r = a\,\mathrm{Cosec}\,\theta + b$ —— (1)

Put $r=\infty$ $\quad \infty = a\,\mathrm{Cosec}\,\theta + b$

$\infty - b = a\,\mathrm{Cosec}\,\theta \Rightarrow \infty = a\,\mathrm{Cosec}\,\theta.$

$\frac{\infty}{a} = \mathrm{Cosec}\,\theta \Rightarrow \infty = \mathrm{Cosec}\,\theta.$

$\infty - a = \infty$
$\infty + a = \infty$
$\frac{\infty}{a} = \infty$

$\frac{1}{0} = \frac{1}{\sin\theta} \Longrightarrow \sin\theta = 0$

$\theta = 0, \pi = \alpha$

Differentiate (1) w.r.t. r.

$1 = a(-\mathrm{Cosec}\,\theta\,\mathrm{Cot}\,\theta)\frac{d\theta}{dr}$

$\frac{1}{-a\,\mathrm{Cosec}\,\theta\,\mathrm{Cot}\,\theta} = \frac{d\theta}{dr}$

$\frac{\sin\theta\,\sin\theta}{-a\cos\theta} = \frac{d\theta}{dr} \Rightarrow \frac{d\theta}{dr} = \frac{-\sin^2\theta}{a\cos\theta}$

$r^2\frac{d\theta}{dr} = (a^2\mathrm{Cosec}^2\theta + b^2 + 2ab\,\mathrm{Cosec}\,\theta)\left(-\frac{\sin^2\theta}{a\cos\theta}\right)$

$= \left(\frac{a^2}{\sin^2\theta} + b^2 + \frac{2ab}{\sin\theta}\right)\left(\frac{-\sin^2\theta}{a\cos\theta}\right)$

$r^2\frac{d\theta}{dr} = \left(\frac{a^2 + b^2\sin^2\theta + 2ab\sin\theta}{\cancel{\sin^2\theta}}\right)\left(\frac{-\cancel{\sin^2\theta}}{a\cos\theta}\right)$ —— (iii)

$P = \lim_{\theta\to\alpha} r^2\frac{d\theta}{dr} = \lim_{\theta\to 0} -\left(\frac{a^2 + b^2\sin^2(0) + 2ab\sin(0)}{a\cos(0)}\right)$

$\therefore P = \frac{-a^2}{a} = \boxed{-a}$

Eq of Asymptote $\quad P_1 = r\sin(\alpha-\theta)$ (iv)

$P_1 = -a \quad \alpha = 0$ $\qquad -a = r\sin(0-\theta)$

$-a = -r\sin(\theta)$

$\boxed{a = r\sin\theta.}$

Now at $\alpha = \pi$

$$P = \lim_{\theta\to\alpha} r^2\frac{d\theta}{dr} \qquad \cos\pi = -1$$

$$= \lim_{\theta\to\pi} -\frac{(a^2+b^2\sin^2(\pi)+2ab\sin(\pi))}{a\cos(\pi)}$$

$\because \sin\pi = 0.$
$\because \cos\pi = -1$

$$P_2 = \frac{-a^2}{-a} = \boxed{a}$$

Eq of Asymptote $\quad P_2 = r\sin(\alpha-\theta)$

$P = a \quad \alpha = \pi$ $\qquad a = r\sin(\pi-\theta)$

$\boxed{a = r\sin\theta}$

Asymptote $\quad a = r\sin\theta.$ (Repeated are written once)

Q16 $\quad r = 2a\sin\theta\tan\theta$ (1)

Put $r=\infty$ $\qquad \infty = 2a\sin\theta\tan\theta.$

$\frac{\infty}{2a} = \sin\theta\tan\theta, \Rightarrow \infty = \sin\theta\tan\theta.$

$\frac{1}{0} = \sin\theta\,\frac{\sin\theta}{\cos\theta}. \Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}. = \alpha$

