So $\ln|x|$

Trignometric functions

$\int \sin x \, dx = -\cos x + c$

$\int \cos x \, dx = \sin x + c$

$\int \sec^2 x \, dx = \tan x + c$

$\int \operatorname{cosec} x \cot x = -\operatorname{cosec} x + c$

$\int \sec x \tan x = \sec x + c$

$\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

$\int \tan x = \ln|\sec x| + c$

$\int \cot x = \ln|\sin x| + c$

$\int \sec x \, dx = \ln|\sec x + \tan x| + c$

$\int \operatorname{cosec} x \, dx = \ln|\operatorname{cosec} x - \cot x|$

Inverse Trigonometric Functions

$$\int \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x + c$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x + c$$

$$\frac{dx}{1+x^2} = \tan^{-1}x + c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$$

Hyperbolic functions.

$$\int \sinh x \, dx = \cosh x + c$$

$$\int \cosh x \, dx = \sinh x + c$$

$$\int \operatorname{sech}^2 x \, dx = \tanh x + c$$

$$\int \operatorname{cosech}^2 x \, d = -\coth x + c$$

$$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + c$$

$$\int \operatorname{cosech} x \coth x \, dx = -\coth x + c$$

$$\int \tanh x \, dx = \ln|\cosh x| + c$$

$$\int \coth x \, dx = \ln|\sinh x| + c$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1}x = \ln\left(x+\sqrt{x^2+1}\right)$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1}x = \ln\left(x+\sqrt{x^2-1}\right)$$

$$\sinh^{-1}\left(\frac{x}{a}\right) = \ln\left(\frac{x+\sqrt{x^2+a^2}}{a}\right)$$

$$\cosh^{-1}\left(\frac{x}{a}\right) = \ln\left(\frac{x+\sqrt{x^2-a^2}}{a}\right)$$

$\int \sqrt{a^2-x^2}\,dx$ or $\int \frac{1}{\sqrt{a^2-x^2}}\,dx \Rightarrow x = a\sin\theta$

$\int \sqrt{a^2+x^2}\,dx$ or $\int \frac{1}{\sqrt{a^2+x^2}}\,dx$

$x = a\sinh\theta$ $\quad$ $x = a\tan\theta$

$\int \sqrt{x^2-a^2}\,dx$ or $\int \frac{1}{\sqrt{x^2-a^2}}\,dx$

$x = a\cosh\theta$ $\quad$ $x = a\sec\theta$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cosh 2\theta = \cosh^2\theta + \sinh^2\theta$$

$$\cosh^2\theta = \frac{1+\cosh 2\theta}{2}$$

$$\sinh^2\theta = \frac{\cosh 2\theta - 1}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$$

Exercise No 4.2

Evaluate

$$\int \frac{e^{\arctan x}}{1+x^2}\,dx$$

let $z = \tan^{-1}x$

$dz = \frac{1}{1+x^2}\,dx$

$= \int e^z\,dz = e^z + c$

$= e^{\tan^{-1}x} + c$ Ans.

$$\int \sqrt{\sin x}\,\cos x\,dx$$

let $\sin x = z$

$\cos x\,dx = dz$

so

$\int \sqrt{z}\,dz$

$= \frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c \Rightarrow \frac{z^{3/2}}{3/2} + c$

$= \frac{2}{3}(\sin x)^{3/2} + c$

$= \frac{2}{3}\sqrt{\sin^3 x} + c$

$= \frac{2}{3}\sqrt{\sin^3 x} + c$ Ans.

$$\int \frac{dx}{1+\sqrt{x+1}}$$

let $\sqrt{x+1} = z$

so that

$$z^2 = x+1 \quad , \text{ then}$$

$$2z\,dz = dx$$

$$= \int \frac{2z}{1+z}\,dz$$

$$\int \left[2 - \frac{2}{1+z}\right] dz$$

$$= 2z - 2\ln(1+z)$$

$$= 2\sqrt{x+1} - 2\ln(1+\sqrt{x+1}) \text{ Ans.}$$

$$\int \frac{\cos^6 2\theta}{\sin^8 2\theta}\,d\theta$$

$$= \int \frac{\cos^6 2\theta}{\sin^6 2\theta} \cdot \operatorname{cosec}^2 2\theta\,d\theta$$

$$= \int \cot^6 2\theta \cdot \operatorname{cosec}^2 2\theta\,d\theta$$

let

$$\cot^6 2\theta = z$$

$$-2\operatorname{cosec}^2 2\theta\,d\theta = dz$$

$$\operatorname{cosec}^2 2\theta\,d\theta = -\frac{dz}{2}$$

so

$$= \int z^6 - \frac{dz}{2}$$

$$= \frac{-1}{2}\int z^6\,dz$$

$$= -\frac{1}{2} \cdot \frac{z^7}{7} + c$$

$$= -\frac{1}{14}\cot^7 2\theta + c \text{ Ans.}$$

$$\int \frac{2x+1}{(x^2+x+1)^{5/2}}\,dx$$

let $x^2+x+1 = z$

$(2x+1)\,dx = dz$

$= \int \frac{dz}{z^{5/2}}$

$= \int z^{-5/2}\,dz$

$= \frac{z^{-3/2}}{-3/2} \Rightarrow \frac{-2}{3z^{3/2}}$

$-\frac{2}{3}\frac{1}{(x^2+x+1)^{3/2}}$ Ans.

