Exercise Set 5.3

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Determine whether the following improper integrals converge.

Evaluate the integrals that converge (Problems 1 – 33):

Notes of Chapter 05
Calculus with Analytic Geometry
by Ilmi Kitab Khana, Lahore.

$$\int_{0}^{\infty} e^{-x} dx$$

$$\int_{0}^{0} \frac{dx}{1+x^2}$$

$$\int_{-\infty}^{0} \frac{dx}{(2x-1)^3}$$

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$

$$9. \qquad \int_{0}^{\infty} x^3 \ dx$$

11.
$$\int_{0}^{1} \frac{dx}{x}$$

$$13. \qquad \int\limits_0^{\infty} \frac{dx}{(x-1)^2}$$

15.
$$\int_{0}^{x} x^{2} \ln x \ dx$$

$$17. \qquad \int_{-2}^{2} \frac{dx}{x}$$

19.
$$\int_{0}^{\pi/2} \frac{\cos x}{\sqrt{1-\sin x}} dx$$

21.
$$\int_{0}^{\infty} \frac{\ln{(1+x^2)}}{1+x^2} dx$$

$$2. \int_{0}^{\infty} e^{-x} \sin x \ dx$$

 $4. \qquad \int e^{-2x} \cos 2x \ dx$

$$8. \int_{-\infty}^{\infty} \frac{x \, dx}{\sqrt{x^2 + 2}}$$

10.
$$\int_{-\infty}^{\infty} \frac{x}{(x^4+1)} dx$$

$$12. \int_0^{\pi} \frac{dx}{x\sqrt{a^2-x^2}}$$

$$14. \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$16. \int \frac{dx}{x^{1/3}}$$

18.
$$\int_{0}^{\infty} \frac{dx}{x^2 + 2x - 3}$$

20.
$$\int_{0}^{\infty} \frac{x}{x^2 - 5x + 6} \ dx$$

22.
$$\int_{0}^{\infty} \frac{x \ dx}{(1+x)(1+x^2)}$$

$$23. \qquad \int_{-\infty}^{0} \frac{e^{x}}{1+e^{x}} \ dx$$

24.
$$\int_{0}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$$

$$25. \qquad \int_{\pi/4}^{\pi/2} \frac{\sin x}{\sqrt{\cos x}} \ dx$$

$$26. \int_{-\infty}^{\infty} xe^{-x^2} dx$$

27.
$$\int_{-\infty}^{\infty} \frac{dx}{x^2 + 6x + 12}$$
 28.
$$\int_{-1}^{1} \frac{dx}{x^2}$$

$$28. \int_{-1}^{1} \frac{dx}{x^2}$$

$$29. \qquad \int\limits_{2}^{\infty} \frac{dx}{x (\ln x)^3}$$

$$30. \int_{0}^{\infty} xe^{-x} dx$$

31.
$$\int_{1}^{\infty} \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$$
 32.
$$\int_{0}^{1} \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

32.
$$\int_{0}^{1} \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$33. \qquad \int\limits_0^\infty \frac{x^3}{x^3+1} dx$$

Let
$$I_n = \int_0^\infty x^n e^{-x} dx$$
, where n is a positive integer. Prove that $I_n = nI_{n-1}$.

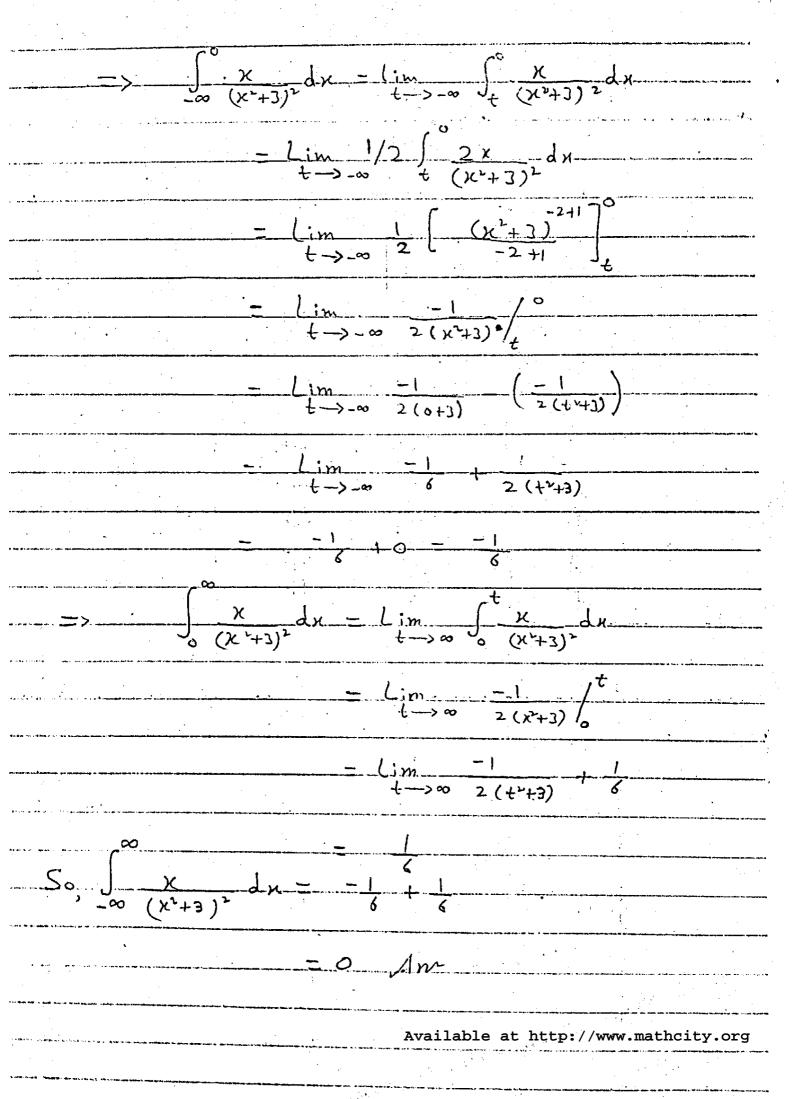
Hence show that $I_n = n!$.

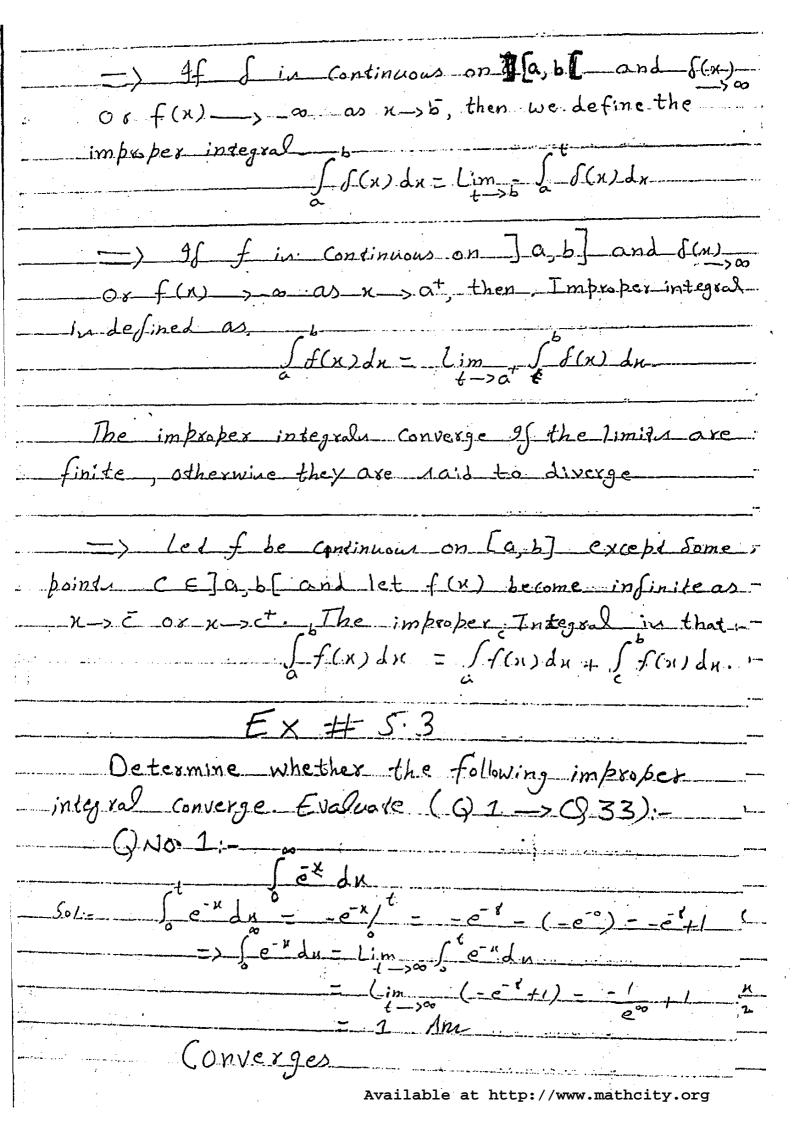
Evaluate
$$\int_{1}^{5} [x] dx$$
, where [x] denotes the greatest integer less than or equal to x.