Diff ① w.r.t. r

$$1 = 2a\left[\cos\theta\tan\theta\frac{d\theta}{dr} + \sin\theta\sec^2\theta\frac{d\theta}{dr}\right]$$

$$= 2a\left[\cos\theta\cdot\frac{\sin\theta}{\cos\theta} + \frac{\sin\theta}{\cos^2\theta}\right]\frac{d\theta}{dr}$$

$$1 = 2a\left[\frac{\sin\theta\cos^2\theta + \sin\theta}{\cos^2\theta}\right]\frac{d\theta}{dr}$$

$$\cos^2\theta = 2a\left[\sin\theta(\cos^2\theta+1)\right]\frac{d\theta}{dr}$$

$$\frac{\cos^2\theta}{2a\sin\theta(\cos^2\theta+1)} = \frac{d\theta}{dr}$$

$$(2a\sin\theta\tan\theta)^2\ \frac{\cos^2\theta}{2a\sin\theta(\cos^2\theta+1)} = r^2\frac{d\theta}{dr}$$ using ①

$$4a^2\sin^2\theta\ \frac{\sin^2\theta}{\cos^2\theta}\ \frac{\cos^2\theta}{2a\sin\theta(\cos^2\theta+1)} = r^2\frac{d\theta}{dr}$$

$$\frac{2a\sin^3\theta}{\cos^2\theta+1} = r^2\frac{d\theta}{dr}$$

$$p = \lim_{\theta\to\alpha} r^2\frac{d\theta}{dr} \quad ②$$

At $\alpha = \pi/2$

$$p = \lim_{\theta\to\frac{\pi}{2}} \frac{2a\sin^3\theta}{\cos^2\theta+1} = \frac{2a\sin^2(\frac{\pi}{2})}{\cos^2(\frac{\pi}{2})+1} = \frac{2a(1)}{0+1}$$

$$\therefore p = 2a$$

7-1-16

Eq of Asymptote $\quad p = r\sin(\alpha-\theta)$ ——— (3)

$p = 2a \quad \alpha = \frac{\pi}{2} \qquad \therefore 2a = r\sin\left(\frac{\pi}{2}-\theta\right)$

$$\boxed{2a = r\cos\theta}$$

$\sin\frac{3\pi}{2} = -1$

$3\cos\frac{3\pi}{2} = 0$

For $\alpha = \frac{3\pi}{2} \quad p = \lim_{\theta\to\frac{3\pi}{2}} \frac{2a\sin^3\theta}{\cos^3\theta+1} = \frac{2a\sin^3\left(\frac{3\pi}{2}\right)}{\cos^3\left(\frac{3\pi}{2}\right)+1} = \frac{2a(-1)}{0+1}$

$\alpha = \frac{3\pi}{2} \quad p = -2a \qquad p = r\sin(\alpha-\theta)$

$-2a = r\sin\left(\frac{3\pi}{2}-\theta\right)$

$-2a = -r\cos\theta \implies \boxed{2a = r\cos\theta}$

Q.17 $\quad r\sin 2\theta = a\cos 3\theta$.

$r = \frac{a\cos 3\theta}{\sin 2\theta}$ ——— (1)

$\frac{1}{0} = \frac{a\cos 3\theta}{\sin 2\theta} \implies \sin 2\theta = 0 \implies 2\theta = 0, \pi, 2\pi, 3\pi, \dots$

$\implies \theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \dots$

$\theta = \begin{cases} 0, 2\pi, 4\pi, \dots, & \pi, 3\pi, 5\pi, \dots \\ \frac{\pi}{2}, \frac{5\pi}{2}, \dots, & \frac{3\pi}{2}, \frac{7\pi}{2}, \dots \end{cases}$