$\int \tan^2\theta \sec^4\theta \, d\theta$

let

$\tan\theta = z$

$\sec^2\theta\, d\theta = dz$

so

$= \int \tan^2\theta\,(1+\tan^2\theta)\,.\sec^2\theta\, d\theta$

on substitution

$= \int z^2(1+z^2)\,dz$

$= \int (z^2+z^4)\,dz$

$= \frac{z^3}{3} + \frac{z^5}{5} + c$

$= \frac{\tan^3\theta}{3} + \frac{\tan^5\theta}{5} + c$ Ans.

Exercise No 4.2

Evaluate

$$\int \frac{dx}{\sqrt{a^2+x^2}}$$

$$x = a\tan\theta$$

$$dx = a\sec^2 d\theta$$

$$= \int \frac{a\sec^2\theta\, d\theta}{\sqrt{a^2 + a^2\tan^2\theta}}$$

$$= \int \frac{\cancel{a}\sec^{\cancel{2}}\theta\, d\theta}{\cancel{a}\cancel{\sec\theta}}$$

$$= \int \sec\theta\, d\theta$$

$$\ln|\sec\theta + \tan\theta|$$

$$\because \frac{x}{a} = \tan\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \frac{x^2}{a^2} = \sec^2\theta$$

$$\frac{a^2 + x^2}{a^2} = \sec^2\theta$$

$$\frac{\sqrt{a^2+x^2}}{a} = \sec\theta$$

$$= \ln\left|\frac{\sqrt{a^2+x^2}}{a} + \frac{x}{a}\right|$$

$$= \ln\left|\frac{x + \sqrt{a^2+x^2}}{a}\right| \text{ Ans.}$$

$$\int \frac{dx}{\sqrt{x^2-a^2}}$$

$$x = a\sec\theta$$

$$dx = a\sec\theta\tan\theta\, d\theta$$

$$= \int \frac{a\sec\theta\tan\theta\, d\theta}{\sqrt{a^2\sec^2\theta - a^2}}$$

$$= \int \frac{a\sec\theta\cancel{\tan\theta}\, d\theta}{a\cancel{\tan\theta}}$$

$$= \int \sec\theta\, d\theta$$

$$\ln|\sec\theta + \tan\theta| + c$$

$$\because \frac{x}{a} = \sec\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

$$= \frac{x^2}{a^2} - 1$$

$$= \frac{x^2 - a^2}{a^2}$$

$$\tan\theta = \frac{\sqrt{x^2-a^2}}{a}$$

$$= \ln\left|\frac{x}{a} + \frac{\sqrt{x^2-a^2}}{a}\right|$$

$$= \ln\left|\frac{x + \sqrt{x^2-a^2}}{a}\right| \text{ Ans.}$$

$$\int \tan x\, dx$$

$$= \int \frac{\sin x}{\cos x}\, dx$$

$$- \int \frac{-\sin x}{\cos x} \, dx$$

$$= -\ln|\cos x| + c$$

$$= \ln|\cos x|^{-1} + c$$

$$= \ln\left|\frac{1}{\cos x}\right| + c$$ 6

$\ln \sec x + c$ Ans.

4 $$\int \cot x \, dx$$

$$= \int \frac{\cos x}{\sin x} \, dx$$

$$= \ln|\sin x| + c$$ Ans.

5 $$\int \sec x \, dx$$

$$= \int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)}$$

$$= \ln|\sec x + \tan x| + c$$ 7

$$= \ln\left|\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right| + c$$

$$= \ln\left|\frac{1 + \sin x}{\cos x}\right| + c$$

$$= \ln \frac{\sin^2 x/2 + \cos^2 x/2 + 2\sin x/2 \cos x/2}{\cos^2 x/2 - \sin^2 x/2} + c$$

$$= \frac{\ln(\cos x/2 + \sin x/2)^{\cancel{2}}}{\cancel{(\cos x/2 + \sin x/2)}(\cos x/2 - \sin x/2)} + c$$ 8

$$= \ln \frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2} + c$$

$$= \ln \frac{1 + \tan x/2}{1 - \tan x/2}$$

$$= \ln \frac{\tan \pi/4 + \tan x/2}{1 - \tan \pi/4 \tan x/2}$$

$$= \ln |\tan(\pi/4 + x/2)| + c \quad \text{Ans.}$$

6 $$\int \operatorname{Cosec} x \, dx$$

$$= \int \frac{\operatorname{Cosec} x (\operatorname{Cosec} x - \operatorname{Cot} x)}{(\operatorname{Cosec} x - \operatorname{Cot} x)} dx$$