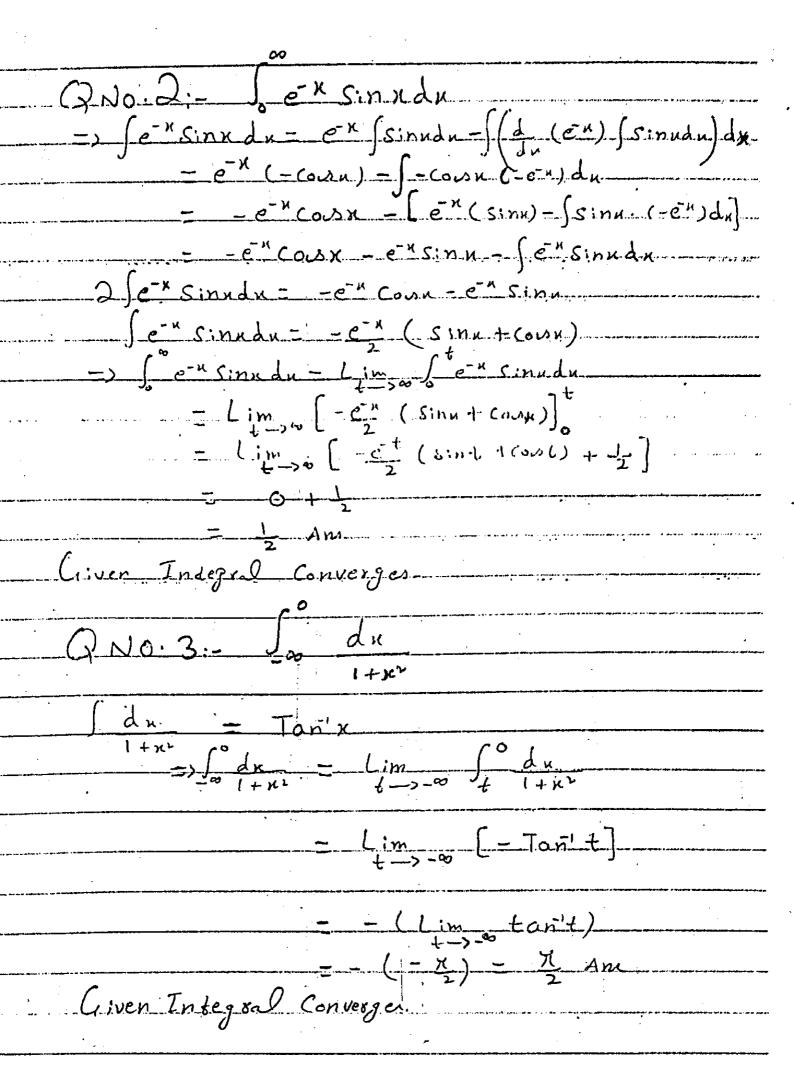
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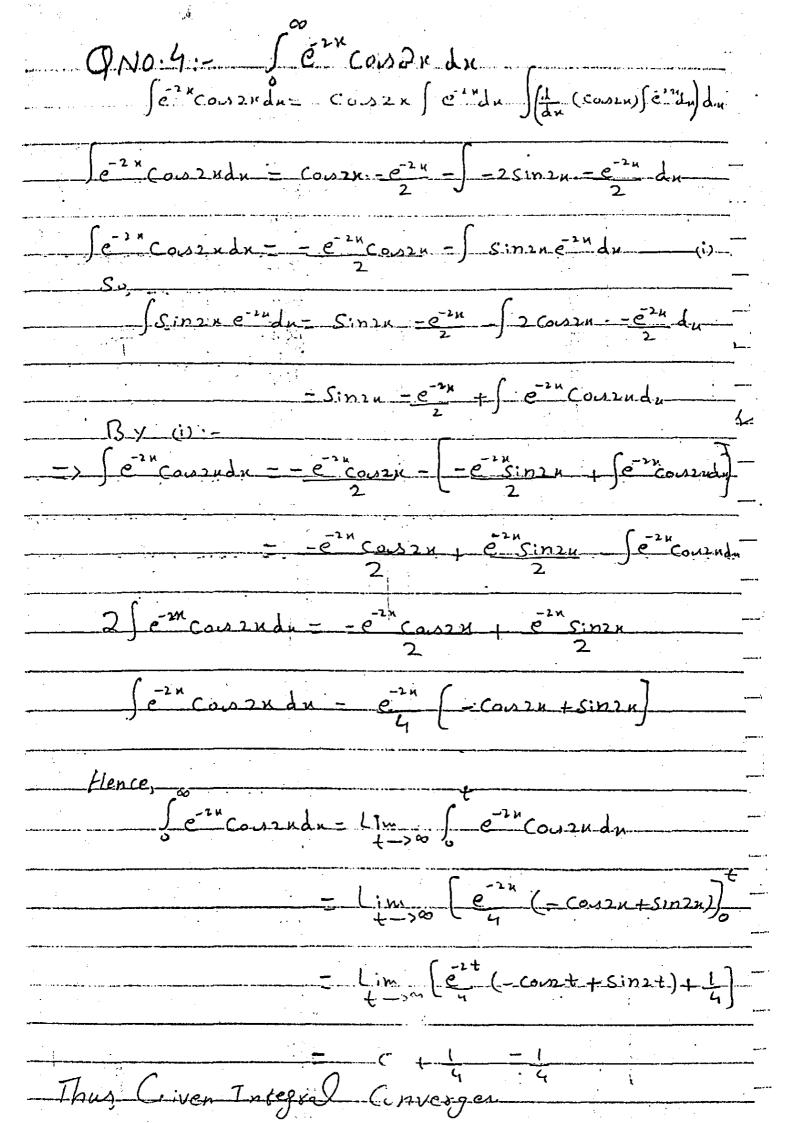
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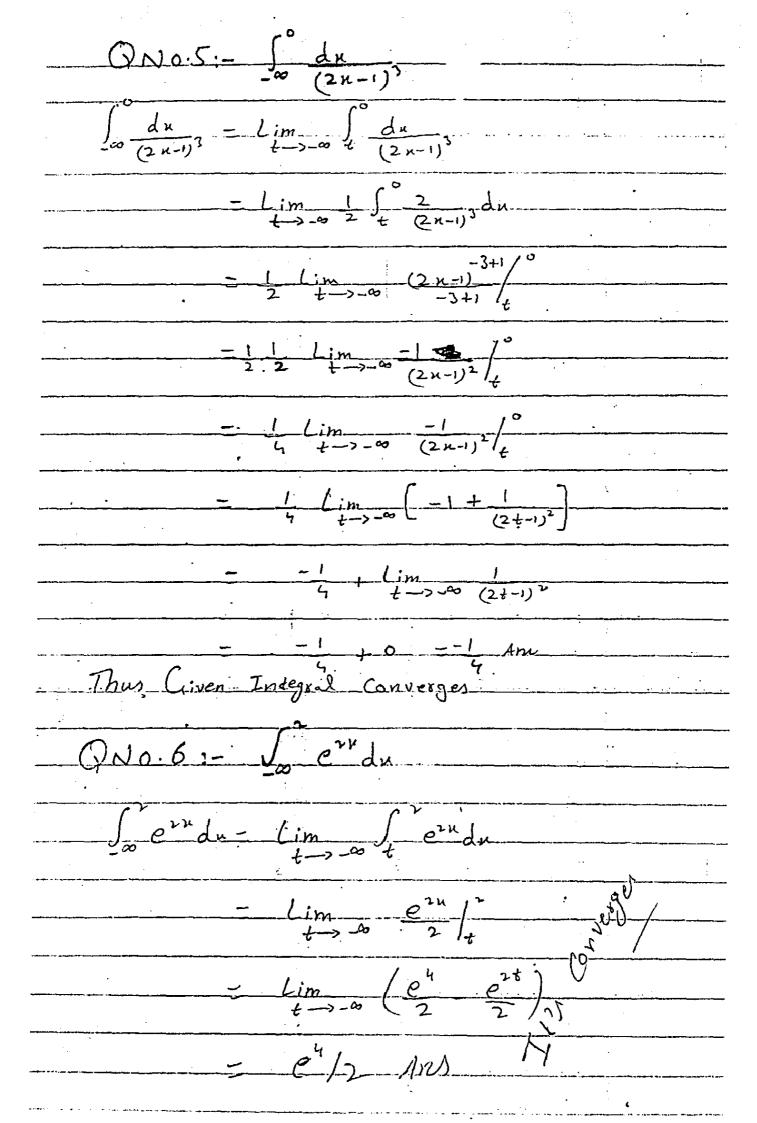
Improper Integrals:
1) :- Integrals which have infinite Intervals
of Integration.
ii) 1- Integrals in which the integrands become
Infinite within the intervaly of integration.
These are called Improper Integrals
Definition:
Let $\int be$ continuous on $[a, \infty]$. The improper Integral $\int_{a}^{\infty} f(x) dx$ in defined as the limit
Integral of f(n) dx in defined as the limit
$\int_{\infty}^{\infty} f(x) dx - \lim_{t \to \infty} \int_{\infty}^{t} f(x) dx$ $= \int_{\infty}^{\infty} \int_{\infty}^{\infty} f(x) dx - \lim_{t \to \infty} \int_{\infty}^{\infty} f(x) dx$
In this case, the Integral of (x) dx is said to
In this case the Integral I T(x) dx soid to
Converge If the limit does not exist (Infinite),
the Integral in said to diverge
$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^{\infty} f(x) dx$
The improper indegral of (x) dx in defined as \[\int_{\infty} S(x) dx - \int_{\infty} S(x) dx + \int_{\infty} S(x) dx \\ \(i) \]
- John Children (i)
$T_{\alpha} = T_{\alpha}(\alpha + \beta) = \begin{cases} f(x) + f(x) \\ f(x) + f(x) \end{cases}$
The Integral So & (x) dx in said to converge only
When both the integrals on the right Side of entire Converge. otherwise it is said to diverge.
converge synerwise in said to diverge
Fox Example: - Ca Ocado / X 14
For Example: - Calculate & X dx
Solin $\int_{-\infty}^{\infty} \frac{\chi}{(x^2+3)^2} dx = \int_{-\infty}^{\infty} \frac{\chi}{(x^2+3)^2} dx + \int_{-\infty}^{\infty} \frac{\chi}{(x^2+3$
$-\infty$ $(x+3)^{2}$ $-\infty$ $(x+3)^{2}$ $0/x+3/2$