Diff (1)

$$1 = \frac{\sin 2\theta\left[a(-\sin 3\theta)3\frac{d\theta}{dr}\right] - (a\cos 3\theta)(\cos 2\theta)2\frac{d\theta}{dr}}{(\sin 2\theta)^2}$$

$$\frac{d\theta}{dr} = \frac{(\sin 2\theta)^2}{-\left[3a\sin 2\theta\sin 3\theta + 2a\cos 2\theta\cos 3\theta\right]}$$

$$r^2\frac{d\theta}{dr} = \frac{(a\cos 3\theta)^2}{\cancel{(\sin 2\theta)^2}}\left(\frac{\cancel{(\sin 2\theta)^2}}{-(3a\sin 2\theta\sin 3\theta + 2a\cos 2\theta\cos 3\theta)}\right)$$

$$p = \lim_{\theta\to\alpha} r^2\frac{d\theta}{dr} = \lim_{\theta\to 0} \frac{-a^2\cos^2 3\theta}{3a\sin 2\theta\sin 3\theta + 2a\cos 2\theta\cos 3\theta} = \frac{-a^2}{0+2a}$$

$\therefore P = -\frac{a^2}{2a} = -\frac{a}{2}$

Eq of Asymptote $\quad p = r\sin(\alpha-\theta)$

$\alpha = 0,\ P = -\frac{a}{2}$

$\Rightarrow -\frac{a}{2} = r\sin(0-\theta)$

$\Rightarrow -\frac{a}{2} = -r\sin\theta$

$\Rightarrow \boxed{\frac{a}{2} = r\sin\theta}$

when $\alpha = \frac{\pi}{2}$

$$P = \lim_{\theta\to\frac{\pi}{2}} r^2\frac{d\theta}{dr} = \lim_{\theta\to\frac{\pi}{2}}\left[\frac{-a^2\cos^2 3\theta}{a(3\sin3\theta\sin2\theta + 2\cos3\theta\cos2\theta)}\right] \quad \frac{0}{0}\text{ form}$$

$$P = \lim_{\theta\to\frac{\pi}{2}} \frac{-a\cos^2 3\theta}{\frac{3}{-2}\left[-2\sin3\theta\sin2\theta\right] + \left[2\cos3\theta\cos2\theta\right]}$$

$\times$ & $\div$ by -2

$$P = \lim_{\theta\to\frac{\pi}{2}} \frac{-a\cos^2 3\theta}{\frac{3}{-2}\left[(\cos(5\theta) - \cos\theta) + (\cos5\theta + \cos\theta)\right]} \quad \frac{0}{0}\text{ form.}$$

Apply L Hospital Rule (M)

$$P = \lim_{\theta\to\frac{\pi}{2}} \frac{-2a\cos3\theta(-\sin3\theta)3}{\frac{3}{-2}\left[(-\sin5\theta(5) + \sin\theta) + (-\sin5\theta(5) + (-\sin\theta))\right]}$$

$\therefore \sin\frac{5\pi}{2} = \sin\frac{\pi}{2} = 1$

$\cos\frac{3\pi}{2} = 0$

$$P = \frac{0}{\frac{3}{-2}\left[(-5+1) + (-5-1)\right]} = 0$$

Eq of Asymptote $\quad P = r\sin(\alpha-\theta)$

$P = 0,\ \alpha = \frac{\pi}{2}$

$0 = r\sin(\frac{\pi}{2}-\theta)$

$0 = r\cos\theta$

$0 = \cos\theta \quad \Rightarrow \boxed{\theta = \frac{\pi}{2}}$

L. Hospital Rule

$\cos\pi = -1$
$\sin\pi = 0$
$\sin\frac{3\pi}{2} = -$
$\cos\frac{3\pi}{2} = 0$

We get $P = \frac{a}{2}$ by putting $\alpha = \pi$ in (11) So $\frac{a}{2} = r\sin(\pi - \theta)$

$\boxed{\frac{a}{2} = r\sin\theta}$

we get $P = 0$ by putting $\alpha = \frac{3\pi}{2}$ in (11) So $0 = r\sin(\frac{3\pi}{2} - \theta)$

$\Rightarrow 0 = -r\cos\theta$

Hence Required Asymptotes are $\boxed{\theta = \frac{\pi}{2}}$, $\boxed{\frac{a}{2} = r\sin\theta}$ $\Rightarrow$ $\boxed{\theta = \frac{\pi}{2}}$

Q18 $r = \frac{a}{1 - \cos\theta}$ (1)

$\frac{1}{0} = \frac{a}{1 - \cos\theta}$ $\Rightarrow 1 - \cos\theta = 0$.