$$= \ln |\operatorname{Cosec} x - \operatorname{Cot} x| + c$$

$$= \ln \left| \frac{1}{\sin x} - \frac{\cos x}{\sin x} \right| + c$$

$$= \ln \left| \frac{1 - \cos x}{\sin x} \right| + c$$

$$= \ln \left| \frac{2\sin^2 x/2}{2\sin x/2 \cos x/2} \right| + c$$

$$= \ln |\tan x/2| + c \quad \text{Ans.}$$

7 $$\int (ax^2 + 2bx + c)^n (ax + b) dx$$

$$= \frac{1}{2} \int (ax^2 + 2bx + c)^n (2ax + 2b) \, dx$$

$$= \frac{1}{2} \frac{(ax^2 + 2bx + c)^{n+1}}{n+1} + c$$

+c $$= \frac{1}{2(n+1)} (ax^2 + 2bx + c)^{n+1} + c \quad \text{Ans.}$$

8 $$\int \sqrt{\frac{1+x}{1-x}} \, dx$$

Let

$$I = \int \sqrt{\frac{1+x}{1-x}} \, dx$$

Multiplying and dividing by $\sqrt{1+x}$

$$= \int \frac{\sqrt{1+x}}{\sqrt{1-x}} \times \frac{\sqrt{1+x}}{\sqrt{1+x}}\, dx$$

$$= \int \frac{(\sqrt{1+x})^2}{\sqrt{1-x^2}}\, dx$$ 10

$$= \int \frac{1+x}{\sqrt{1-x^2}}$$

$$= \int \frac{1}{\sqrt{1-x^2}}\, dx + \int \frac{x}{\sqrt{1-x^2}}\, dx$$

$$= \sin^{-1} x + \frac{1}{(-2)} \int \frac{-2x}{\sqrt{1-x^2}}\, dx$$

$$= \sin^{-1} x - \frac{1}{2} \left[\frac{(1-x^2)^{-1/2+1}}{-1/2+1} \right] + c$$

$$= \sin^{-1} x - (1-x^2)^{1/2} + c$$

$$= \sin^{-1} x - \sqrt{1-x^2} + c \quad \text{Ans.}$$ 11

$$\int \frac{dx}{a+\sqrt{bx+c}}$$

$$a + \sqrt{bx+c} = t$$

$$\frac{1}{2}(bx+c)^{-1/2} b\, dx = dt$$

$$dx = \frac{2}{b}(bx+c)^{1/2}\, dt$$

$$dx = \frac{2}{b}(t-a)\, dt$$

$$\because \sqrt{bx+c} = t-a$$

$$\frac{2}{b} \int \frac{t-a}{t}\, dt$$ 12

$$= \frac{2}{b}\int (1 - a/t)\, dt$$

$$= \frac{2}{b}\left[t - a\ln t\right] + c \quad \text{Ans.}$$

$$= \frac{2}{b}\left[a + \sqrt{bx+c} - a\ln(a + \sqrt{bx+c})\right] + c$$

10 $$\int \frac{dx}{(1+x^2)\tan^{-1}x}$$

$$\tan^{-1}x = t$$

$$\frac{1}{1+x^2}\, dx = dt$$

$$= \int \frac{1}{t}\, dt$$

$$= \ln|t| + c$$

$$= \ln|\tan^{-1}x| + c \quad \text{Ans.}$$

11 $$\int \frac{\sin x + \cos x}{\sin x - \cos x}\, dx$$

Let

$$\sin x - \cos x = t$$

$$(\cos x + \sin x)\, dx = dt$$

$$= \int \frac{(\cos x + \sin x)\, dx}{(\sin x - \cos x)}$$

$$= \int \frac{1}{t}\, dt$$

$$= \ln|t| + c$$

$$= \ln|\sin x - \cos x| + c \quad \text{Ans.}$$

12 $$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

Let $\sqrt{x} = t$

$$\frac{1}{2\sqrt{x}}\,dx = dt$$

$$= 2\int \sin\sqrt{x} \cdot \frac{1}{2\sqrt{x}}\,dx$$

$$= 2\int \sin\sqrt{x} \cdot \frac{1}{2\sqrt{x}}\,dx$$

$$= 2\int \sin t \cdot dt$$ 15

$$= -2\cos t + c$$

$$= -2\cos\sqrt{x} + c \quad \text{Ans.}$$

13 $$\int \sqrt{e^{2x} + e^{3x}}\,dx$$

$$= \int \sqrt{e^{2x}(1 + e^{x})}\,dx$$

$$= \int (1 + e^{x})^{1/2}\,e^{x}\,dx$$

$$= \frac{(1 + e^{x})^{\frac{1}{2}+1}}{\frac{1}{2} + 1} + c$$

$$= \frac{2}{3}(1 + e^{x})^{3/2} + c \quad \text{Ans.}$$

14

$$\int \frac{dx}{e^{x} + e^{-x}}$$ 16

$$= \int \frac{dx}{e^{x} + 1/e^{x}}$$

$$= \int \frac{e^{x}}{e^{2x} + 1}\,dx$$

Let $e^x = t$

$e^x dx = dt$

$= \int \frac{dt}{t^2+1}$

$= \tan^{-1}(t) + c$

$= \tan^{-1}(e^x) + c$ Ans.

