PPSC Paper 2011 (Lecturer in Mathematics)

Available at: http://www.mathcity.org/ppsc

Time Allowed: Two Hours   Maximum Marks: 100

We are very thankful to *Ms. Iqra Liaqat* for providing solution of this paper.

Instructions:

• Read QUESTION PAPER carefully and mark your answers on the ANSWER SHEET.

• Each question has four options. Fill only one box that you think is the correct answer. Each question carries 1 mark. 0.25 mark will be deducted for each incorrect answer.

• Use of calculator is NOT allowed.

1. A ring $\mathbb{R}$ is a Boolean Ring if for all $x \in \mathbb{R}$.
   (A) $x^2 = x$ (B) $x^2 = -x$ (C) $x^2 = 0$ (D) $x^2 = 1$

2. The group of Quaterminons is a non-abelian group of order ———
   (A) 6 (B) 8 (C) 10 (D) 4

3. Every group of prime order is ——-
   (A) an abelian but not cyclic (B) an abelian group (C) a non-abelian group (D) a cyclic group

4. Any two conjugate subgroup of a group G are
   (A) Equivalent (B) Similar (C) Isomorphic (D) None of these

5. If H is a subgroup of index —— then H is a normal subgroup of G
   (A) 2 (B) 4 (C) Prime number (D) None of these

6. nZ is a maximal ideal of a ring Z if and only if n is ——
   (A) Prime number (B) Composite number (C) Natural number (D) None of these

7. Let G be a cyclic group of order 24 generated by a then order of a^{10} is ——
   (A) 2 (B) 12 (C) 10 (D) None of these

8. If a vector space V has a basis of n vectors, then every basis of V must consist of exactly —— vectors.
   (A) $n+1$ (B) n (C) $n-1$ (D) None of these

9. An indexed set of a vectors $v_1, v_2, v_3, ..., v_r$ in R^n is said to be —— if the vector equation $x_1v_1 + x_2v_2 + ... + x_pv_p = 0$ has only trivial solution.
   (A) Linearly independent (B) Basis (C) Linearly dependent (D) None of these

10. The set C_n of all, nth roots of unity for a fixed positive integer n is a group under ——
    (A) Addition (B) Addition modulo n (C) Multiplication (D) Multiplication modulo n

11. Intersection of any collection of normal subgroups of a group G ——
(A) is normal subgroup (B) may not be normal subgroup (C) is cyclic subgroup (D) is abelian subgroup

12. $\mathbb{Z}/2\mathbb{Z}$ is a quotient group of order ——-
(A) 1 (B) 2 (C) infinite (D) none of these

13. A group G having order ———, where p is prime, is always abelian.
(A) p^4 (B) p^2 (C) $2p$ (D) p^3

14. The number of conjugacy classes of symmetric group of degree 3 is ————
(A) 6 (B) 2 (C) 3 (D) 4

15. ———— is a set of all those elements of a group G which commutes with all other elements of G
(A) commutator subgroup (B) centre of group (C) automorphism of G (D) None of these

16. What are zero divisors in the ring of integers modulo 6
(A) $\bar{1}, \bar{2}, \bar{4}$ (B) $\bar{0}, \bar{2}, \bar{3}$ (C) $\bar{0}, \bar{2}, \bar{4}$ (D) $\bar{2}, \bar{3}, \bar{4}$

17. If H is a normal subgroup of G, then $Na(H) =$ ————
(A) H (B) G (C) $\{e\}$ (D) None of these

18. An $n \times n$ matrix with n distinct eigenvalues is ————-
(A) Diagonalization (B) Similar matrix (C) Not diagonalizable (D) None of these

19. Let $T : U \longrightarrow V$ be a linear transformation from an n dimensional vector space $U(F)$ to a vector space $V(F)$ then
(A) $\dim N(T) + \dim R(T) = 0$ (B) $\dim N(T) + \dim R(T) = 2n$
(C) $\dim N(T) + \dim R(T) = n^2$ (D) $\dim N(T) + \dim R(T) = n$

