" We always calculate limit at a point "

LIMITS

" Something which you can approach but can't achieve "

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LIMITS

"A limit $\lim_{x \to a} f(x)$ captures how $f(x)$ behaves as x gets arbitrarily close to a. Whether $f(a)$ itself is undefined (a hole or vertical asymptote) or defined but possibly discontinuous, the limit lets us 'zoom in' and see if there's a single value L that $f(x)$ approaches. Formally, for every $\varepsilon > 0$ there's a $\delta > 0$ so that whenever $\boxed{0 < |x - a| < \delta}$, we have $|f(x) - L| < \varepsilon$."

$$x \to a \Rightarrow |x - a| < \delta$$

then

$$|f(x) - L| < \varepsilon$$

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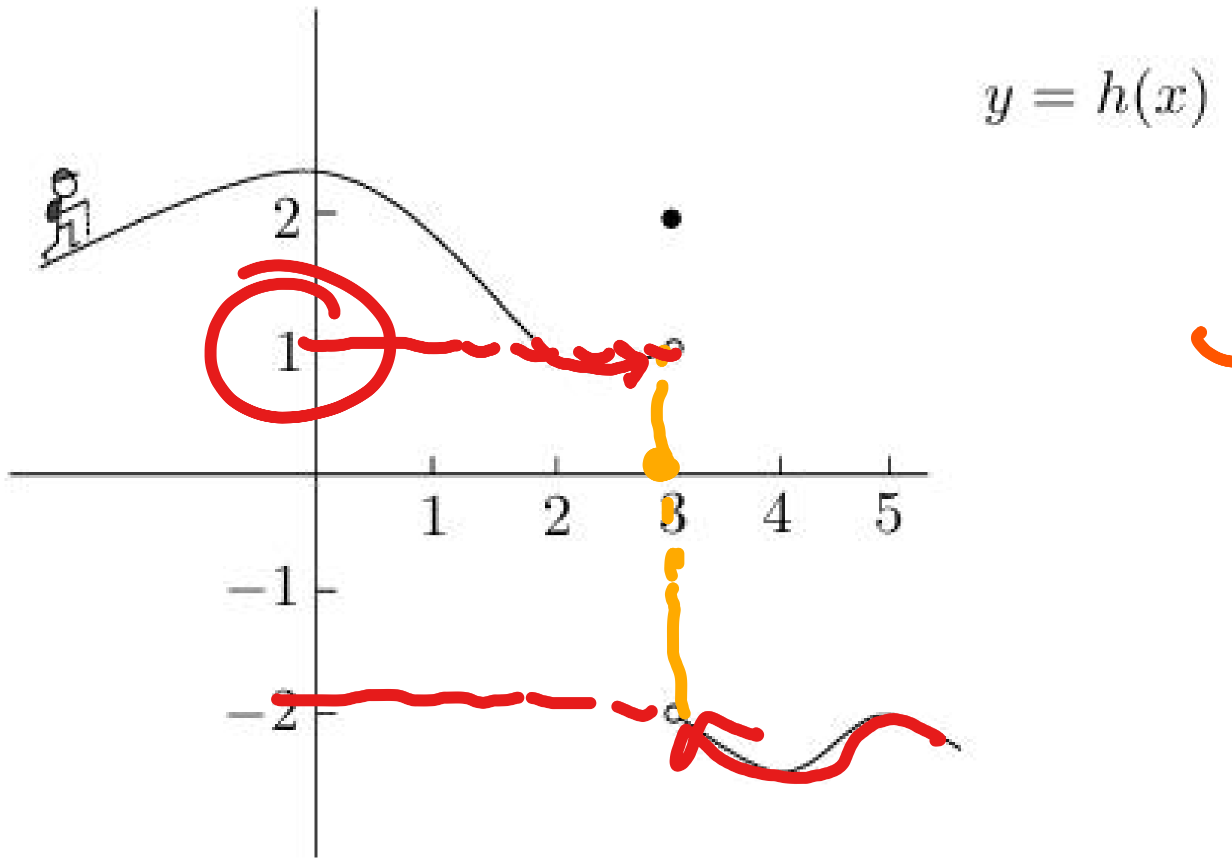
LIMITS