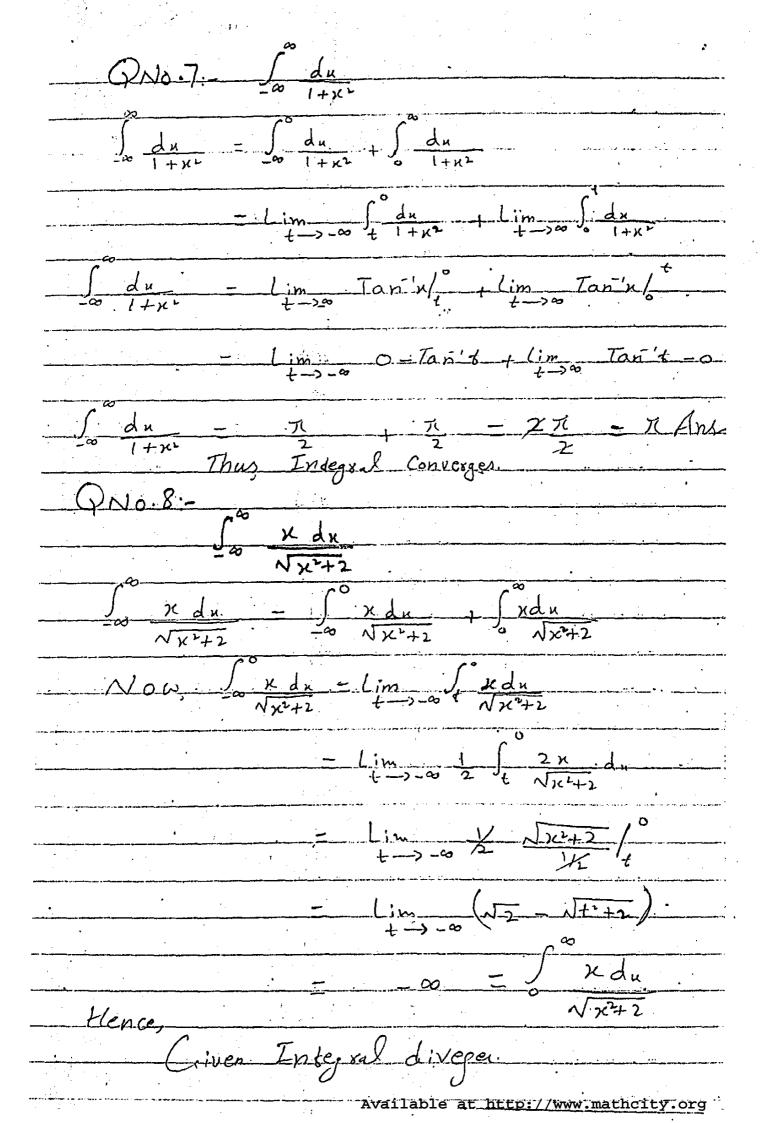


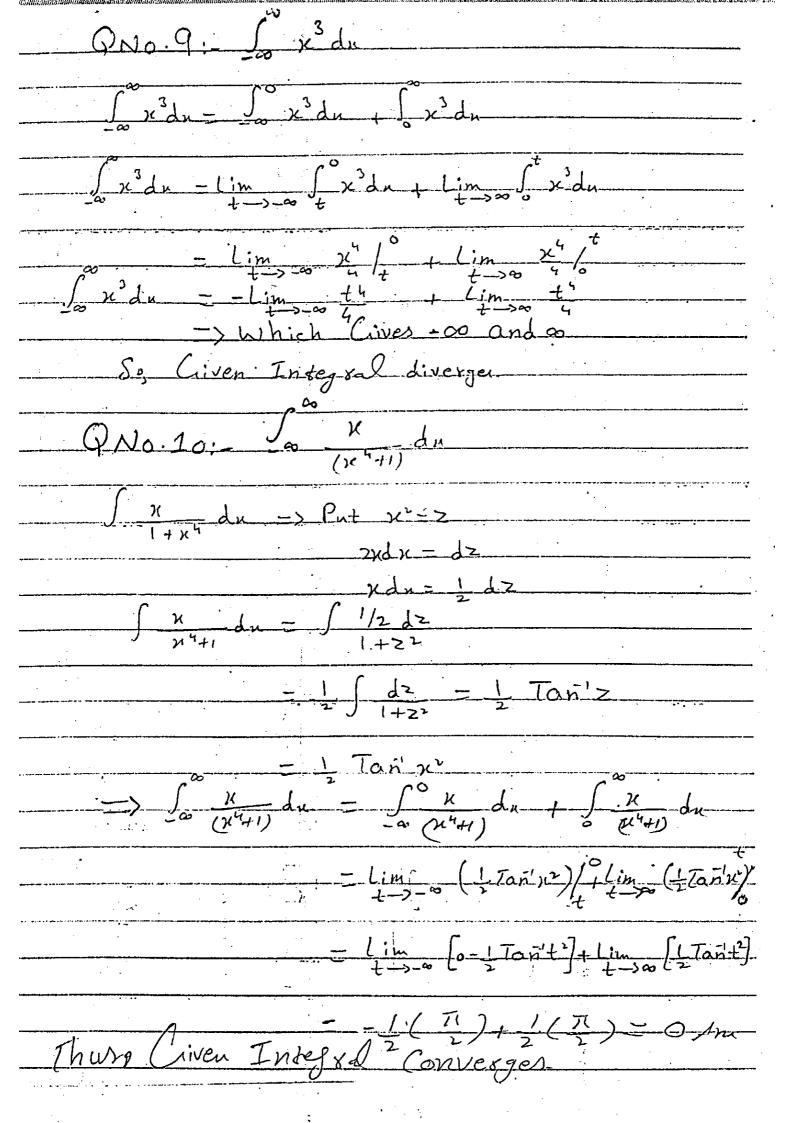


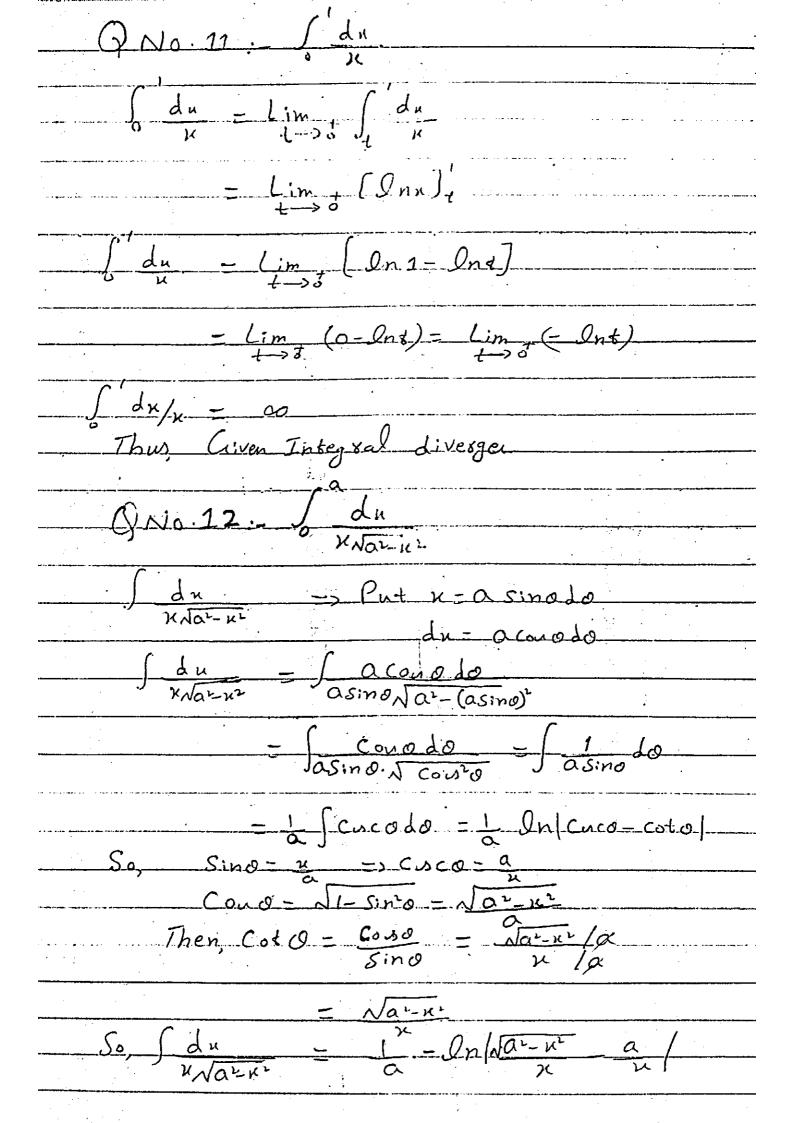


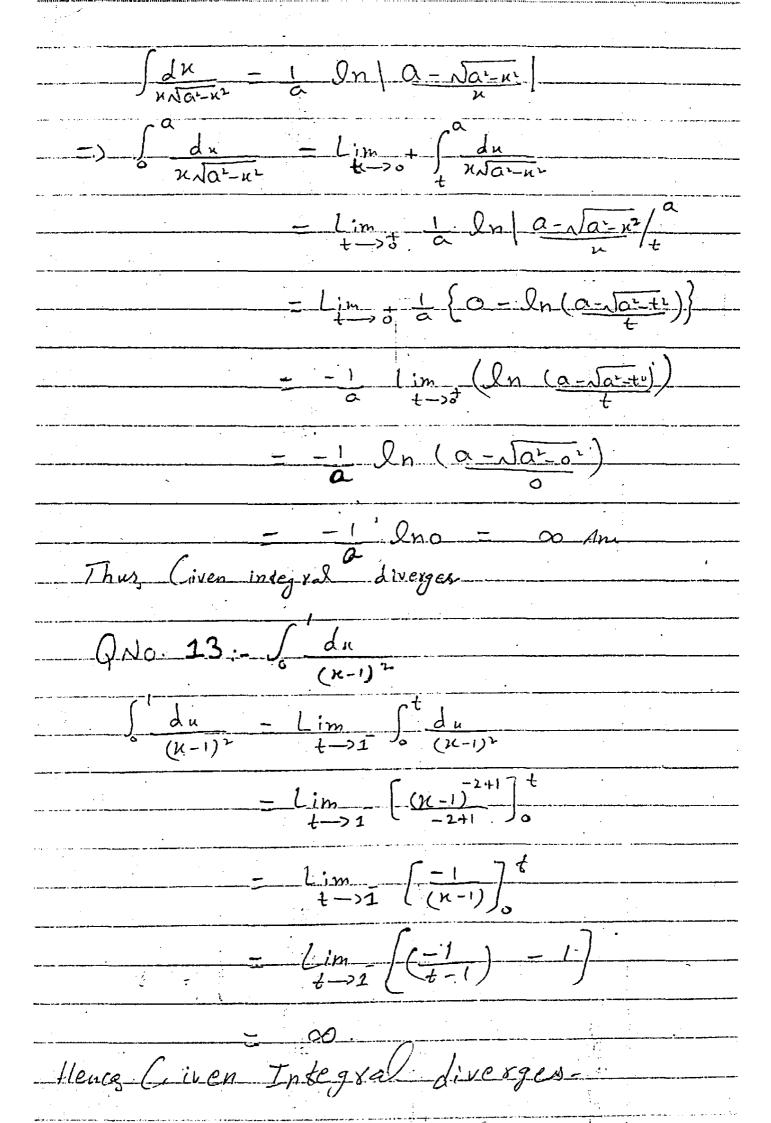




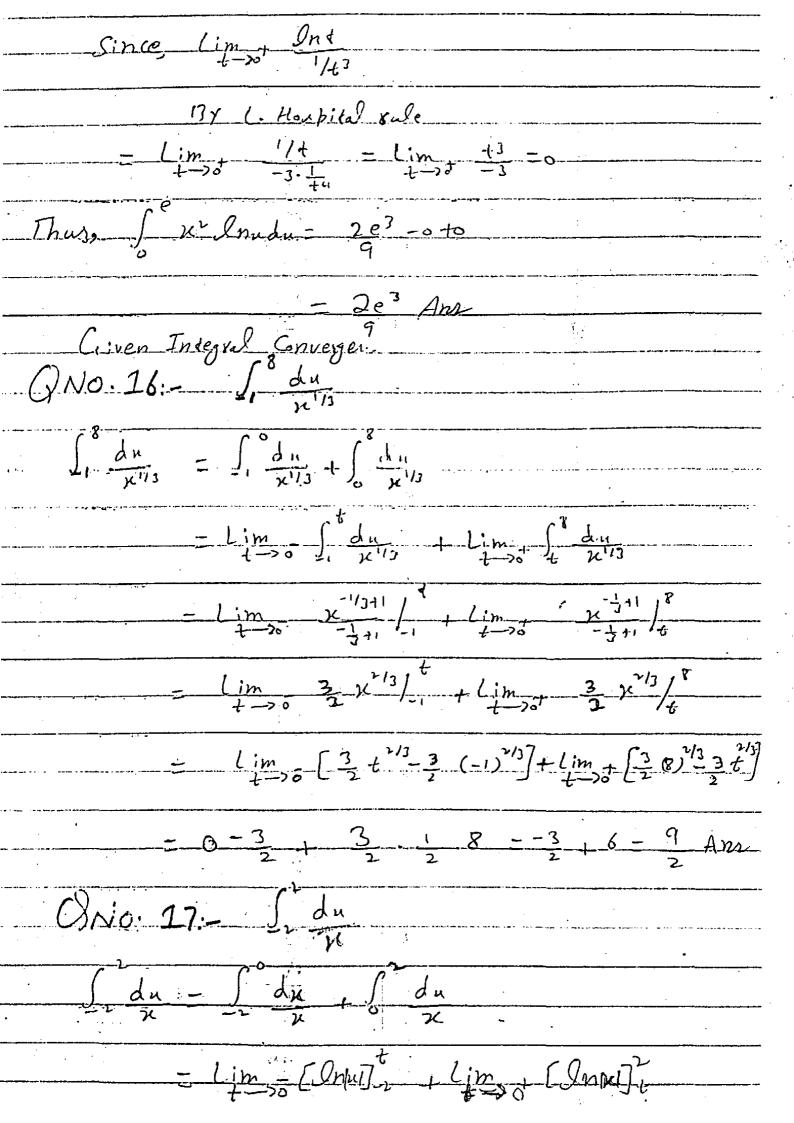


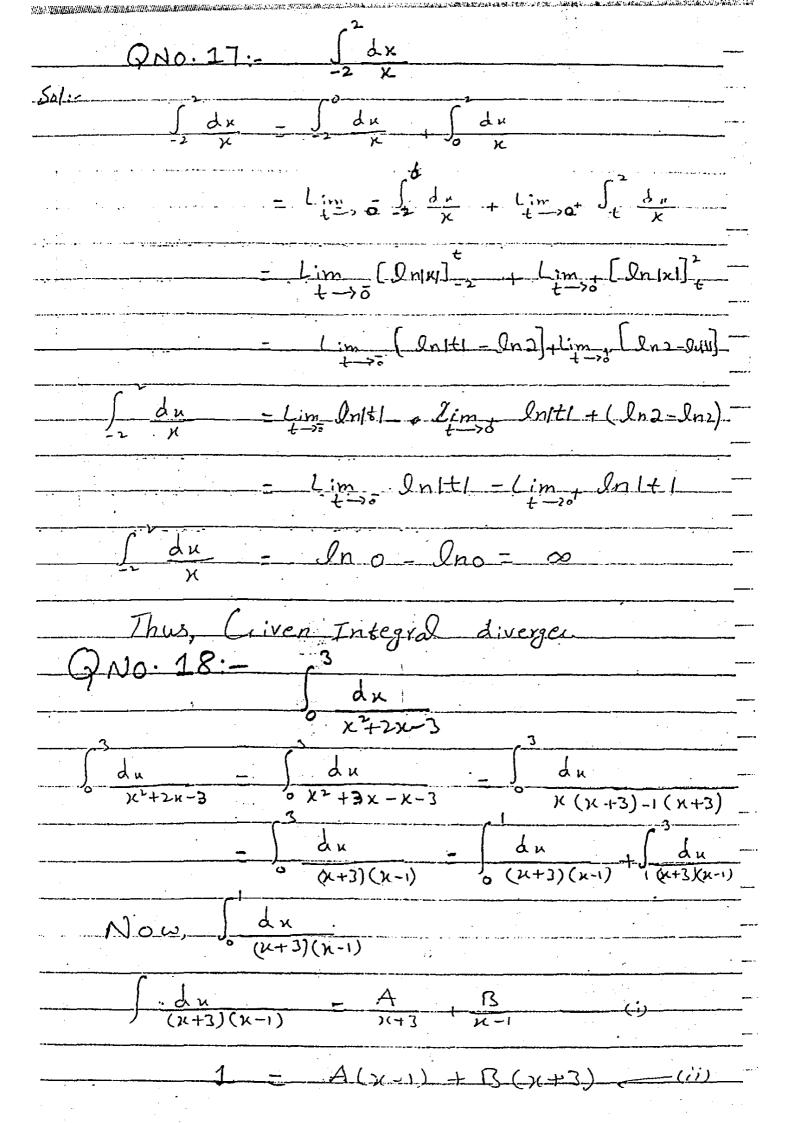






QNO. 24: 5 dn	Written	by Abrar	Mustafa
		·	.,
$\int_{0}^{1} \frac{du}{\sqrt{1-x^{2}}} - \lim_{t \to 1} \int_{0}^{t} \frac{du}{\sqrt{1-x^{2}}}$		د د مین در این به این این این این در	
- <u>Lim</u> [Sin'x] _o			
- Lim Sirit =	- Sin'o		
$\int du - Sin'1 - 0 - \frac{1}{2}$	An	,	
Thus, Criven integral			
QNO.15:- e Sx Inndu			· · · · · · · · · · · · · · · · · · ·
>		· · · · · · · · · · · · · · · · · · ·	
$\int (\Omega nx) x^2 dx = \Omega nx \cdot \frac{x^3}{3}$	$\frac{1}{3}$	du	
- n ³ Ink -	= 1 5 x dn		
- x³ Onx	$-\frac{1}{2}\frac{\chi^3}{3}$	&	
=> \(\sigma^2 \ln\nd\n = \lim_{t->0}^{\text{e}} \\ \frac{\tannant \lambda}{\tannant \lambda}	e xv Inu.	Lu	
- Lim	[x3 ln	$\left[\frac{1}{9}\right]^{\frac{2}{3}}$	
- <u>Lim</u> t->3	$\left[\left(\frac{e^{3}}{J}\ln B\right)\right]$	$\frac{e^{7}}{9}$ $-\left(\frac{t^{7}}{3}\right)$	$Ont-\frac{6^3}{4}$