20. The dimension of the row space or column space of a matrix is called the ——- of the matrix.
(A) Basis (B) Null space (C) Rank (D) None of those

21. $\underline{a} \times (\underline{b} \times \underline{c})$ is a vector lying in the plane containing vectors
(A) $\underline{a}, \underline{b}$ and $\underline{c}$ (B) $\underline{a}$ and $\underline{c}$ (C) $\underline{b}$ and $\underline{c}$ (D) $\underline{b}$ and $\underline{a}$

22. The square matrix A and its transpose have the ——— eigenvalues.
(A) Same (B) Different (C) Unique (D) None of these

23. The set $S = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ of vectors in $\mathbb{R}^2$ is ————
(A) linearly independent (B) linearly dependent (C) basis of $\mathbb{R}^2$ (D) none of these

24. Let X and Y vectors spaces over the field F with $\dim X = m$ and $\dim Y = n$ then the $\dim Hom(X, Y) =$
(A) mn (B) n (C) n^m (D) m^2

25. All subgroups of an abelian group are ——— subgroups.
    (A) cyclic (B) normal (C) characteristic (D) None of these

26. The set of all solutions to the homogenous equation $Ax = 0$ when A is an $m \times n$ matrix is ———
    (A) Null space (B) Column space (C) Rank (D) None of these

27. If 7 cards are dealt from an ordinary deck of 52 playing cards, what is the probability that at least 1 of them will be a queen?
    (A) 0.4773 (B) 0.4774 (C) 0.4775 (D) 0.4776

28. Let G be an abelian group. Then which one of the following is not true.
    (A) every commutator of G is identity
    (B) iF m is divisor of order of G then G must have subgroup of order m
    (C) centre of G is G itself
    (D) every subgroup of G is cyclic

29. Every group of order ≤ 5 is
    (A) cyclic (B) abelian (C) non abelian (D) none of these

30. Number of non-isomorphic groups of order 8 is ——
    (A) 4 (B) 2 (C) 3 (D) 5

31. Centre of the group of quaternions Q_8 is of order
    (A) 1 (B) 2 (C) 8 (D) 4

32. $\underline{a} \cdot (\underline{b} \times \underline{c})$ is not equal to
    (A) $\underline{a} \cdot (\underline{c} \times \underline{b})$ (B) $(\underline{a} \times \underline{b}) \cdot \underline{c}$ (C) $\underline{b} \cdot (\underline{c} \times \underline{a})$ (D) $\underline{a} \cdot (\underline{a} \times \underline{b})$

33. Let G be a group. Then the derived group G' is subgroup of G
    (A) cyclic (B) abelian (C) normal (D) none of these

34. Let G be a group. Then the factor group G/G is ——-
    (A) abelian (B) cyclic (C) normal (D) none of these

35. Finite simple abelian group are of order
    (A) 4 (B) prime power (C) power of 2 (D) prime number

36. Set of integers Z is
    (A) Field (B) group under multiplication (C) integral domain (D) division ring

37. Set of integers $\mathbb{Z}$ is ——- of the set $\mathbb{Q}$ of rationals.
    (A) prime ideal (B) sub ring (C) maximal ideal (D) none of these

38. Solution set of the equation $1 + \cos x = 0$ is
    (A) $\{\pi + n\pi : n \in \mathbb{Z}\}$ (B) $\{2n\pi : n \in \mathbb{Z}\}$ (C) $\{\frac{\pi}{2} + n\pi : n \in \mathbb{Z}\}$ (D) $\{\pi + 2n\pi : n \in \mathbb{Z}\}$

39. Non-zero elements of a field from a group under
(A) addition (B) multiplication (C) subtraction (D) division

40. Let $\mathbb{Q}$ be the set of rational numbers. Then $\mathbb{Q}(\sqrt{3}) = \{a + b\sqrt{3} : a, b \in \mathbb{Q}\}$ is a vector space over g with dimension
(A) 1 (B) 2 (C) 3 (D) 4