$\Rightarrow \cos\theta = 1$.

$\theta = 0, 2\pi = \alpha$.

Diff. (1) w.r.t. 'r'

$1 = -a(1-\cos\theta)^{-2}[-(-\sin\theta)]\frac{d\theta}{dr}$

$1 = \frac{-a\sin\theta}{(1-\cos\theta)^2}\frac{d\theta}{dr}$

$-\frac{(1-\cos\theta)^2}{a\sin\theta} = \frac{d\theta}{dr}$

$r^2\frac{d\theta}{dr} = \frac{a^2}{(1-\cos\theta)^2}\left[-\frac{(1-\cos\theta)^2}{a\sin\theta}\right]$

$r^2\frac{d\theta}{dr} = \frac{-a}{\sin\theta}$.

$P = \lim_{\theta\to\alpha} r^2\frac{d\theta}{dr} = \lim_{\theta\to 0}\frac{-a}{\sin\theta}$

7-1-19

$$p = \frac{-a}{\sin(0)} = \frac{-a}{0} = -\infty.$$

Hence there is no asymptote of the curve.

Similarly for $\alpha = 2\pi$ there is no asymptote of the curve.

V. Imp

(19) : $r\sin n\theta = a$ — (1)

$$r = \frac{a}{\sin n\theta}.$$

$$\frac{1}{0} = \frac{a}{\sin n\theta}$$

$\Rightarrow \sin n\theta = 0.$

$$n\theta = 0, \pi, 2\pi, \ldots K\pi$$

$$\theta = 0, \frac{\pi}{n}, \frac{2\pi}{n}, \ldots \frac{K\pi}{n}$$

where $K = 0, 1, 2, 3 \ldots$

Diff (1) w.r.t. r

$$1 = -a(\sin n\theta)^{-2}\cos n\theta (n)\frac{d\theta}{dr}.$$

$$1 = \frac{-a\cos n\theta \, n}{(\sin n\theta)^2}\frac{d\theta}{dr}$$

$$\frac{(\sin n\theta)^2}{-an\cos n\theta} = \frac{d\theta}{dr}.$$

$$r^2\frac{d\theta}{dr} = \frac{a^2}{(\sin n\theta)^2} \cdot \left[-\frac{(\sin n\theta)^2}{an\cos n\theta}\right]$$

$$r^2\frac{d\theta}{dr} = \frac{-a}{n\cos n\theta}.$$

7·1-20

$$P = \lim_{\theta \to \alpha} r^2 \frac{d\theta}{dr}$$

$$= \lim_{\theta \to \frac{K\pi}{n}} \left[\frac{-a}{n\cos n\theta}\right]$$

$$= \frac{-a}{n\cos \cancel{n}\left(\frac{K\pi}{\cancel{n}}\right)}$$

$$P = \frac{-a}{n\cos K\pi}$$

Eq of Asymptote $P = r\sin(\alpha-\theta)$

$\because P = \frac{-a}{n\cos K\pi}$, $\alpha = \frac{K\pi}{n}$

$$\frac{-a}{n\cos K\pi} = r\sin\left(\frac{K\pi}{n}-\theta\right)$$

$$\frac{-a}{n\cos K\pi} = -r\sin\left(\theta-\frac{K\pi}{n}\right)$$

$$\frac{a}{n\cos K\pi} = r\sin\left(\theta-\frac{K\pi}{n}\right)$$

$$\frac{a}{n} \sec\;) = r\sin\left(\theta-\frac{K\pi}{n}\right)$$

where $K = 0,1,2,3---$

Q.20 $r(e^{\theta}-1) = a(e^{\theta}+1)$

$$r = \frac{a(e^{\theta}+1)}{(e^{\theta}-1)} \quad \text{——— (1)}$$

$$\frac{1}{0} = \frac{a(e^{\theta}+1)}{(e^{\theta}-1)} \qquad \Rightarrow e^{\theta}-1=0.$$