15 $\int \frac{e^{2x}}{\sqrt{e^x - 1}} dx$

$e^x = t$

$e^x dx = dt$

$= \int \frac{e^x \cdot e^x dx}{\sqrt{e^x - 1}}$

$= \int \frac{t}{\sqrt{t-1}} \cdot dt$

$= \int \frac{t - 1 + 1}{\sqrt{t - 1}} dt$

$= \int (t-1)^{1/2} dt + \int (t-1)^{-1/2} dt$

$= \frac{(t-1)^{1/2+1}}{1/2+1} + \frac{(t-1)^{-1/2+1}}{-1/2+1} + c$

$= \frac{2}{3}(t-1)^{3/2} + 2(t-1)^{1/2} + c$

$= \frac{2}{3}(e^2 - 1)^{3/2} + 2(e^x - 1)^{1/2} + c$ Ans.

16 $\int \frac{\cos(\ln x)}{x} dx$

$\ln x = t$

$\frac{1}{x} dx = dt$

$\int \cos t \cdot dt$

$= \sin t + c$

$= \sin(\ln x) + c$ Ans.

17 $\displaystyle\int \frac{2x+5}{\sqrt{x^2+5x+7}}\,dx$

$= \int (x^2+5x+7)^{-1/2}(2x+5)\,dx$

$= \dfrac{(x^2+5x+7)^{-1/2+1}}{-1/2+1} + c$

$= 2(x^2+5x+7)^{1/2} + c$ Ans.

18 $\displaystyle\int \frac{(x+2)\,dx}{\sqrt{2x^2+8x+5}}$

$2x^2+8x+5 = t$

$(4x+8)dx = dt$

$4(x+2)dx = dt$

$(x+2)\,dx = \dfrac{dt}{4}$

$\Rightarrow \dfrac{1}{4}\displaystyle\int \frac{dt}{\sqrt{t}}$

$= \dfrac{1}{4}\displaystyle\int (t)^{-1/2}\,dt$

$= 1/4 \dfrac{(t)^{-1/2+1}}{-1/2+1} + c$

$= \dfrac{1}{2} t^{1/2} + c$

$= \dfrac{1}{2}\sqrt{2x^2+8x+5} + c$ Ans.

19 $\displaystyle\int \frac{\sqrt{x^2-a^2}}{x^4}\,dx$

$$x = a\cosh\theta$$
$$dx = a\sinh\theta\, d\theta$$

$$= \int \frac{\sqrt{a^2\cosh^2\theta - a^2}}{a^4\cosh^4\theta} \cdot a\sinh\theta\, d\theta$$

$$= \frac{a^2}{a^4}\int \frac{\sinh^2\theta\, d\theta}{\cosh^4\theta}$$

$$= \frac{1}{a^2}\int \tanh^2\theta \cdot \operatorname{sech}^2\theta\, d\theta$$

$$= \frac{1}{a^2}\,\frac{\tanh^3\theta}{3}$$

$$= \frac{1}{3a^2}\tanh^3\theta$$

$$= \frac{1}{3a^2}\left[\frac{x^2 - a^2}{x}\right]^{3/2} + c$$

$$= \frac{(x^2 - a^2)^{3/2}}{3a^2x^3} + c \text{ Ans.}$$

20 $$\int \cos^6 x \sin^3 x\, dx$$

$$= \int \cos^6 x \cdot \sin^2 x \cdot \sin x\, dx$$

$$= \int \cos^6 x\,(1 - \cos^2 x)\, \sin x\, dx$$

Let

$$\cos x = t$$
$$-\sin x\, dx = dt$$
$$\sin x\, dx = -dt$$

$$= \int t^6(1 - t^2) \cdot -dt$$

$$= -\int t^6(1 - t^2)\, dt$$

$$= \int (t^6 - t^8)\, dt$$

$$= -\left(\frac{t^7}{7} - \frac{t^9}{9}\right) + c$$

$$= -\left(\frac{\cos^7 x}{7} - \frac{\cos^9 x}{9}\right) + c$$

21 $\int \tan^3 x \sec^3 x \, dx$

$$= \int \tan^2 x \sec^2 x \,(\sec x \tan x)\, dx$$

$$= \int \sec^2 x (\sec^2 x - 1)(\sec x \tan x)\, dx$$

Let

$$\sec x = t$$
$$\sec x \tan x \, dx = dt$$

$$= \int t^2 (t^2 - 1)\, dt$$

$$= \int (t^4 - t^2)\, dt$$

$$= \frac{t^5}{5} - \frac{t^3}{3} + c$$

$$= \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + c \quad \text{Ans.}$$

22 $\int \cot^3 x \operatorname{cosec}^4 x \, dx$

$$= \int \cot^3 x \operatorname{cosec}^2 x \cdot \operatorname{cosec}^2 x \, dx$$

$$= \int \cot^3 x (1 + \cot^2) \operatorname{cosec}^2 x \, dx$$

$$= \int \cot^3 x + \cot^5 x \cdot \operatorname{cosec}^2 x \, dx$$

$$\cot x = t$$
$$-\operatorname{cosec}^2 x \, dx = dt$$

23

$$\operatorname{Cosec}^2 x = -dt$$

$$= \int (t^3 + t^5)(-dt)$$

$$= -\int (t^3 + t^5)\,dt$$

$$= -\left[\frac{t^4}{4} + \frac{t^6}{6}\right] + c$$

$$= -\frac{\operatorname{Cot}^4 x}{4} - \frac{\operatorname{Cot}^6 x}{6} + c \text{ Ans.}$$