41. Let W be a subspace of the space $\mathbb{R}^3$. if $\dim W = 0$ then W is a
(A) line through the origin 0 (B) plane through the origin 0 (C) entire space R^3 (D) a point

42. Let $P_n(t)$ be a vector space of all polynomials of degree $\leq n$. Then
(A) $\dim P_n(t) = n - 1$ (B) $\dim P_n(t) = n$ (C) $\dim P_n(t) = n + 1$ (D) 2

43. A one to one linear transformation preserves ——————
(A) basis but not dimension (B) basis and dimension (C) dimension but not basis (D) none of these

44. In a group $(\mathbb{Z}, \circ)$ of all integers where $a \circ b = a + b + 1$ for $a, b \in \mathbb{Z}$, the inverse of -3 is
(A) -3 (B) 0 (C) 3 (D) 1

45. The set $\mathbb{Z}$ of all integers is not a vector space over the field $\mathbb{R}$ of real numbers under ordinary addition '$+$', multiplication '$\times$' of real numbers, because
(A) $(\mathbb{Z}, +)$ is a ring (B) $(\mathbb{Z}, +, \times)$ is not a field (C) $(\mathbb{R}, \times)$ is not a group (D) ordinary multiplication of real numbers does not define a scalar multiplication of $\mathbb{Z}$ by $\mathbb{R}$.

46. Let G be an abelian group. Then $\varphi : G \longrightarrow G$ given by ———— is an automorphism.
(A) $\varphi(x) = x^3$ (B) $\varphi(x) = e$ (C) $\varphi(x) = x^2$ (D) $\varphi(x) = x^{-1}$

47. Let G be a group in which $g^2 = 1$ for all g is G. Then G is ————-
(A) Abelian (B) cyclic (C) abelian but not cyclic (D) non abelian

48. Let $G = \langle a, b : b^2 = 1 = a^2, ab = ba^{-1} \rangle$. Then the number of distinct left cosets of $H = \langle b \rangle$ in G is —————
(A) 1 (B) 2 (C) 4 (D) 3

49. A linear transformation $T : U \to V$ is one-to-one if and only if kernel of T is equal to
(A) U (B) V (C) $\{0\}$ (D) $\Im(T)$

50. For a scalar point function $\varphi(x, y, z)$, $textdivgrad\varphi$ is
(A) scalar point function (B) vector point function (C) guage function (D) neither

51. A particle moves along a curve $F = (e^{-1}, 2\cos 3t, 2\sin 3t)$, where t is time. The velocity at $t = 0$ is
(A) $(-1, 0, 6)$ (B) $(-1, -6, 0)$ (C) $(1, 2, 0)$ (D) $(-1, 2, 2)$

52. The coordinates surface for the cylindrical coordinates $x = r\cos\varphi, y = r\sin\varphi, z = z$ are given by
(A) $r = c, \varphi = c$ (B) $r = c_1, \varphi = c, z = c_3$ (C) $r = c_1, z = c_3$ (D) $\varphi = c_2, z = c_3$

53. The metric coefficients in cylindrical coordinates are
(A) $(1, 1, 1)$ (B) $(1, 0, 1)$ (C) $(1, r, 1)$ (D) neither

54. The value of the quantity $\delta_i x_i x_j$ is
(A) x_i (B) zero (C) x_{ij} (D) $x_i x_j$

55. A tensor of rank 5 in a space of 4 dimensions has components
(A) 5 (B) 4 (C) 625 (D) 1024

56. A vector is said to be irrational if
(A) $\bigtriangledown\bar{F} = 1$ (B) $\bigtriangledown\bar{F} = 0$ (C) $\bigtriangledown \times \bar{F} = 0$ (D) none

57. The moment of inertia of a rigid hemisphere of mass M and radius a about a diameter of a base is
(A) $Ma^2/5$ (B) $Ma^2/2$ (C) $2Ma^2/5$ (D) more information needed