$$\Rightarrow e^{\theta}=1$$

$$\Rightarrow e^{\theta}=e^{0}$$

$$\Rightarrow \theta = 0 = \alpha$$

Diff (1) w.r.t. 'r'

$$1 = \left(\frac{(e^{\theta}-1)a(e^{\theta}) - a(e^{\theta}+1)(e^{\theta})}{(e^{\theta}-1)^2}\right)\frac{d\theta}{dr}.$$

$$1 = \left(\frac{\cancel{e^{\theta}ae^{\theta}} - ae^{\theta} - ae^{\theta} - \cancel{ae^{\theta}e^{\theta}}}{(e^{\theta}-1)^2}\right)\frac{d\theta}{dr}$$

$$1 = \left(\frac{-2ae^{\theta}}{(e^{\theta}-1)^2}\right)\frac{d\theta}{dr}$$

$$-\frac{(e^{\theta}-1)^2}{2ae^{\theta}} = \frac{d\theta}{dr}.$$

$$\frac{a^2(e^{\theta}+1)^2}{\cancel{(e^{\theta}-1)^2}}\left(-\frac{\cancel{(e^{\theta}-1)^2}}{2ae^{\theta}}\right) = r^2\frac{d\theta}{dr} \qquad \text{using (1)}$$

$$-\frac{a}{2e^{\theta}}(e^{\theta}+1)^2 = r^2\frac{d\theta}{dr}.$$

$$p = \lim_{\theta \to \alpha} r^2 \frac{d\theta}{dr} = \lim_{\theta \to 0} \frac{-a(e^{\theta}+1)^2}{2e^{\theta}}$$

$$= \frac{-a(e^{0}+1)^2}{2e^{0}} = -\frac{a(1+1)^2}{2} = -\frac{4a}{2}$$

$$p = -2a$$

Eq of Asymptote $\quad p = r\sin(\alpha-\theta)$

$p = -2a \quad \alpha = 0 \qquad -2a = r\sin(0-\theta)$

$$2a = r\sin(-\theta)$$

$$\boxed{2a = r\sin\theta}$$

Imp

(21) $r^n \sin n\theta = a^n$

$$r^n = \frac{a^n}{\sin n\theta} \quad \text{——— (1)} \qquad \infty^n = \infty$$

$$\infty^n = \frac{a^n}{\sin n\theta} \Rightarrow \frac{1}{0} = \frac{a^n}{\sin n\theta}$$

$$\Rightarrow \sin n\theta = 0 \Rightarrow n\theta = 0, \pi, 2\pi, \ldots\ldots k\pi$$

$$\theta = 0, \frac{\pi}{n}, \frac{2\pi}{n} \ldots \frac{k\pi}{n}$$

where $K = 0, 1, 2, 3 \ldots$

Diff (1) w.r.t 'r'

$$n r^{n-1} = -a^n (\sin n\theta)^{-2} \cos n\theta \, (n) \frac{d\theta}{dr}$$

$$\cancel{n} r^{n-1} = -\frac{\cancel{n} a^n \cos n\theta}{(\sin n\theta)^2} \frac{d\theta}{dr}$$

$$-\frac{r^{n-1}(\sin n\theta)^2}{a^n \cos n\theta} = \frac{d\theta}{dr}$$

$r^n = \dfrac{a^n}{\sin n\theta}$

$r = \dfrac{a}{(\sin n\theta)^{\frac{1}{n}}}$

$$\frac{r^2 d\theta}{dr} = r^2\left[\frac{-r^{n-1}(\sin n\theta)^2}{a^n \cos n\theta}\right]$$

$$= -\frac{r^{n+1}(\sin n\theta)^2}{a^n \cos n\theta}$$

$$= -\frac{r^n \cdot r(\sin n\theta)^2}{a^n \cos n\theta}$$

$$= -\frac{\cancel{a^n}}{\sin n\theta} \cdot \frac{r(\sin n\theta)^2}{\cancel{a^n}\cos n\theta} = -\frac{r(\sin n\theta)}{\cos n\theta}$$