2.3

$$\int \frac{1}{2(x^2 + 3x/2 + 2)}\,dx$$

$$= \int \frac{1}{\sqrt{2}} \frac{1}{x^2 + 3x/2 + 2}\,dx$$

$$\frac{1}{\sqrt{2}} \int \frac{1}{(x)^2 + 2(x)(3/4) + (3/4)^2 - (3/4)^2 + 2}\,dx$$

$$\frac{1}{\sqrt{2}} \int \frac{dx}{(x+3/4)^2 + (23/16)}$$

$$\because -\frac{9}{16} + 2$$

$$\frac{-9+32}{16}$$

$$\frac{1}{\sqrt{2}} \int \frac{dx}{(x+3/4)^2 + (\sqrt{23}/4)^2}$$

$$\frac{23}{16}$$

$$= \frac{1}{\sqrt{2}} \sinh^{-1}\left(\frac{x + 3/4}{\sqrt{23}/4}\right) + c$$

2.4

$$\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{4x+3/4}{\sqrt{23}/4}\right)+c$$

$$\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{4x+3}{\sqrt{23}}\right)+c$$

24. $\sqrt{a^2 - x^2}$

Let

$$I = \int \sqrt{a^2 - x^2}\, dx$$

$$x = a\sin\theta$$
$$dx = a\cos\theta\, d\theta$$

$$= \int \sqrt{a^2 - a^2\sin^2\theta}\, . a\cos\theta\, d\theta$$

$$= \int \sqrt{a^2(1 - \sin^2\theta)}\, a\cos\theta\, d\theta$$

$$= a^2\int \cos^2\theta\, d\theta$$

$$= a^2\int \frac{1 + \cos 2\theta}{2}\, d\theta$$

$$= a^2\left[\frac{1}{2}\int 1\, d\theta + \frac{1}{2}\int \cos 2\theta\, d\theta\right]$$

$$= \frac{a^2}{2}\theta + \frac{a^2 \sin 2\theta}{4} + c$$

$$= \frac{a^2}{2}\theta + \frac{a^2 2\sin\theta\cos\theta}{4} + c$$

$$= \frac{a^2}{2}\theta + \frac{a^2 \sin\theta\cos\theta}{2} + c$$

$$= \frac{a^2}{2}\theta + a^2\left[\frac{\sin\theta\sqrt{1 - \sin^2\theta}}{2}\right] + c$$

$$= \frac{a^2}{2}\sin^{-1}\left[\frac{x}{a}\right] + a^2\left[\frac{x/a\sqrt{1 - x^2/a^2}}{2}\right] + c$$

$$= \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{\cancel{a^2}\, x\sqrt{a^2 - x^2}}{2\cancel{a^2}} + c$$

$$= \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c \quad \text{Ans.}$$

25 $(2x+3)\sqrt{2x+1}$

$\sqrt{2x+1} = t$

$2x+1 = t^2$

$2x = t^2 - 1$ 27

$2\,dx = 2t\,dt$

$dx = t\,dt$

$= \int (t^2 - 1 + 3)\,t \cdot t\,dt$

$= (t^2 + 2)\,t^2\,dt$

$= (t^4 + 2t^2)\,dt$

$= \frac{t^5}{5} + \frac{2t^3}{3} + c$

$= \frac{(2x+1)^{5/2}}{5} + \frac{2}{3}(2x+1)^{3/2} + c$

26 $(1+x^2)^{-3/2}\,dx$

$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$= \int (1+\tan^2\theta)^{3/2}\sec^2\theta\,d\theta$

$= \int (\sec^{-3}\theta)\,\sec^2\theta\,d\theta$

$= \int \sec^{-1}\theta\,d\theta$

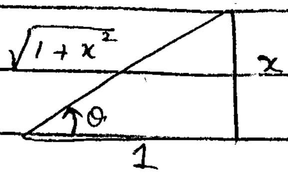
$= \int \frac{1}{\sec\theta}\,d\theta$

$\int \cos\theta\,d\theta$

$= \sin\theta + c$

(29) 4.2

$$= \frac{x}{\sqrt{1+x^2}} + c \quad \text{Ans.}$$



27 $\dfrac{x^2}{\sqrt{x^2+1}}$ $\qquad \sin\theta = \dfrac{x}{\sqrt{1+x^2}}$

$$\int \frac{x^2+1-1}{\sqrt{x^2+1}}\,dx$$

$$\int \sqrt{x^2+1}\,dx - \int \frac{1}{\sqrt{x^2+1}}\,dx$$

let $\quad I_1 = \int \sqrt{x^2+1}.\,dx$

$$x = a\sinh\theta$$

$$dx = (1)\cosh\theta\,d\theta$$

$$dx = \cosh\theta\,d\theta$$

$$\Rightarrow \int \sqrt{\sinh^2\theta+1}\,\cosh\theta\,d\theta$$

$$= \int \cosh\theta \cdot \cosh\theta\,d\theta$$

$$= \int \cosh^2\theta\,d\theta$$

$$= \int \frac{1+\cosh 2\theta}{2}\,d\theta$$

$$= \frac{1}{2}\int 1\,d\theta + \frac{1}{2}\int \cosh 2\theta\,d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{2}(\sinh\theta * \cosh\theta)$$