58. Radius of gyration of a rigid body of mass $4gm$ having moment of inertia $32gm(cm)^2$ is:
(A) $8(cm)^2$ (B) $2\sqrt{2}cm$ (C) $\sqrt{2}$ (D) $2\sqrt{2}gm$

59. Equation for the ellipsoid of inertia for a rigid body having moments and products of inerti $1_{xx} = 18$units, $1_{yy} = 18$units, $1_{zz} = 36$units, $1_{xy} = -13.5$units, $1_{xz} = 0$, $1_{yz} = 0$
(A) $18(x^2 + y^2 + z^2) - 27xy = 1$ (B) $18(x^2 + y^2 + 2z^2) - 27xy = 1$ (C) $18(x^2 + y^2) + 2z^2 - 27xy = 1$ (D) more information needed

60. The neighbourhood of 0, under the usual topology for the real line r, is
(A) $]\frac{-1}{2}, \frac{1}{2}]$ (B) $]-1, 0]$ (C) $]0, 1]$ (D) $[0, \frac{1}{2}[$

61. Let$A = [0, 1]$ be a subset of R with Euclidean metric Then interior of A is
(A) $[0, 1[$ (B) $]0, 1[$ (C) $[0, 1]$ (D) $]0, 1]$

62. Number of non-isomorphic groups of order 8 is
(A) 5 (B) 2 (C) 3 (D) 4

63. Suppose a and c are real numbers, $c > 0$, and f is defined on $[-1, 1]$ by

$$f(x) = \begin{cases} x^a \sin(x^{-c}) & (if\ x \neq 0), \\ 0 & (if\ x = 0). \end{cases}$$

f is bounded if and only if
(A) $a > 1 + c$ (B) $b > 2 + c$ (C) $a \geq 1 + c$ (D) $a \geq 2 + c$

64. Let $M_{2,3}$ be a vector space of all 2×3 matrices over R. Then dimension of $Hom(M_{2,3}, \mathbb{R}^4)$
(A) 12 (B) 6 (C) 8 (D) 24

65. Let $X = \{a, b, c, d, e\}$. Which one of the following classes of subsets of X is a topology on X.

(A) $T_1 = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ (B) $T_2 = \{X, \phi, \{a, b, c\}, \{a, b, d\}, \{a, b, c, d\}\}$

(C) $T_3 = \{X, \phi, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}\}$ (D) $T_4 = \{\phi, \{a\}, \{a, b\}, \{a, c\}\}$

66. Let $T = \{X, \phi, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}\}$ be a topology on $X = \{a, b, c, d, e\}$ and $A = \{a, b, c\}$ be the subset of X. Then interior of A is

(A) $\{a, b, c\}$ (B) $\{a, b\}$ (C) $\{a\}$ (D) $\{b, c\}$

67. The value of $\sin(\cos^{-1}\frac{\sqrt{3}}{2})$ is

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) 1

68. The smallest field containing set of integers $\mathbb{Z}$ is

(A) $\mathbb{Q}\sqrt{2}$ (B) $\mathbb{Q}$ (C) $\mathbb{Q}\sqrt{6}$ (D) $\mathbb{Q}\sqrt{3}$

69. Let $\mathbb{R}$ be the usual metric space. Then which of the following set is not closed.

(A) Set of integers (B) Set of rational numbers (C) $[0, 1]$ (D) $\left\{1, \frac{1}{2}, \frac{1}{3}, \dots\right\}$

70. Let $\mathbb{R}$ be the usual metric space and $\mathbb{Z}$ be the set of integers, then clouser of $\mathbb{Z}$ is

(A) $\mathbb{Z}$ (B) set of rational number $\mathbb{Q}$ (C) set of real number $\mathbb{R}$ (D) set of natural number $\mathbb{N}$.

