VECTOR SPACE

SUBSPACES

Let V be V.S over field F then $W \subseteq V$ is Subspace if it is itself V.S under SAME operation.

- Every V.S contains two Subspaces always!
- $W \subseteq V$ is subspace of $V(F)$ iff
  - $w_1 + w_2 \in W$
  - $aw_1 \in W ; a \in F$
- $W \subseteq V$ is subspace of $V(F)$ iff $aw_1 + bw_2 \in W ; a, b \in F$
- Intersection of Any Number of Subspace is Subspace
- Sum of two subspaces is also Subspace

EXAMPLES

F (field), $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{C}$, $P(x)$, $P_n(x)$, $F(x)$, M_{mn}, ...

DEFINITION

Let $V \neq \phi$ set with two operations

- vector addition
- Scalar Multiplication

Then V is called vector space over field F if following axioms are satisfied

- $(V, +)$ is abelian group
- $k(v, w) = kv + kw , k \in F$
- $(a+b)v = av + bv , a, b \in F$
- $(ab)v = a(bv) , a, b \in F$
- $1 \cdot v = v , 1 \in F$

VECTOR SPACE

LINER SPAN

Let $S \subseteq V(F)$ then the set of all L.C of elements of S is called linear span of S

Denoted by $\langle S \rangle$

$\langle S \rangle$ is a Subspace of $V(F)$ containing S

And it is "SMALLEST SUBSPACE" containin S.

LINEAR DEPENDENCE

Let $V(F)$ and $v_1, v_2, \ldots v_m \in V$ are said to L.D if "$a_1v_1 + a_2v_2 + \cdots + a_mv_m = 0$" implies not all a_i's are zero

LINEAR COMBINATION

Let $V(F)$ and $v_1, v_2, \ldots v_m \in V$ Any vector of form $a_1v_1 + a_2v_2 + \cdots + a_mv_m$ is L.C of $v_1, v_2, \ldots v_m$.

LINEARLY INDEPENDENT

If $v_1, v_2, \ldots v_m \in V$ are not L.D then called L.I → "all a_i's $= 0$"

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Dimensions

"Number of elements in a basis set of V(F)"

Now we have

- finite Dimension V(F)
- Infinite Dimension V(F)

BASIS

"A set of L·I vectors spanning a V(F) called basis of V(F)"

VECTOR SPACE

IMP. THEOREMS:

- All bases of finite Dimension V(F) have SAME no. of elements!
- Let $\dim V = n$ the any set having $n+1$ vectors are L·D
- Any finite Dim V(F) contains a basis
- Any L·I set of vector in finite V(F) can be extend to basis
- Let $W \subseteq V$ then $\dim W \leq \dim V$ if $\dim W = \dim V$ then $W = V$
- Any Infinite set is L·I if each of its finite subset is L·I
- empty set ϕ is L·I
- Any non-zero vector is L·I