$$= -\frac{a}{(\sin n\theta)^{\frac{1}{n}}}\,\frac{\sin n\theta}{\cos n\theta} = -\frac{a(\sin n\theta)^{1-\frac{1}{n}}}{\cos n\theta}$$

$$p = \lim_{\theta \to \alpha} \frac{r^2 d\theta}{dr} = \lim_{\theta \to \frac{K\pi}{n}}\left[-a\frac{(\sin n\theta)^{1-\frac{1}{n}}}{\cos n\theta}\right]$$

$$p = -a\frac{\left[\sin \cancel{n}\left(\frac{K\pi}{\cancel{n}}\right)\right]^{1-\frac{1}{n}}}{\cos \cancel{n}\left(\frac{K\pi}{\cancel{n}}\right)} = -\frac{a(0)}{\pm 1} = 0$$

$p = 0$

Eq of Asymptote

$p = 0 \quad \alpha = \dfrac{K\pi}{n}$

$p = r\sin(\alpha - \theta)$

$0 = r\sin\left(\dfrac{K\pi}{n} - \theta\right)$

$0 = \sin\left(\dfrac{K\pi}{n} - \theta\right)$

$0 = \dfrac{K\pi}{n} - \theta \Rightarrow \boxed{\theta = \dfrac{K\pi}{n}}$

where $K = 0, 1, 2, \ldots$

Note $p = 0, \alpha = 0 \Rightarrow 0 = r\sin(-\theta)$
$\Rightarrow 0 = \sin\theta$
$\boxed{\theta = 0}$

which is contained in $\theta = \dfrac{K\pi}{n}$ where $K = 0, 1, 2, 3, \ldots$

$\phi_1(m) = 0$

$\phi_1'(\frac{1}{2}) = \ldots \neq 0$ at $m = \frac{1}{2}$

7.1-24

Q22 $r^2 \sin\theta = a^2 \cos 2\theta$ ——— (1)

$$r^2 = \frac{a^2 \cos 2\theta}{\sin\theta}$$

$\infty^2 = \infty$

$$\frac{1}{0} = \frac{a^2 \cos 2\theta}{\sin\theta} \Rightarrow \sin\theta = 0$$

$\theta = 0, \pi = \alpha$

Diff (1) w.r.t. r

$$2r = \left[\frac{\sin\theta\,(a^2(-\sin 2\theta)2) - a^2\cos 2\theta \cos\theta}{(\sin\theta)^2}\right]\frac{d\theta}{dr}$$

$$2r = \frac{-2a^2 \sin\theta \sin 2\theta - a^2 \cos 2\theta \cos\theta}{(\sin\theta)^2}\frac{d\theta}{dr}$$

$$\frac{2r(\sin\theta)^2}{-(2a^2\sin\theta\sin 2\theta + a^2\cos 2\theta\cos\theta)} = \frac{d\theta}{dr}$$

$$\frac{a^2\cos 2\theta}{\sin\theta}\left(\frac{-2r(\sin\theta)^2}{\cancel{a^2}(2\sin\theta\sin 2\theta + \cos 2\theta\cos\theta)}\right) = r^2\frac{d\theta}{dr}$$

$$\frac{-2\cos 2\theta \sin\theta}{(2\sin\theta\sin 2\theta + \cos 2\theta\cos\theta)}\left(\frac{a^2\cos 2\theta}{\sin\theta}\right)^{\frac{1}{2}} = r^2\frac{d\theta}{dr}$$

$P = \lim\limits_{\theta\to\alpha}$ $r^2\frac{d\theta}{dr} = \dfrac{-2\cos 2(0)(\sin(0))^{\frac{1}{2}}}{2\sin(0)\sin 2(0) + \cos 2(0)\cos(0)}(a^2\cos 2(0))^{\frac{1}{2}}$

$P = 0$

Eq. of Asymptote

$P = 0 \quad \alpha = 0$

Note At $\alpha = \pi$ also $P = 0$

$\theta = 0$ is Asymptote

$0 = r\sin(\theta - 0)$

$0 = -r\sin\theta$

$0 = r\sin\theta$

$0 = \sin\theta$

$\boxed{\theta = 0}$ Required Asymptote