71. A subspace A of a complete metric space X is complete if and only if A is

(A) X (B) open (C) closed (D) empty set

72. A subset A of a topological space x is open if and only if A is

(A) A is neighbourhood of each of its point (B) A is neighbourhood of some of its point

(C) A contain all of its limits points (D) A contain all of its boundary points

73. Non-zero elements of a finite filed form ———— group.

(A) non-cyclic (B) An abelian group but not cyclic (C) Non-abelian (D) a cyclic

74. Let R be the co-finite topology. Then R is a

(A) T_0 but not T_1 (B) T_1 but not T_2 (C) T_2 but not T_3 (D) T_3 but not T_1

75. Let $f(x) = \dfrac{x+5}{(x-1)(x-2)}$ then range of f is

(A) set of all real numbers R (B) $R - \{1, 2\}$ (C) R^+ (D) R^-

76. The value of $\int_0^1 xe^y\,dx$ is

(A) -1 (B) 1 (C) c (D) $2c$

77. The solution of the congruence $4x \equiv 5 \pmod 9$ is

(A) $x \equiv 6 \pmod 9$ (B) $x \equiv 7 \pmod 9$ (C) $x \equiv 8 \pmod 9$ (D) $x \equiv 2 \pmod 9$

78. The series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ is convergent for

(A) $|x| < 1$ only (B) $|x| \leq 1$ (C) $-1 < x \leq 1$ (D) all real values of x

79. The general solution of the differential equation $(x^2+y^2)dx - 2xdy = 0$ is
(A) $x^2 - cx - y^2 = 0$, c is an arbitrary constant. (B) $(x-y)^2 = cx$, c is an arbitrary constant. (C) $x+y+2xy = c$, c is an arbitrary constant. (D) $y = x^2 - 2x + c$, c is an arbitrary constant.

80. Let f be defined on $\mathbb{R}$ by setting $f(x) = x$, if x is rational and $f(x) = 1 - x$ if x is irrational. Then
(A) f is continuous on $\mathbb{R}$ (B) f is continuous only at $x = \frac{1}{2}$ (C) f is continuous everywhere except at $x = \frac{1}{2}$ (D) f is discontinuous everywhere.

81. The differential equation $ydx - 2xdy = 0$ represents
(A) a family of straight lines (B) a family of parabola (C) a family of hyperbola (D) a family of circles

82. A particular integral of the differential equation $(D^2+4)y = x$ is
(A) xc^{-2x} (B) $x\cos 2x$ (C) $x\sin 2x$ (D) $\frac{x}{4}$

83. The area of the cardioid $r = a(1+\cos\theta)$ is equal to
(A) $4\pi a^2$ (B) $8\pi a$ (C) $\frac{3\pi a^2}{4}$ (D) $2\pi a^2$

84. The value of $\sqrt{3}\sin x + \cos x$ will be greatest when x is equal to
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{8}$

85. If a particle in equilibrium is subjected to four forces $F_1 = 2\hat{i} - 5\hat{j} + 6\hat{k}$, $F_2 = \hat{i} + 3\hat{j} - 7\hat{k}$, $F_3 = 2\hat{i} - 2\hat{j} - 3\hat{k}$ and F_4 then F_4 is equal to
(A) $-5\hat{i} + 4\hat{j} + 4\hat{k}$ (B) $5\hat{i} - 4\hat{j} - 4\hat{k}$ (C) $3\hat{i} - 2\hat{j} - \hat{k}$ (D) $3\hat{i} + \hat{j} - 10\hat{k}$

86. The function $f(x) = |x| + |x-1|$ is
(A) Continuous and differentiable for $x = 0, x = 1$
(B) Continuous but not differentiable for $x = 0, x = 1$
(C) Discontinuous but differentiable for $x = 0, x = 1$
(D) Neither continuous nor differentiable for $x = 0, x = 1$

87. Evaluate $\lim\limits_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{3}{x^3}}$
(A) 0 (B) e (C) $e^{\frac{1}{3}}$ (D) e^3