We've seen that limits describe the behavior of a function near a certain point. Think about how you would describe the behavior of $h(x)$ near $x = 3$:



limit of $h(x)$ = DNE

We can summarize our findings from above by writing

$$\lim_{x \to 3^-} h(x) = 1 \quad \text{and} \quad \lim_{x \to 3^+} h(x) = -2.$$

This is taken from BOOK the CALCULUS LIFESAVER

LIMITS

Now, limits don't always exist, as we'll see in the next section. But here's something important: the regular two-sided limit at $x = a$ exists **exactly when** both left-hand and right-hand limits at $x = a$ exist **and are equal to each other**! In that case, all three limits—two-sided, left-hand, and right-hand—are the same. In math-speak, I'm saying that

$$\boxed{\lim_{x \to a^-} f(x) = L} \quad \textbf{and} \quad \boxed{\lim_{x \to a^+} f(x) = L}$$

is the same thing as

$$\lim_{x \to a} f(x) = L.$$

If the left-hand and right-hand limits are not equal, as in the case of our function h from above, then the two-sided limit does not exist. We'd just write

$$\lim_{x \to 3} h(x) \text{ does not exist}$$

or you could even write "DNE" instead of "does not exist."

$a \quad x_n^n = x_1, x_2, x_3, \cdots x_n$

This is taken from BOOK the CALCULUS LIFESAVER

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LIMIT OF SEQUENCE

Seq. is a func. whose domain is the set of +ve integers

$\{x_n\} \to L$

for every $\varepsilon > 0$ then

$\boxed{|x_n - L| < \varepsilon}$

we say that "L" is limit of $\{x_n\}$ $\exists$ natural no N s.t $n > N$

$x_n = \frac{2n-1}{n} = 2$ → covergent seq.

$= \lim_{n\to\infty} \frac{2n}{n} - \frac{1}{n}$

$= \lim_{n\to\infty} 2 - \frac{1}{n}$

x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 ... L

$\boxed{x_n = (-1)^n}$ ⇒ −1, 1, −1, 1, −1, 1, −1, ... Divergent seq.

* $\lim_{n\to\infty} 2n-1$ = deg. to ∞

$= 2(\infty) - 1$

$= \infty - 1$

$= \boxed{\infty}$

* $\lim_{n\to\infty} 2n - n^2$?

deg to −∞

→ Convergent Seq.

$\lim_{\boxed{x\to a}} \{x_n\} = L$ (Koi finite no.)

unique limit

→ Divergent seq.

$\lim_{x\to a} \{x_n\} = \infty / -\infty /$ L is not unique

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LIMIT OF SEQUENCE

Some of the most useful rules about convergent sequences are summarized below:

1. Every convergent sequence is **bounded**; that is, there exists a positive number, M, such that the absolute value of every term of the sequence is no greater than M. [The converse of this statement is not true; for example, the sequence (x_n) with $x_n = (-1)^n$ is bounded, but not convergent.]
2. If a sequence is monotonic and bounded, then it's convergent.
3. If k is a constant, and (a_n) converges to A, then $(ka_n) \to kA$.
4. If (a_n) converges to A and (b_n) converges to B, then

$$\boxed{(a_n + b_n)} \to \boxed{A + B},$$
$$\boxed{(a_n - b_n)} \to \boxed{A - B},$$
$$\boxed{(a_n b_n)} \to AB, \text{ and}$$
$$\boxed{\left(\frac{a_n}{b_n}\right)} \to \frac{A}{B} \quad \text{(assuming that } \boxed{B \neq 0}\text{).}$$

5. (a) If k is a positive constant, then $\left(\frac{1}{n^k}\right) \to 0$.
   (b) If $|k| > 1$, then $\left(\frac{1}{k^n}\right) \to 0$.

$$\lim_{n\to\infty} \frac{2}{n} = 2 \left[\lim_{n\to\infty} \frac{1}{n}\right]$$
$$\boxed{= 2(0)}$$

$$\left\{\frac{1}{2^n}\right\} \to 0 \qquad \left\{\frac{1}{3^n}\right\} \to 0$$

This is taken from BOOK Cracking the GRE Subject Mathematics

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LIMITS

Find the value of each of these limits (if they exist):

(a) $\lim\limits_{x\to\infty} \dfrac{2x^2-x+1}{x^2+4}$ (b) $\lim\limits_{x\to\infty} \dfrac{2x^2-x+1}{x^3+4}$ (c) $\lim\limits_{x\to-\infty} (\arctan x)$ $\boxed{\text{(d) } \lim\limits_{x\to 0} \dfrac{1}{x}}$ $= \text{DNE}$

$$\lim_{x\to 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x\to 0^+} \frac{1}{x} = \infty$$

