

## SUBSPACES

Let  $V$  be V.S over field  $F$

then  $W \subseteq V$  is Subspace if it is itself V.S under same operation-



- Every V.S contains two Subspaces always!
- $W \subseteq V$  is Subspace of  $V(F)$  iff
  - $w_1, w_2 \in W$
  - $aw_1, cw_2 \in W; a, c \in F$
- $W \subseteq V$  is Subspace of  $V(F)$  iff  $aw_1 + bw_2 \in W; a, b \in F$
- Intersection of Any Number of subSpace is Subspace
- Sum of two subspaces is also SubSpace

## EXAMPLES

$F$  (field),  $\mathbb{R}, \mathbb{Q}, \mathbb{C}, P(x),$   
 $P_n(x) \cap F(x), M_{mn}, \dots$

## DEFINITION

Let  $V \neq \emptyset$  set with two operations

- Vector addition
- Scalar multiplication

Then  $V$  is called

Vector Space over field  $F$  if following axioms are satisfied

- $(V, +)$  is abelian group
- $k(v, w) = kv + kw, k \in F$
- $(a+b)v = av + bv, a, b \in F$
- $(ab)v = a(bv), a, b \in F$
- $1 \cdot v = v, 1 \in F$

# LINER SPAN

Let  $S \subseteq V(F)$   
then the set  
of all L.C  
of elements of  $S$  is called  
linear Span of  $S$



Denoted by  $\langle S \rangle$

# LINEAR DEPENDENCE

Let  $V(F)$  and  $v_1, v_2, \dots, v_m \in V$   
are said to L.D if  
 $a_1v_1 + a_2v_2 + \dots + a_mv_m = 0$   
implies not all  $a_i$ 's are zero



If  $v_1, v_2, \dots, v_m \in V$  are not L.D then called L.I  $\rightarrow$  "all  $a_i$ 's = 0"

$\langle S \rangle$  is a SubSpace  
of  $V(F)$  containing  $S$   
And it is "SMALLEST SUBSPACE"  
containing  $S$ .

# LINER COMBINATION

Let  $V(F)$  and  $v_1, v_2, \dots, v_m \in V$   
Any vector of form  
 $a_1v_1 + a_2v_2 + \dots + a_mv_m$   
is L.C of  $v_1, v_2, \dots, v_m$ .

# Dimensions

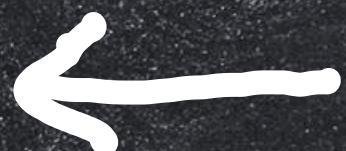
ee Number of elements

in a basis set of  $V(F)$ "

Now we have

→ finite Dimension  $V(F)$

→ Infinite Dimension  $V(F)$



# BASIS

ee A Set of L.I

vectors Spanning

a  $V(F)$  called basis

of  $V(F)_{aa}$

## VECTOR SPACE →

### IMP. THEOREMS:

→ All bases of finite Dimension  $V(F)$  have SAME no. of elements!

→ Let  $\dim V = n$  the any set having  $n+1$  vectors are L.D

→ Any finite Dim  $V(F)$  contains a basis

→ Any L.I set of vectors in finite  $V(F)$

can be extend to basis

→ Let  $W \subseteq V$  then  $\dim W \leq \dim V$   
if  $\dim W = \dim V$  then  $W = V$

→ Any infinite set is L.I  
if each of its finite subset is L.I

→ empty set  $\emptyset$  is L.I

→ Any non-zero vector is L.I

MforMathodology