$$= \frac{1}{2}\left[\theta + \sinh\theta\cosh\theta\right] + c$$

$$= \frac{1}{2}\left[\sinh^{-1}x + x.\sqrt{1+x^2}\right] = \frac{\sinh^{-1}x}{2} + \frac{x\sqrt{1+x^2}}{2}$$

Now $\quad I_2 = \int \frac{1}{\sqrt{x^2+1}}\,dx$

$x = \sinh\theta$

$dx = \cosh\theta\, d\theta$

$= \int \frac{\cosh\theta\, d\theta}{\sqrt{\sinh^2\theta+1}}$

$= \int \frac{\cosh\theta\, d\theta}{\cosh\theta}$

$= \int 1\, d\theta$

$= \theta + c$

(i) $\Rightarrow \quad \frac{\sinh^{-1}x}{2} + \frac{x\sqrt{1+x^2}}{2} - \sinh^{-1}x$

$= \sinh^{-1}x\left[\frac{1}{2} - 1\right] + \frac{x\sqrt{1+x^2}}{2}$

$= \frac{x\sqrt{x^2+1}}{2} - \frac{1}{2}\sinh^{-1}x$ Ans.

(28) $\quad \int (2x+4)\sqrt{2x^2+3x+1}\,dx$

$\frac{1}{2}\int (2x^2+3x+1)^{1/2}(4x+8)\,dx$

$= \frac{1}{2}\int (2x^2+3x+1)^{1/2}(4x+3+5)\,dx$

$= \frac{1}{2}\int \frac{(4x+3) + 5}{(2x^2+3x+1)^{-1/2}}\,dx$

$\frac{1}{2}\left[\int \frac{4x+3}{(2x^2+3x+1)^{-1/2}}\,dx + 5\int (2x^2+3x+1)^{1/2}\,dx\right]$

$\frac{1}{2}\,\frac{(2x^2+3x+1)^{+1/2+1}}{+1/2+1} + \frac{5}{2}\int (2x^2+3x+1)^{1/2}\,dx$

$\frac{1}{2} \times \frac{2}{3}(2x^2+3x+1)^{3/2} + \frac{5}{2}\left[\int 2\left(x^2+\frac{3x}{2}+\frac{1}{2}\right)^{1/2}dx\right]$

(31) 4.2

$$= \frac{1}{3}(2x^2+3x+1)^{3/2} + \frac{5}{2}\left[\sqrt{2}\int\left[\left(x^2+\frac{3}{2}x+\frac{1}{2}\right)^{1/2}dx\right]\right]$$

$$\frac{1}{3}(2x^2+3x+1)^{3/2} + \frac{5}{\sqrt{2}}\left[\int\left(x^2+2(x)(3/4)+(3/4)^2-(3/4)^2+(1/2)\right)^{1/2}dx\right]$$

$$\frac{1}{3}(2x^2+3x+1)^{3/2} + \frac{5}{\sqrt{2}}\left[\int\sqrt{(x+3/4)^2-(1/4)^2}\,dx\right] \longrightarrow (1)$$

let

Take 2nd Term

$$x+\frac{3}{4} = \frac{1}{4}\cosh\theta$$

$$dx = \frac{1}{4}\sinh\theta\, d\theta$$

$$= \frac{5}{\sqrt{2}}\int\sqrt{(1/4\cosh\theta)^2-(1/4)^2}\;\frac{1}{4}\sinh\theta$$

$$= \frac{5}{16\sqrt{2}}\int\sqrt{(\cosh^2\theta-1)}\;\sinh\theta$$

$$\frac{5}{16\sqrt{2}}\int\sinh^2\theta\, d\theta$$

$$\frac{5}{16\sqrt{2}}\int\left(\frac{\cosh 2\theta-1}{2}\right)d\theta \Rightarrow \frac{5}{2\times16\sqrt{2}}\left[\frac{\sinh 2\theta}{2}-\theta\right]$$

$$= \frac{5}{2\times16\sqrt{2}}(\sinh\theta\cosh\theta-\theta)$$

$$\frac{5}{\cancel{2}\times16\cancel{\sqrt{2}}}\left[\cancel{2}\cancel{\sqrt{2}}\sqrt{2x^2+3x+1}\,(4x+3)-\cosh^{-1}(4x+3)\right]$$

$$\because x+\frac{3}{4} = \frac{1}{4}\cosh\theta \Rightarrow \cosh\theta = 4x+3$$

$$\theta = \cosh^{-1}(4x+3) \quad ; \quad dx = \frac{1}{4}\sinh\theta\, d\theta$$

$$\sinh\theta = \sqrt{\cosh^2\theta-1}$$

$$\sinh\theta = \sqrt{(4x+3)^2-1} \Rightarrow \sqrt{16x^2+9+24x-1}$$

$$\sqrt{16x^2+24x+8} \Rightarrow \sqrt{8}\sqrt{2x^2+3x+1}$$

$$= \frac{5}{16}\sqrt{2x^2+3x+1}\,(4x+3)$$

x]

x]