Sulnudu = limot [e3 Ina -e	$\frac{3}{3} - \frac{13}{3}$ On $\frac{3}{3}$	4 + + 3 }
- 2e/q-L	im + (t)	Int) + 0	
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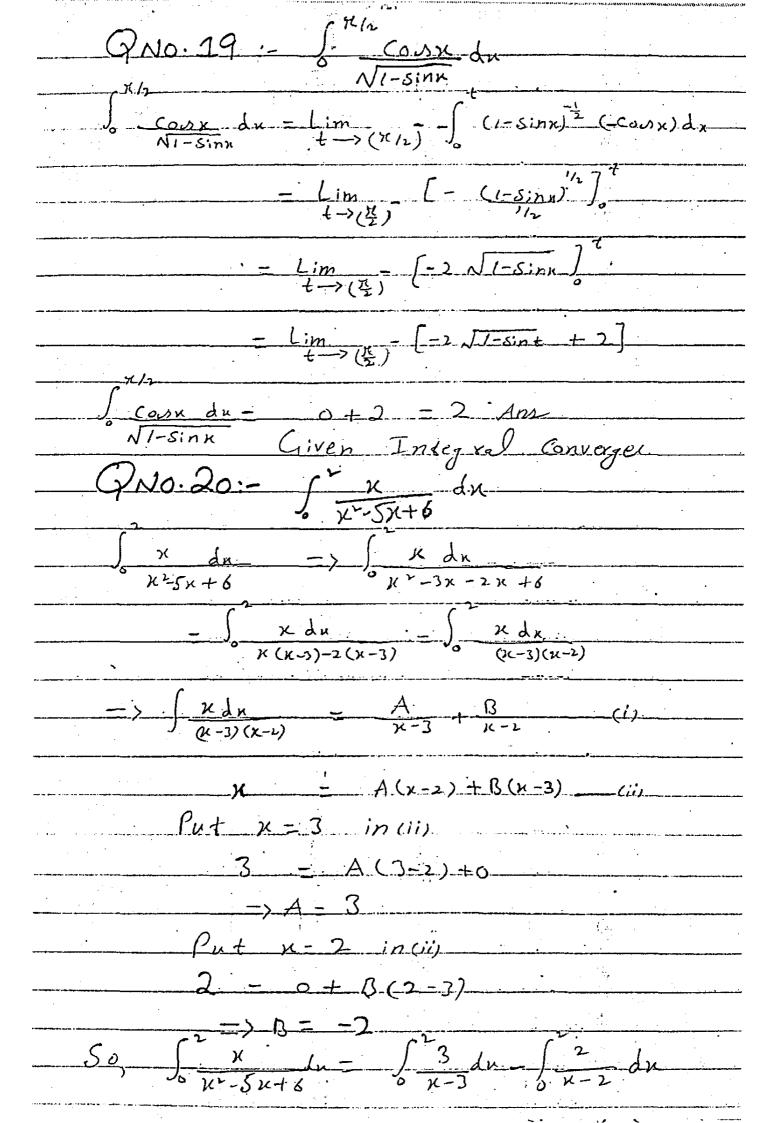
	Written by Ab	rar Mustafa
Put x=-3 in (i)		
A = A(-3-1) + 0	en e	
=) A 1		
Put x=1 in (11)		
1 = 0 + B(1+3)		•
tlence, 4		
du -lim	ſ (- , <u> </u>	7 1
$\int du - \lim_{t \to 1} \frac{1}{t}$	4 (x+3)	4 (x-1) d/)
- Lim +->1	S dic -1	im f du t->T 0 4(n+3)
- Lim t>T	(-in-ln-11)-Li	m (- lu (x+)
- Lim - (1/4	In (1-+1)-Lim	- (- In/++3)- - Ln3)
= Lim = (1 d		In4-In3)
and the second of the second		

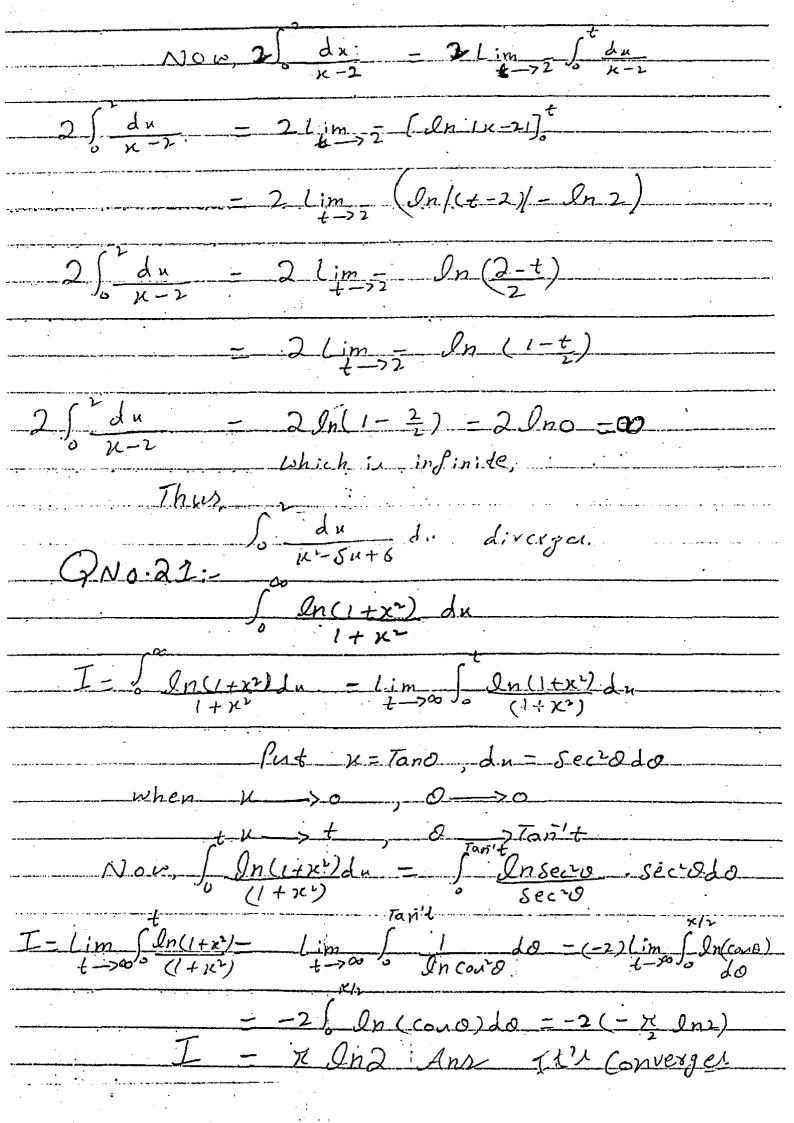
= lim - (-| ln11-t1) - | ln 4

= 1/4 In 0 = 1/4 In 4/3 = 00 which word finite

Hence, 3

du diverger





QNO.22:- 5 x dx (1+x)(1+x2) $I = \int_{0}^{\infty} \frac{\chi dx}{(1+\kappa)(1+\kappa^{2})} = \lim_{t \to \infty} \int_{0}^{\infty} \frac{\chi dx}{(1+\kappa)(1+\kappa^{2})}$ Put x= Tano, dn = Secrodo when 1->0, 0->0 + > 1-> 1-, 10-> Tan't $\sqrt{O \omega}$, $\int \frac{\chi \, dn}{(1+\chi)(1+\chi^2)} = \int \frac{Tano}{o} \frac{Seco.do}{(1+Tano)Seco.}$ $\frac{Tanit}{\sqrt{Tanit}}$ Tanit

