LAPLACE TRANSFORM

"1'K f n . change by Integeation" → Say Main

$f(x,y) = x^2y^3$

$\int_0^1 x^2y^3\,dx = y^3\left|\frac{x^3}{3}\right|_0^1$

$= \frac{y^3}{3}\{(1)^3 - 0^3\}$

$\boxed{= y^3/3}$

T_ie of = - gal Tq s_e ati n

Constant term bhar

linearity property

$\mathcal{L}\{1\}$

$\mathcal{L}\{1\} = \int_0^\infty e^{-st}(1)\,dt$

$= \frac{e^{-t\infty}}{-s}$

$= \frac{e^{-\infty}}{s} \quad \frac{e^{-0}}{}$

$\boxed{\frac{1}{s}}$

t}

$\mathcal{L}\{t\} = \int_0^\infty \underset{\text{II}}{e^{-st}}\ \underset{\text{I}}{t}\,dt$

$= t\,\frac{e^{-st}}{-s}\Big|_0^\infty - \int_0^\infty \frac{e^{-st}}{-s}\,dt$

$= \frac{1}{s}\left(\frac{1}{s}\right) \quad \boxed{= \frac{1}{s^2}}$

$\mathcal{L}\{e^{-3t}\}$

$e^{-3t}\}$

$= \int_0^\infty e^{-st}\,e^{-3t}\,dt$

$= \int_0^\infty e^{-st-3t}\,dt$

$= \int_0^\infty e^{-(s+3)t}\,dt$

$\left(\frac{e^{-(s+3)}}{-s+3)}\right)_0^\infty$

$\boxed{= \frac{1}{s+3}}$

If f is defined on $[0,\infty)$ then integr

$$\boxed{\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st}\,\boxed{f(t)}\,dt}$$

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LAPLACE TRANSFORM

THEOREM 7.1.1 **Transforms of Some Basic Functions**

(a) $\mathscr{L}\{1\} = \dfrac{1}{s}$

(b) $\mathscr{L}\{t^n\} = \dfrac{n!}{s^{n+1}}, \quad n = 1, 2, 3, \ldots$

(c) $\mathscr{L}\{e^{at}\} = \dfrac{1}{s - a}$

(d) $\mathscr{L}\{\sin kt\} = \dfrac{k}{s^2 + k^2}$

(e) $\mathscr{L}\{\cos kt\} = \dfrac{s}{s^2 + k^2}$

(f) $\mathscr{L}\{\sinh kt\} = \dfrac{k}{s^2 - k^2}$

(g) $\mathscr{L}\{\cosh kt\} = \dfrac{s}{s^2 - k^2}$

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LAPLACE TRANSFORM

THEOREM 7.2.1 **Some Inverse Transforms**

$$\textbf{(a)}\quad 1 = \mathscr{L}^{-1}\left\{\frac{1}{s}\right\}$$

$$\textbf{(b)}\quad t^n = \mathscr{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\},\quad n = 1, 2, 3, \dots \qquad \textbf{(c)}\quad e^{at} = \mathscr{L}^{-1}\left\{\frac{1}{s-a}\right\}$$

$$\textbf{(d)}\quad \sin kt = \mathscr{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\} \qquad \textbf{(e)}\quad \cos kt = \mathscr{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\}$$

$$\textbf{(f)}\quad \sinh kt = \mathscr{L}^{-1}\left\{\frac{k}{s^2-k^2}\right\} \qquad \textbf{(g)}\quad \cosh kt = \mathscr{L}^{-1}\left\{\frac{s}{s^2-k^2}\right\}$$

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LAPLACE TRANSFORM

- $f(t) = \begin{cases} 4 & 0<t<2 \\ 0 & t \geq 2 \end{cases}$

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

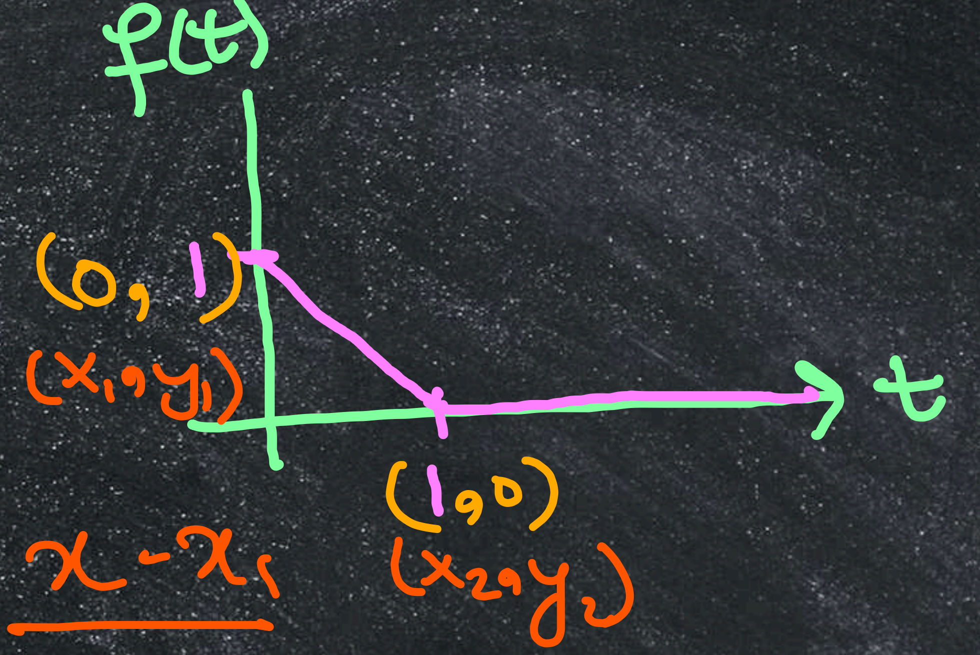
$$= \int_0^2 e^{-st} f(t)\, dt + \cancel{\int_2^\infty e^{-st} f(t)\, dt}$$

$$= \int_0^2 e^{-st}\, 4\, dt$$

$$= 4 \int_0^2 e^{-st}\, dt$$

$$= 4 \left| \frac{e^{-st}}{-s} \right|_0^2$$

$$= 4 \left[\frac{e^{-2s}}{-s} + \frac{1}{s} \right] \Rightarrow \boxed{\frac{4}{s}\left[1 - e^{-2s}\right]}$$

- 

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-1}{0-1} = \frac{x-0}{1-0}$$

$$\frac{y-1}{-1} = x$$

$$y - 1 = -x$$

$$y = -x + 1$$

$$\boxed{f(t) = 1 - t}$$

$-(t) - \{ \; - \; 0 \quad t<1$

$t \geq 1$

$$\boxed{\int_0^1 e^{-} \; (\; -t)\, dt} -$$

$$\cancel{\int^\infty e^{-st} (\;)} -$$

$$\boxed{- \frac{\;}{2} + \frac{1}{s} - \frac{1}{s^2}}$$

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