(1) $\Rightarrow \int (2x+4)\sqrt{2x^2+3x+1}\,dx$

$$= \frac{1}{3}\left(2x^2+3x+1\right)^{3/2} + \frac{5}{16}(4x+3)\sqrt{2x^2+3x+1}$$

$$- \cosh^{-1}(4x+3) + C$$

30

29

$$\int \frac{dx}{3\sin x + 4\cos x}$$

$3 = r\cos\theta$ ----- (i)

$4 = r\sin\theta$ (ii)

$r = \sqrt{16+9}$

$r = 5$

31

$\theta = \tan^{-1}(4/3)$

$$= \int \frac{dx}{r\cos\theta\sin x + r\sin\theta\cos x}$$

$$= \frac{1}{r}\int \frac{dx}{\sin(x+\theta)}$$

$$= \frac{1}{r}\int \operatorname{cosec}(x+\theta)\,dx$$

$$= \frac{1}{r}\ln\left|\operatorname{cosec}(x+\theta) - \cot(x+\theta)\right| + c$$

$$= \frac{1}{r}\ln\tan\left(\frac{x+\theta}{2}\right) + c$$

$$= \frac{1}{5}\ln\tan\left(x/2 + \frac{1}{2}\tan^{-1}(4/3)\right) + c \text{ Ans.}$$

$$\int \frac{\tan x}{\cos x + \sec x}\, dx$$

$$I = \int \frac{\sin x/\cos x}{\cos x + 1/\cos x}\, dx$$

$$= \int \frac{\sin x}{\cos^2 x + 1} \times \frac{\cancel{\cos x}}{\cancel{\cos x}}\, dx$$

$$= \int \frac{\sin x}{\cos^2 x + 1}\, dx$$

$$\cos x = t$$

$$-\sin x\, dx = dt$$

$$\sin x\, dx = -dt$$

$$= \int \frac{-dt}{t^2+1}$$

$$= -\int \frac{dt}{1+t^2}$$

$$= -\tan^{-1}(t) + c$$

$$= -\tan^{-1}(\cos x) + c \text{ Ans.}$$

31

$$\int \frac{dx}{\sin(x-a)\sin(x-b)}$$

Multiplying and dividing by $\sin(a-b)$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin(a-b)}{\sin(x-a)\sin(x-b)}\, dx$$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin(x-x+a-b)}{\sin(x-a)\sin(x-b)}\, dx$$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin(x-b-(x-a))}{\sin(x-a)\sin(x-b)}\, dx$$

$$= \frac{1}{\sin(a-b)} \int \left[\frac{\sin(x-b)\cos(x-a)}{\sin(x-a)\sin(x-b)} - \frac{\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)} \right]$$

$$= \frac{1}{\sin(a-b)} \frac{\cancel{\sin(x-b)}\cos(x-a)}{\sin(x-a)\cancel{\sin(x-b)}} - \frac{\cos(x-b)\cancel{\sin(x-a)}}{\cancel{\sin(x-a)}\sin(x-b)}$$

$$= \frac{1}{\sin(a-b)} \int \frac{\cos(x-a)}{\sin(x-a)}\,dx - \int \frac{\cos(x-b)}{\sin(x-b)}\,dx$$

$$= \frac{1}{\sin(a-b)} \left[\ln|\sin(x-a)| - \ln|\sin(x-b)| \right] + c$$

$$= \frac{1}{\sin(a-b)} \ln\left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c \quad \text{Ans.}$$

32 $\tan x \ln(\sec x)$

Let

$$\ln(\sec x) = t$$

$$\frac{\sec x \tan x}{\sec x}\,dx = dt$$

$$\tan x\,dx = dt$$

$$= \int t\,dt$$

$$= \frac{t^2}{2} + c$$

$$= \frac{1}{2}(\ln \sec x)^2 + c$$

33 $$\int \frac{1}{(3\tan x + 1)\cos^2 x}\,dx$$

Let $\tan x = t$

$$\sec^2 x\,dx = dt$$

$$= \int \frac{\sec^2 x}{3\tan x + 1}\,dx$$

$$= \int \frac{dt}{3t+1}$$

$$= \frac{1}{3}\int \frac{1}{3t+1}\,3dt$$

$$= \frac{1}{3}\ln|3t+1| + c$$

-a)
-b)