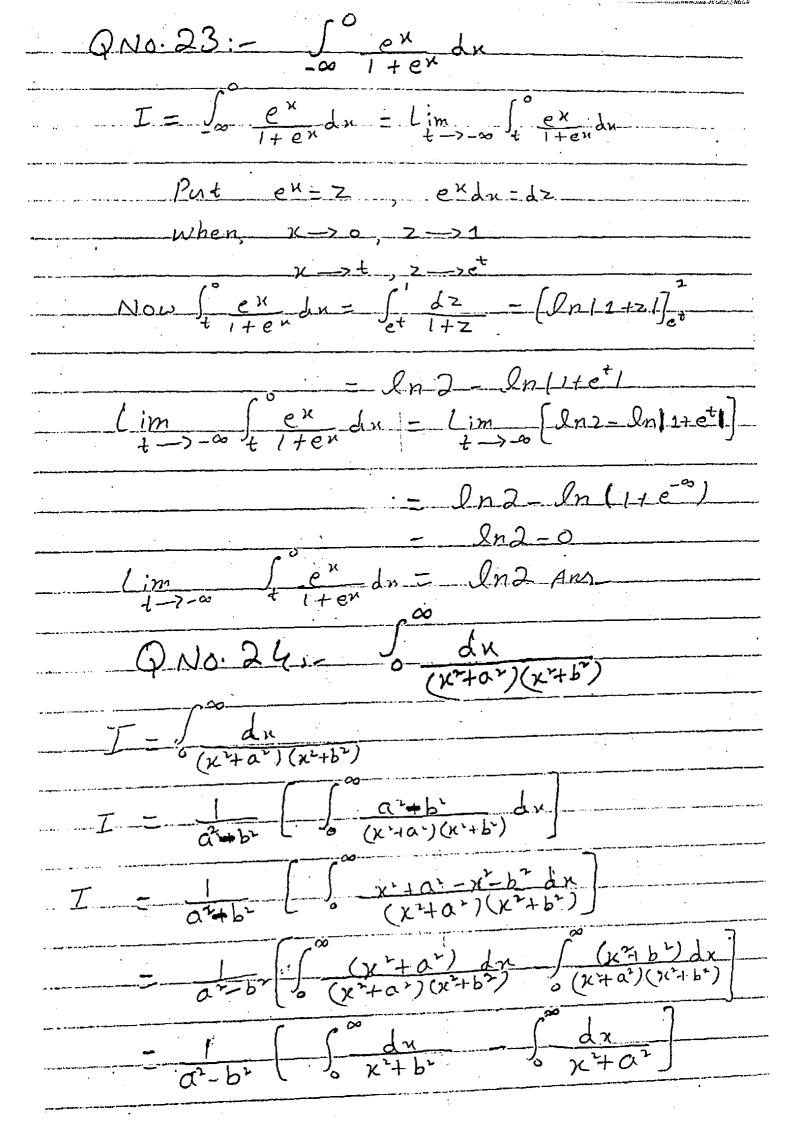
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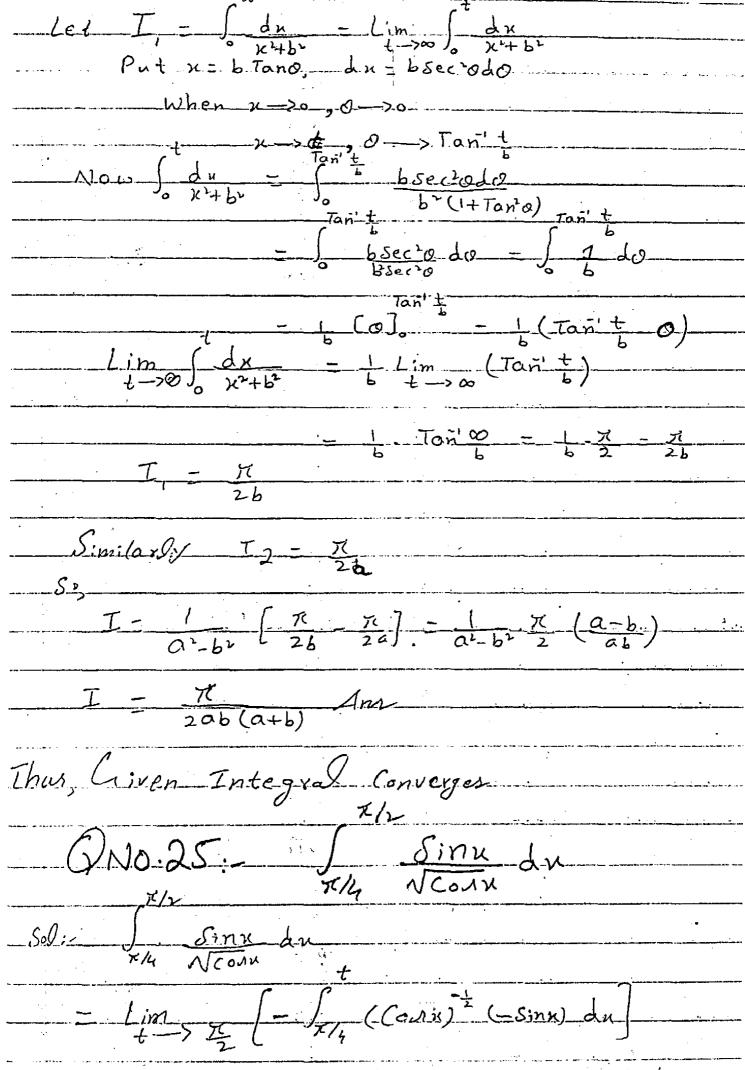
Tanit

Sino/cond

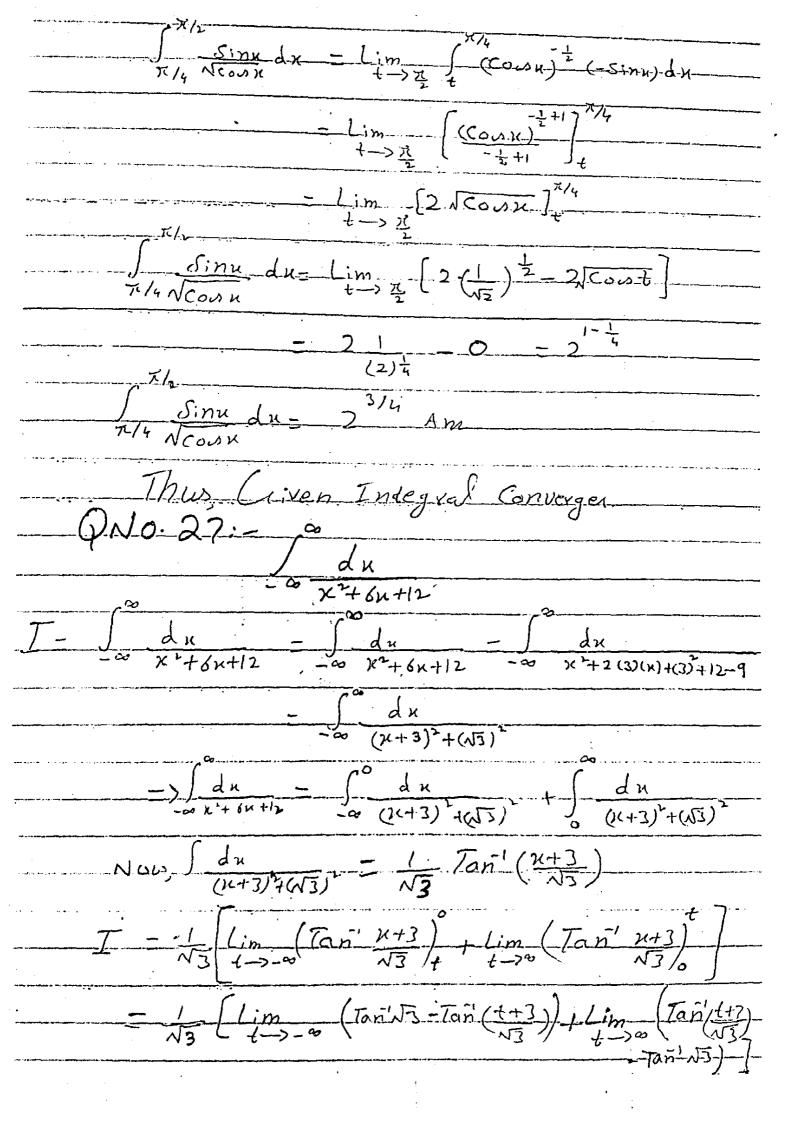
Cond+Sino Sinotional do lim Sino do 1 -> 0 Sino+ (ond $\frac{Sino}{Sino+Coso} = \frac{Sin(\frac{7i}{2}-0)}{Sin(\frac{7i}{2}-0)+Coso} = \frac{Sin(\frac{7i}{2}-0)}{Sin(\frac{7i}{2}-0)}$ Sino + Cono Sino + Cono do 5×12 -1 do = [0]. - 7/4 Ans Thus Civen Integral converges.