88. If $z = x2\tan^{-1}(\frac{x}{y}) - y^2 tan^{-1}(\frac{x}{y})$, then $\frac{d^2z}{dxdy}$ is
(A) $\frac{x^2}{y^2}$ (B) $\frac{x^2+y^2}{x^2-y^2}$ (C) $\frac{x^2-y^2}{x^2+y^2}$ (D) None of these

89. The radius of curvature is
(A) Double the measure of curvature (B) Square of curvature (C) Reciprocal of curvature (D) None of these

90. Suppose a and c are real numbers, $c > 0$, and f is defined on $[-1, 1]$ by

$$f(x) = \begin{cases} x^a \sin(x^{-e}) & (\text{if } x \neq 0), \\ 0 & (\text{if } x = 0). \end{cases}$$

f is continuous if and only if

(A) $a \geq 1$ (B) $a > 1$ (C) $a \geq 0$ (D) $a > 0$

91. The value of $\displaystyle\int_0^\infty \frac{dx}{1+x^2}$ is

(A) $\dfrac{\pi}{2}$ (B) $\dfrac{\pi}{4}$ (C) 0 (D) ∞

92. Which of the following function is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

(A) $f(x) = x^2 + 1$ (B) $f(x) = x^3$ (C) $f(x) = \dfrac{(x^2+1)}{(x^2+2)}$ (D) $f(x) = x^2$

93. $f(z) = \dfrac{1}{z}$ is not uniformly continuous in the region

(A) $0 \leq |z| \leq 1$ (B) $0 \leq |z| < 1$ (C) $0 < |z| < 1$ (D) $0 < |z| \leq 1$

94. $f(z) = z^3 + 3i$ is ....

(A) analytic everywhere except $z = 3i$ (B) analytic everywhere except $z = 0$ (C) analytic everywhere except $z = -3i$ (D) analytic everywhere

95. If C is the circle $|z| = 3$, then $\displaystyle\int_c \frac{dz}{1+z^2}$ is equal to

(A) 3 (B) 2 (C) 0 (D) 1

96. The series $\displaystyle\sum_{n=0}^{\infty} \frac{n^1}{(2i)^n}$ is

(A) convergent (B) absolutely convergent (C) divergent (D) none of these

97. The radius of convergence of $\sinh z$ is

(A) $R = \infty$ (B) $R = 0$ (C) $R = 1$ (D) $R = 2$

98. Four married couples have bought 8 seats in a concert. In how many different ways can they be seated if each couple is to sit together?

(A) 24 (B) 96 (C) 384 (D) None of these

99. A coin is biased so that a head is twice as likely to occur as a tail. if the coin is tossed 3 times, then the probability of getting 2 tails and 1 head is

(A) $\dfrac{1}{9}$ (B) $\dfrac{2}{9}$ (C) $\dfrac{4}{9}$ (D) none of these

100. If X represents the outcome when a die is tossed. Then the expected value of X is

(A) $\dfrac{1}{2}$ (B) $\dfrac{5}{2}$ (C) $\dfrac{7}{2}$ (D) $\dfrac{3}{2}$

ANSWERS

1. A	2. B	3. D	4. C	5. A	6. A	7. B	8. B	9. A	10. C	11. A
12. C	13. B	14. C	15. B	16. D	17. B	18. A	19. D	20. C	21. A	
22. A	23. B	24. A	25. B	26. D	27. D	28. D	29. B	30. D	31. C	
32. A	33. C	34. A	35. D	36. B	37. C	38. D	39. B	40. B	41. D	
42. C	43. B	44. C	45. D	46. B	47. C	48. C	49. C	50. C	51. A	
52. A	53. D	54. D	55. D	56. C	57. C	58. B	59. B	60. A	61. B	
62. A	63. A	64. D	65. C	66. B	67. C	68. B	69. B	69. D	70. A	
71. A	72. A	73. B	74. B	75. A	76. B	77. C	78. A	79. A	80. A	
81. B	82. D	83. C	84. B	85. A	86. A	87. A	88. C	89. C	90. D	
91. A	92. B	93. A	94. D	95. C	96. C	97. A	98. C	99. D	100. D	