So,

$$\boxed{\text{LHL} \neq \text{RHL}}$$

$$\lim_{x\to\infty} \frac{\dfrac{2x^2-x+1}{x^2}}{\dfrac{x^2+4}{x^2}} = \frac{2 - \cancelto{0}{\dfrac{1}{x}} + \cancelto{0}{\dfrac{1}{x^2}}}{1 + \cancelto{0}{4/x^2}} = \frac{2}{1} = 2$$

$$\lim_{x\to\infty} \frac{\dfrac{2x^2-x+1}{x^3}}{\dfrac{x^3+4}{x^3}} = \frac{\cancelto{0}{\dfrac{2}{x}} - \cancelto{0}{\dfrac{1}{x^2}} + \cancelto{0}{\dfrac{1}{x^3}}}{1 + \cancelto{0}{4/x^3}} = \frac{0}{1} = 0$$

$$\lim_{x\to-\infty} (\tan^{-1} x) = \tan^{-1}(-\infty) = -\frac{\pi}{2}.$$

This table is taken from BOOK Cracking the GRE Subject Mathematics

LIMIT OF FUNCTION

x approaches a from below and $f(x) \to L$ then it is "Left Hand Limit"

x approaches a from above and $f(x) \to L$ then "Right Hand limit"

$f(x) \to L$ as $\boxed{x \to a}$

written as

$$\boxed{\lim_{x \to a} f(x) = L}$$

when

$$\boxed{LHL = RHL}$$

then limit Exists!

LIMIT OF FUNCTION

$x_n = e^{-n} \cos n^n$

$-1 \leq \cos n^n \leq 1$

$\lim_{n\to\infty} -e^{-n} \leq \boxed{e^{-n} \cos n^n} \leq e^{-n}$

When working with limits of functions, the following rules are often used:

1. $\boxed{\lim_{x\to a} x = a}$, $\boxed{\lim_{x\to a} k = k}$ (for any constant k), and $\lim_{x\to a} x^n = a^n$.
2. If $\lim_{x\to a} f(x) = L_1$ and $\lim_{x\to a} g(x) = L_2$, then

$$\lim_{x\to a} [f(x) + g(x)] = L_1 + L_2$$

$$\lim_{x\to a} [f(x) - g(x)] = L_1 - L_2$$

$$\lim_{x\to a} [f(x)g(x)] = L_1 L_2$$

$$\lim_{x\to a} \left[\frac{f(x)}{g(x)}\right] = \frac{L_1}{L_2} \text{ (assuming that } L_2 \neq 0\text{).}$$

3. To say that $\boxed{\lim_{x\to a} f(x) = L}$ means that for every sequence (x_n) converging to a, the sequence $(f(x_n))$ converges to L.
4. Assume that $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} h(x) = L$. If there is a positive number δ such that $f(x) \leq g(x) \leq h(x)$ for all x satisfying $0 < |x - a| < \delta$, then $\lim_{x\to a} g(x) = L$. This, again, is the Sandwich (or Squeeze) theorem.

$x_n \to a$

$f(x_n) \to L$

$\lim_{n\to\infty} e^{-n} \cos n^n = 0$

$f(x) \to L$

$g(x) \to L$

$x)$

This is taken from BOOK Cracking the GRE Subject Mathematics

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LIMITS

Evaluate each of the following limits:

(a) $\lim_{x\to 1} \frac{x-1}{\sqrt{x}+1}$ (b) $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$ (c) $\lim_{x\to 1-} \frac{x-1}{|x-1|}$ (d) $\boxed{\lim_{x\to 1-}[x-1]}$

bracket func

$|a| = a, a>0$

$|a| = -a, a<0$

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x}+1} = \frac{1-1}{\sqrt{1}+1} = \frac{0}{1+1} = \frac{0}{2} = 0$$

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}\frac{(\sqrt{x}+1)}{(\sqrt{x}+1)} = \frac{\cancel{(x-1)}(\sqrt{x}+1}{\cancel{x-1}} = \sqrt{1}+1 = 2$$

$$\lim_{x\to 1^-} \frac{x-1}{|x-1|} = \frac{\cancel{x-1}}{-\cancel{(x-1)}} = -1$$

$$\lim_{x\to 1^-} [x-1] = -1$$