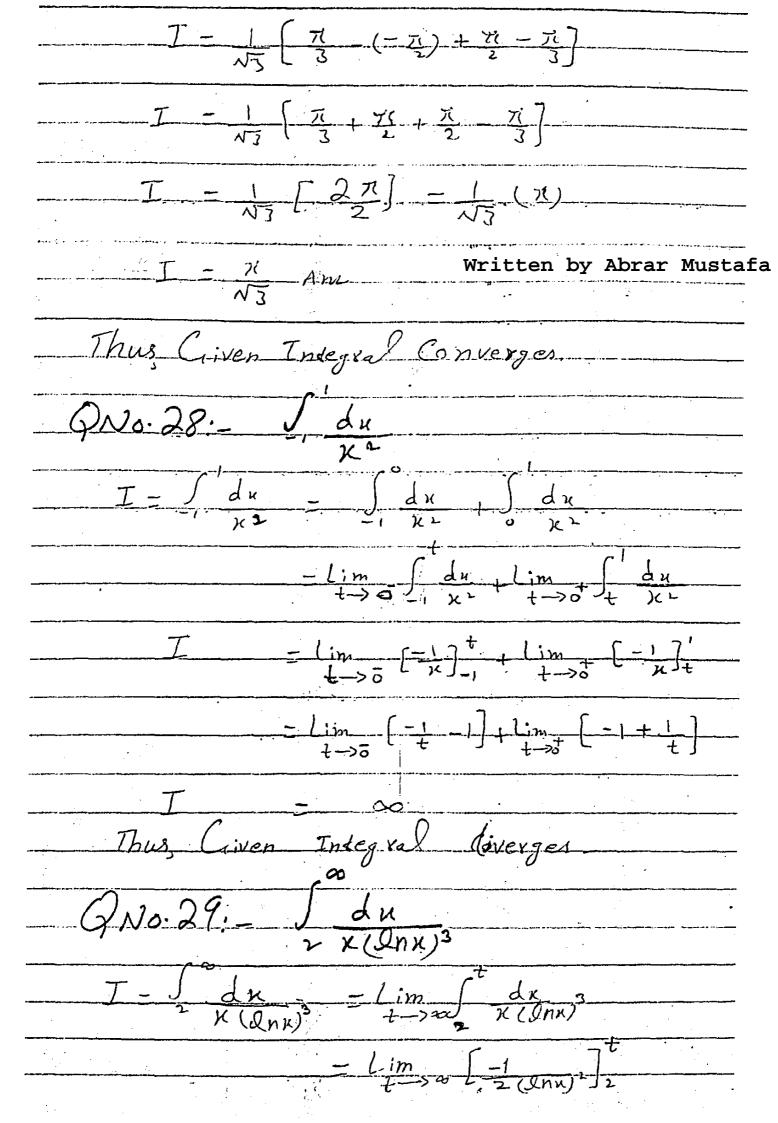
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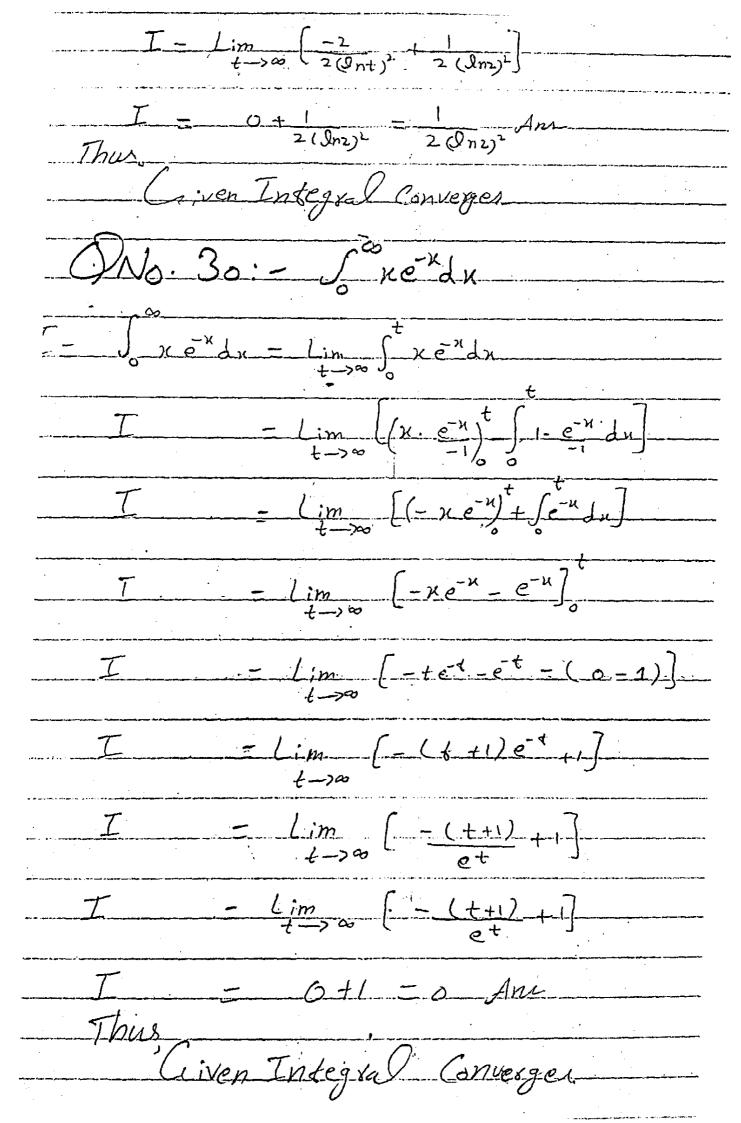


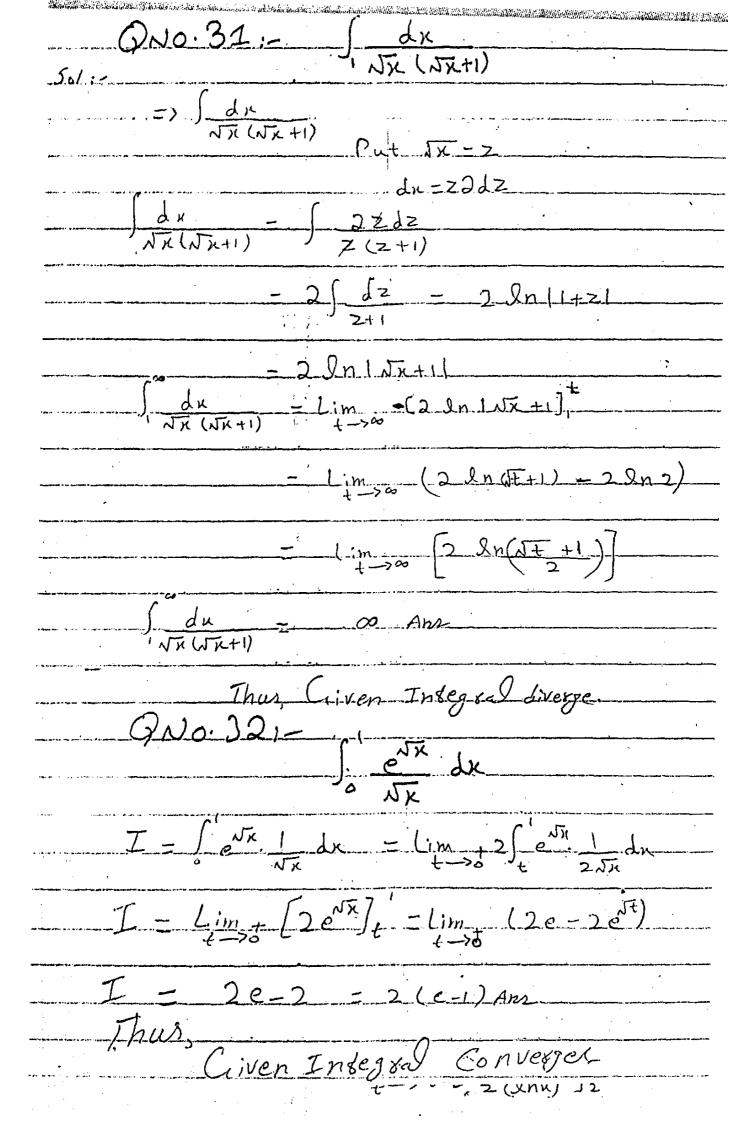


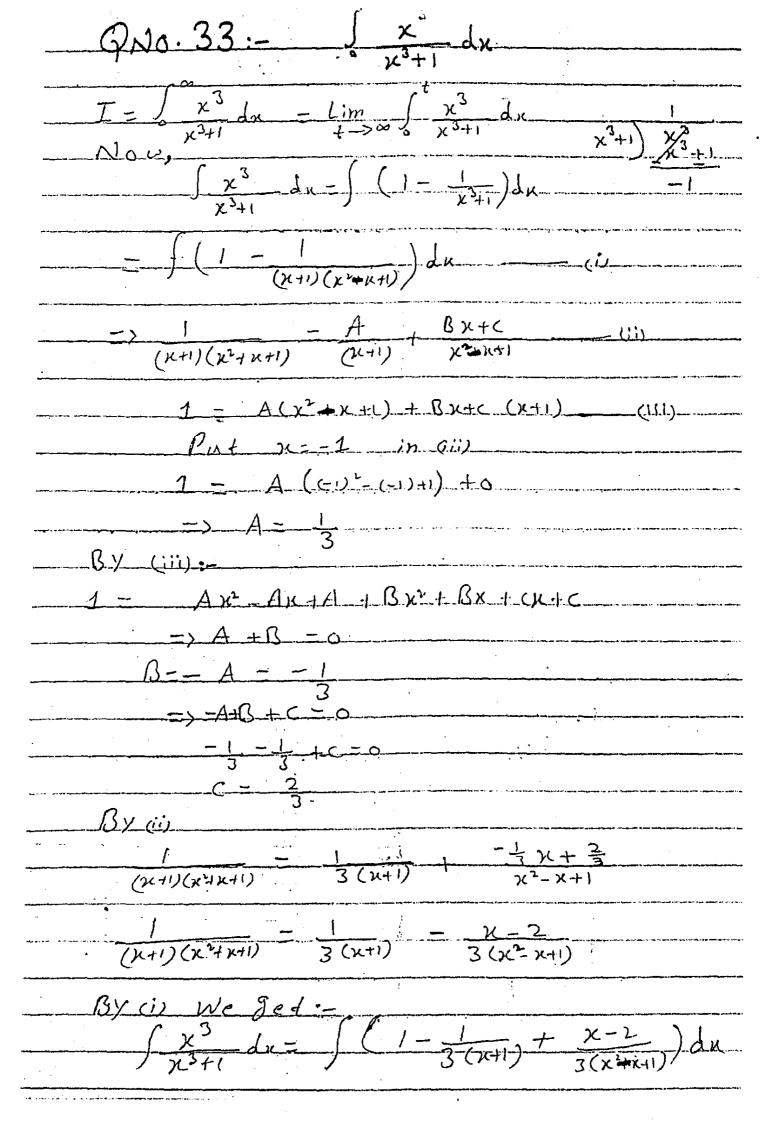
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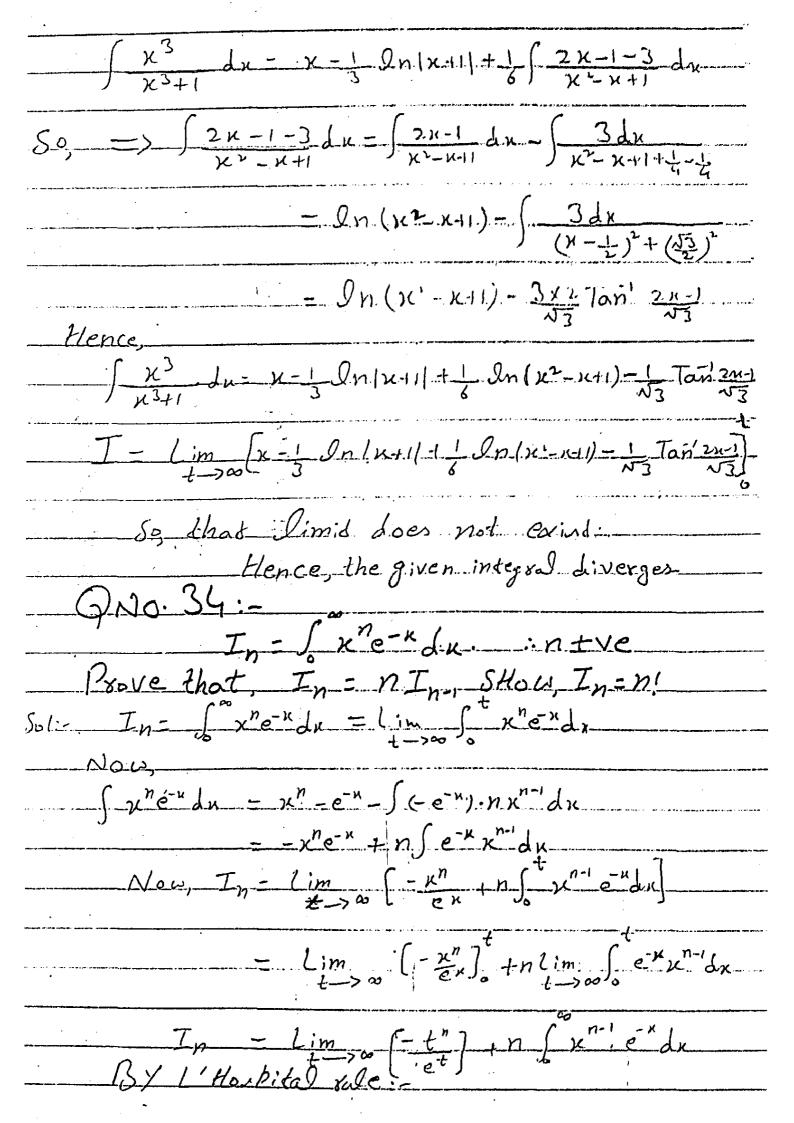