$$= \frac{1}{3} \ln|1 + 3\tan x| + c \text{ Ans.}$$

34 $$\int e^{\sin x} \cos x \, dx$$

let $\sin x = t$

$$\cos x \, dx = dt$$

$$= \int e^{\sin x} \cos x \, dx$$

$$= \int e^t \, dt$$

$$= e^t + c$$

$$= e^{\sin x} + c \text{ Ans.}$$

35 $$\int \sqrt{1 + 3\cos^2 x} \, \sin 2x \, dx$$

let

$$\cos^2 x = t$$

$$2\cos x(-\sin x) \, dx = dt$$

$$-2\sin x \cos x \, dx = dt$$

$$\sin 2x = -dt$$

$$= -\int \sqrt{1 + 3t} \, dt$$

$$= \frac{-1}{3} \int \sqrt{1 + 3t} \, 3dt$$

$$= \frac{-1}{3} \cdot \frac{(1 + 3t)^{\frac{1}{2}+1}}{\frac{1}{2} + 1} + c$$

$$= \frac{-1}{3} \times \frac{2}{3} (1 + 3t)^{3/2} + c$$

$$= -\frac{2}{9} (1 + 3t)^{3/2} + c$$

$$= -\frac{2}{9} (1 + 3\cos^2 x)^{3/2} + c \text{ Ans.}$$

36 $$\int \frac{\sin 2x}{\sqrt{1 + \cos^2 x}} \, dx$$

(36) 4.2

let $\cos^2 x = t$

$-2\sin x \cos x \, dx = dt$

$\sin 2x = -dt$

$$= \int \frac{\sin 2x}{\sqrt{1+\cos^2 x}} dx$$

$$= \int \frac{-dt}{\sqrt{1+t}}$$

$$= -\left[\frac{(1+t)^{-1/2+1}}{-1/2+1} + c \right]$$

$$= -2(1+t)^{1/2} + c$$

$$= -2\sqrt{1+t} + c \text{ Ans.}$$

37 $$\int \frac{1}{2\sin^2 x + 3\cos^2 x} dx$$

Multiply and dividing by $\cos^2 x$

$$= \int \frac{1/\cos^2 x}{2\tan^2 x + 3} dx$$

$$= \int \frac{\sec^2 x}{2\tan^2 x + 3} dx$$

Let $\tan x = t$

$\sec^2 x \, dx = dt$

$$= \int \frac{dt}{2t^2 + 3}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 + (\sqrt{3/2})^2}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{3/2}} \tan^{-1}\left[\frac{t}{\sqrt{3/2}} \right] + c$$

$$= \frac{\sqrt{2}}{2\sqrt{3}} \tan^{-1}\left[\frac{\sqrt{2}\tan x}{\sqrt{3}} \right] + c$$

$= \frac{1}{\sqrt{6}} \tan^{-1}\left[\frac{\sqrt{2}\tan x}{\sqrt{3}}\right] + c$ Ans.

38 $\int \frac{1}{\sqrt{x}} \sec\sqrt{x}\tan\sqrt{x}\,dx$

Let $\sqrt{x} = t$

$\frac{1}{2\sqrt{x}}dx = dt$

$dx = 2\sqrt{x}\,dt$

$dx = 2t\,dt$

$= \int \frac{1}{t}\sec t\tan t \cdot 2t \cdot dt$

$= 2\int \sec t \cdot \tan t\,dt$

$= 2\sec t + c$

$= 2\sec\sqrt{x} + c$ Ans.

39 $\int [\pi^{\sin x} + (\sin x)^{\pi}] \cos x\,dx$

$\sin x = t$

$\cos x\,dx = dt$; $\int \pi^t + (t)^{\pi} dt$

$\Rightarrow \frac{\pi^t}{\ln\pi} + \frac{t^{\pi+1}}{\pi+1} + c$

$= \frac{\pi^{\sin x}}{\ln\pi} + \frac{(\sin x)^{\pi+1}}{\pi+1} + c$ Ans.

40 $\int \frac{\cos x}{3\sin x + 4\sqrt{\sin x}}\,dx$

$\sqrt{\sin x} = t$

$\frac{1}{2\sqrt{\sin x}}\cos x\,dx = dt$

$\cos x\,dx = 2\sqrt{\sin x}\,dt$

$\cos x\,dx = 2t\,dt$

$$= \int \frac{2t\,dt}{3t^2+4t}$$

$$= \int \frac{2t\,dt}{t(3t+4)}$$

$$= \frac{2}{3}\int \frac{3}{(3t+4)}\,dt$$

$$= \frac{2}{3}\ln|3t+4| + c$$

$$= \frac{2}{3}\ln|3\sqrt{\sin x}+4| + c \text{ Answer}$$

3

Exercise No. 4.3

Evaluate

1 $$\int x\sec^2 x\,dx$$

Itegration by parts

$$= x\int \sec^2 x\,dx - \int \frac{d}{dx}(x)\int \sec^2 x\,dx$$

$$= x\tan x - \int 1(\tan x)dx$$

$$= x\tan x - \int \frac{\sin x}{\cos x}dx$$

$$= x\tan x + \int \frac{-\sin x}{\cos x}dx$$

$$= x\tan x + \ln|\cos x| + c \text{ Ans.}$$

2 $$\int x\,\text{Cosec}^2 x\,dx$$

$$= x\int \text{cosec}^2 x - \int \frac{d}{dx}(x)\int \text{cosec}^2 x\,dx$$