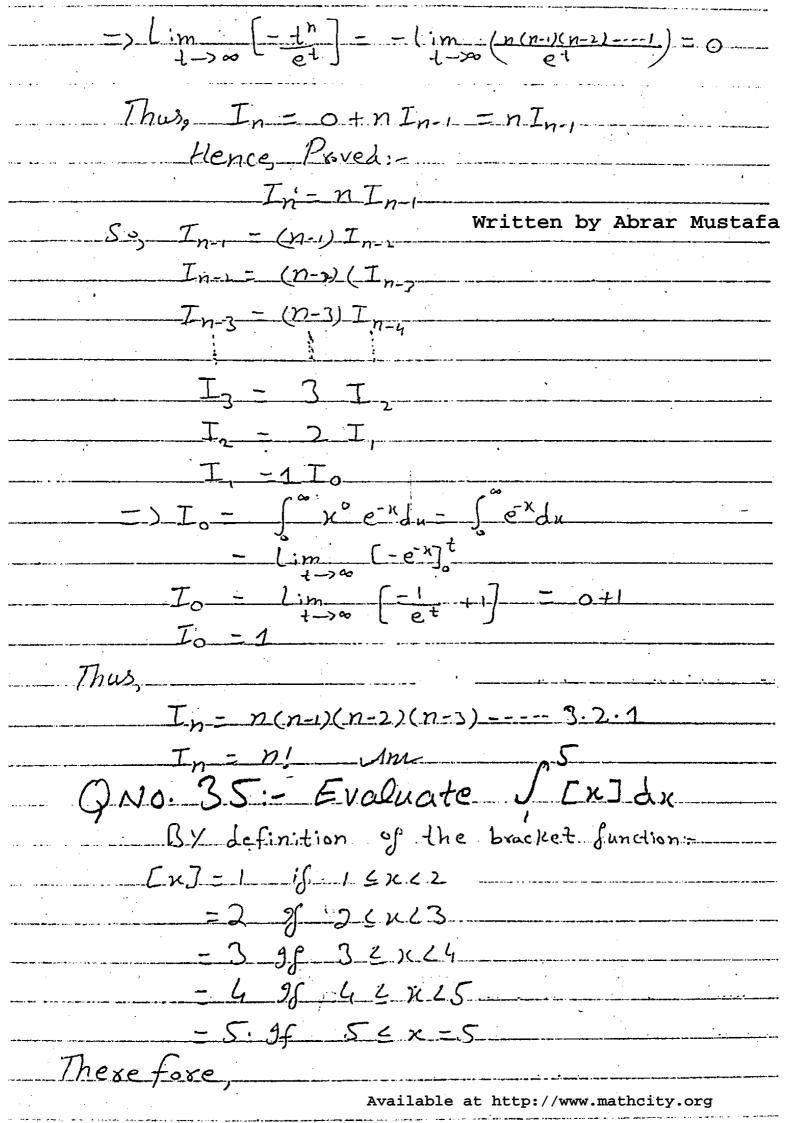


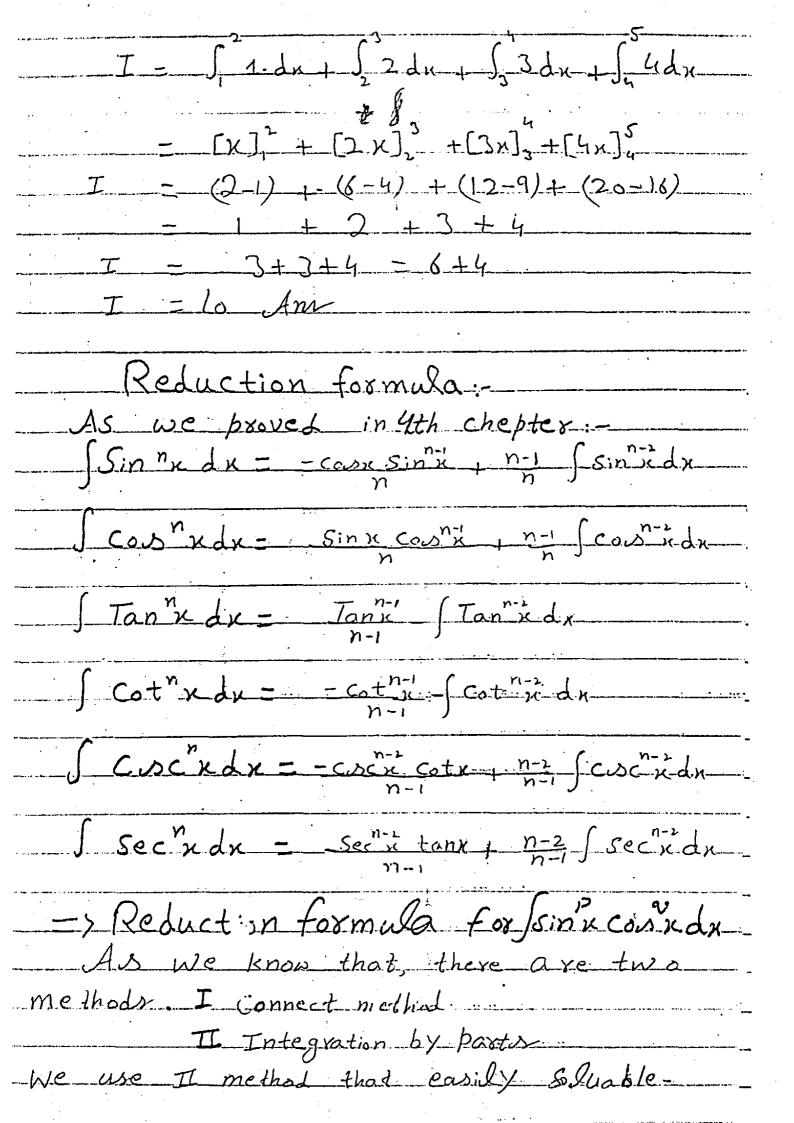


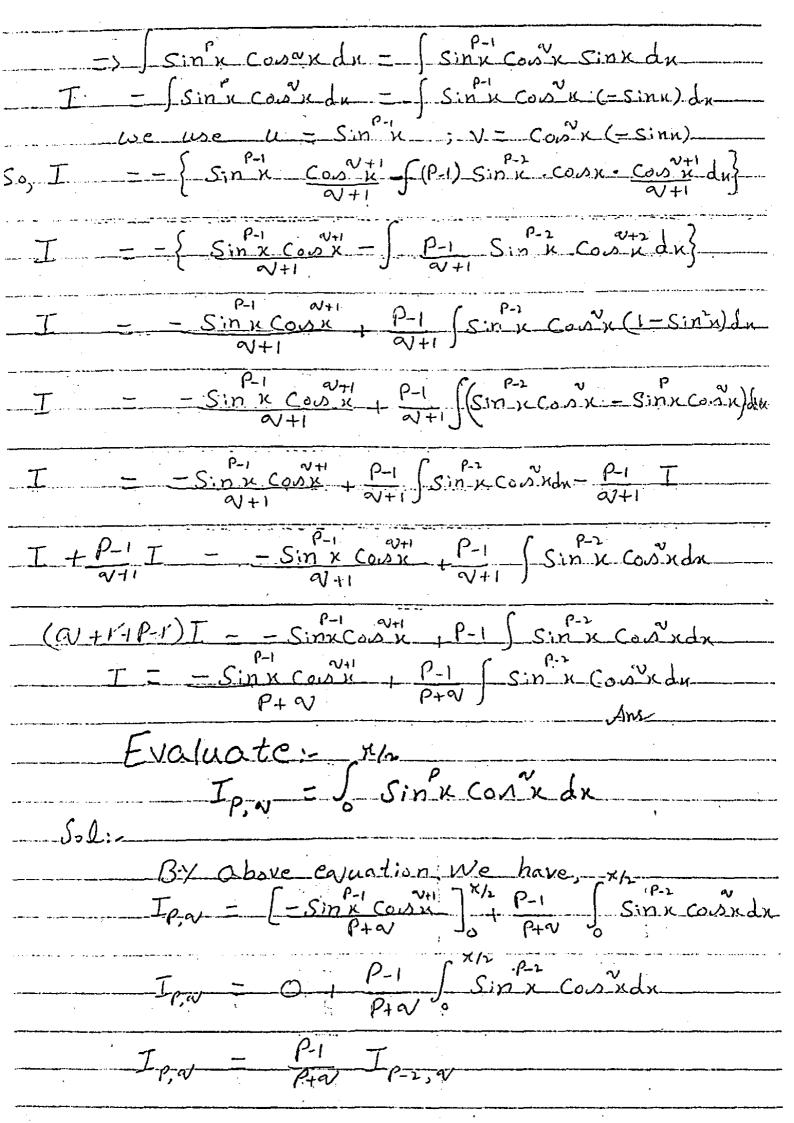


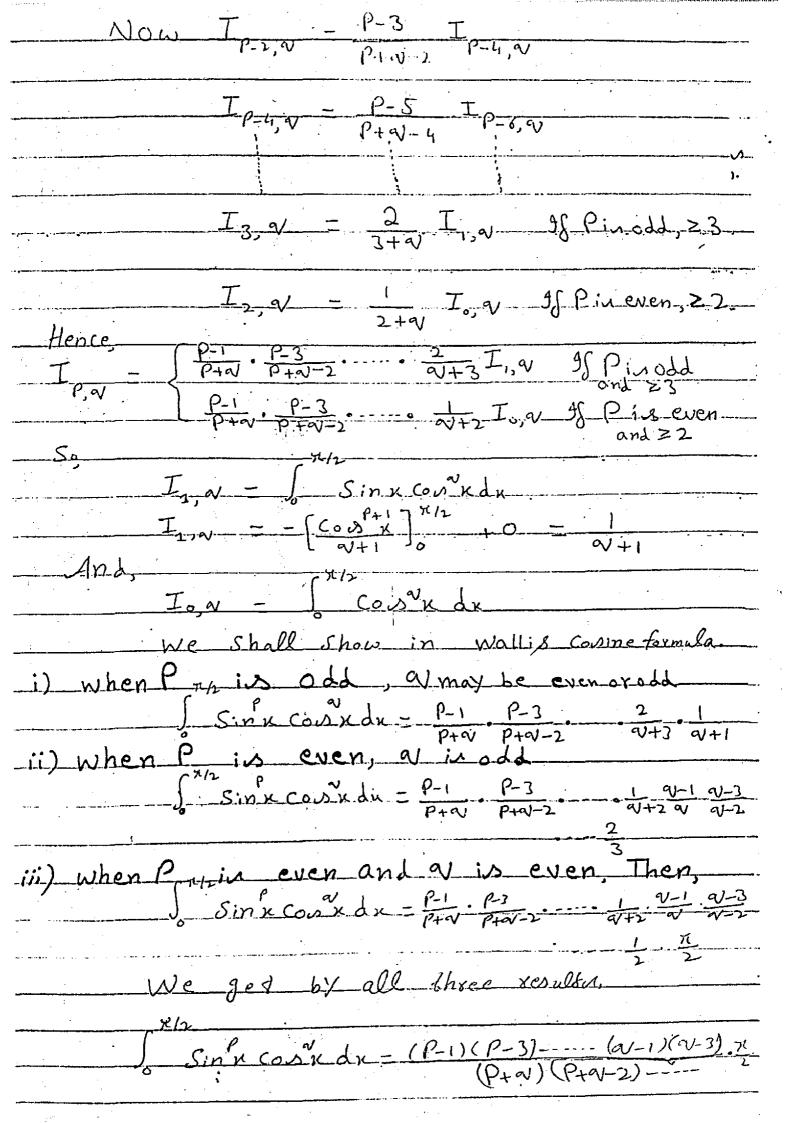


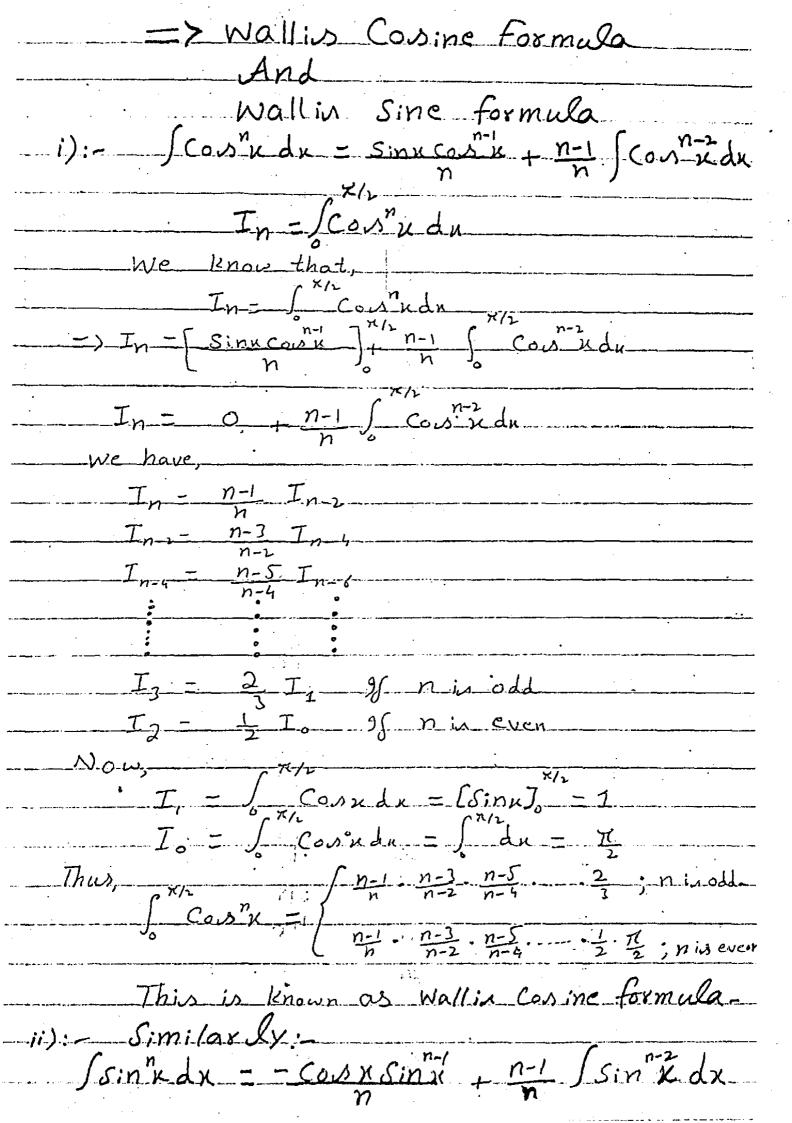


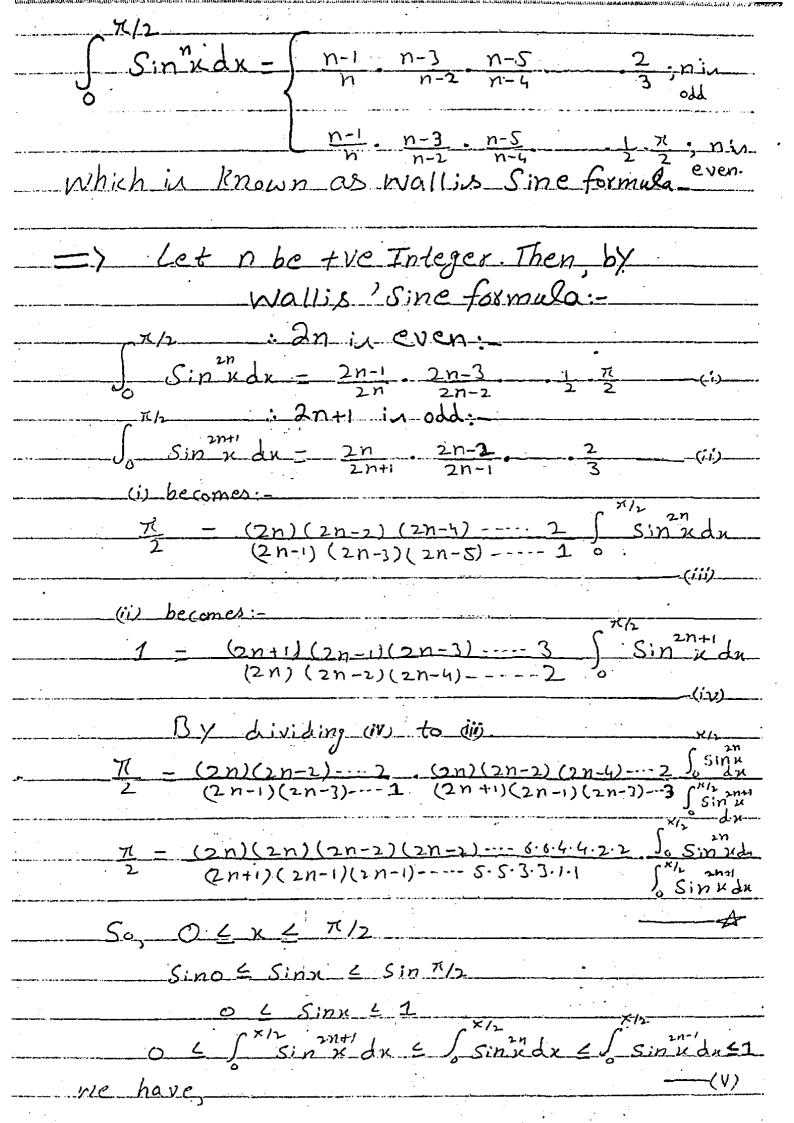












76/2		
$\int_{0}^{\pi/2} Sin^{n} u dx = \left[-Cosu Sin^{n-1} \right] + \frac{n-1}{n} \int_{0}^{\pi/2} Sin^{n-1} dx$		
$= \int_{0}^{\pi/L} \frac{2n+1}{\sin^{2} n} du = \frac{2n+1-1}{2n+1} \int_{0}^{\pi/L} \frac{2n+1-2}{\sin^{2} n} du$		
$\frac{2}{2} \int_{0}^{\infty} \frac{\sin n du}{2n+1} \int_{0}^{\infty} \sin n du$		
$-2n$ $\int \sin^2 x dx$		
$\frac{-2n}{2n+1} \int_{0}^{x/2} \frac{2n-1}{\sin x} dx$ $\frac{2n+1}{2n+1} \int_{0}^{x/2} \frac{\sin^{2} x}{\sin^{2} x} dx = \frac{1}{2n}$ $\frac{1}{2} \int_{0}^{x/2} \frac{\sin^{2} x}{\sin^{2} x} dx = \frac{1}{2n}$ $\int_{0}^{x/2} \sin^{2} x} dx = \frac{1}{2n}$		
we get by (V)		
1 & Jo Sin x dx & 1 + 1		
Sinvak		
mere tore:-		
$\lim_{x \to \infty} \frac{\int_{\mathcal{C}} Sin x dx}{\int_{\mathcal{C}} Sin x dx} = 1$		
Sy the Sandwich theorem, Sink dx		
Taking Limit, of both Sides of (A) as now		
አ/շ		
$\lim_{n\to\infty} \frac{\pi}{2} - \lim_{n\to\infty} \frac{2\cdot 2\cdot 4\cdot 4\cdot 6\cdot 6\cdot - \cdots - (2n)(2n)}{1\cdot 3\cdot 3\cdot 5\cdot 5\cdot 7 - \cdots - (2n-1)(2n+1)} \times \lim_{n\to\infty} \frac{\pi/2}{\int_{0}^{\pi/2} \sin^{2n}x dx}$		
$n-\infty$ 1 $1.3.3.5.5.7$ (2n-1)(2n+1) $n-\infty$ $\int_{0.2n+1}^{\pi/2} \int_{0.2n+1}^{\pi/2} \int_{0.2n+1}^{\pi/$		
$\frac{77 - 2 24}{2 \cdot 1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7} \frac{4 \cdot 6 \cdot 6}{(2n-1)} \frac{2n}{(2n+1)} \times 1$		
$\frac{71 - 2}{2} = \frac{2}{1} = \frac{4}{3} = \frac{4}{5} = \frac{4}{5} = \frac{2n}{2n-1} = \frac{2n}{2n+1}$		
$\frac{7(-2)^{2}}{2} = \frac{2}{1} = \frac{7}{3} = \frac{4}{5} = \frac{6}{5} = \frac{2n}{2n-1} = \frac{2n}{2n+1}$		
1, 10, 10		
This is known as Wallis Product formula		
for 12/2		
Written by Abrar Mustafa		
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The same of the sa		