INTEGRATION

ulta

$\boxed{\text{Anti-Differentiation}}$

$\int f(x) \boxed{dx}$

integral

integrand

variable w.r.t integrate

$\int$ is not unique!

If $F(x)$ And $G(x)$ are both anti-derivative of $f(x)$ then only differ by constant!

$F(x) = x^3 + 4$

$G(x) = x^3 + 5$

$f(x) = 3x^2$

An $\int$ of $f(x)$ is $\boxed{F(x)}$ then $\frac{d}{dx}F(x)$ is $f(x)$ ... !

$$\int f(x) = \boxed{F(x)} + C$$

$\frac{d}{dx}F(x) = f(x)$

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INDEFINITE INTEGRATION
POWER RULE

$$\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C$$

$$\int [x]^2\,dx = \int (x)^2 (1)\,dx = \frac{x^3}{3} + c$$

$$\frac{d}{dx} x = 1$$

$$\int \underbrace{(x^2+1)}_{\frac{d}{dx}(x^2+1) = 2x}{}^3 x\,dx = \frac{1}{2}\int (x^2+1)^3 (2x)\,dx = \frac{(x^2+1)^4}{(2)4} + c$$

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INDEFINITE INTEGRATION

POWER RULE

Evaluate $\int \sqrt{2x+1}\,dx.$

$$= \int (2x+1)^{1/2}\,dx$$

$$= \frac{1}{2}\int (2x+1)^{1/2}(2)\,dx$$

$$= \frac{1}{\cancel{2}}\,\frac{(2x+1)^{\frac{3}{2}}}{\frac{3}{\cancel{2}}} + C$$

$$= \frac{1}{3}(2x+1)^{3/2} + C$$

Find $\int \frac{x}{\sqrt{1-4x^2}}\,dx.$

$$= \int (1-4x^2)^{-1/2}\,x\,dx$$

$$= -\frac{1}{8}\int (1-4x^2)^{-1/2}(-8x)\,dx$$

$$= -\frac{1}{8}\,\frac{(1-4x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$\int \frac{2z\,dz}{\sqrt[3]{z^2+1}}.$

$$= \int (z^2+1)^{-1/3}(2z)\,dz$$

$$= \frac{(z^2+1)^{-1/3+1}}{-\frac{1}{3}+1} + C$$

INDEFINITE INTEGRATION
LN RULE

$$\int \frac{1}{f(x)} f'(x)\, dx = \ln|f(x)| + C$$

$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x}\left(\frac{1}{x}\right) dx = \ln|\ln x| + C$$

$$\int \frac{\cot x}{\ln \sin x} dx = \int \frac{1}{\boxed{\ln \sin x}} (\cot x)\, dx = \ln|\ln \sin x| + C$$

$$\frac{d}{dx} \ln \sin x = \frac{1}{\sin x} \cos x = \cot x$$

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INDEFINITE INTEGRATION
LN RULE

$\int \tan x \, dx.$ $= \int \frac{\sin x}{\cos x} dx = \int \frac{1}{\cos x} (\sin x) \, dx$

$= -\int \frac{1}{\cos x} (-\sin x) \, dx$

$= -\ln|\cos x| + C$

$= \ln|\cos x|^{-1} + C$

$\boxed{= \ln|\sec x| + C}$

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INDEFINITE INTEGRATION
BY SUBTITUTION

Jab power Rule Aur ln Rule se
Baat na banay tou Yeh lagao!

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INDEFINITE INTEGRATION
BY SUBTITUTION

Find $\int \sqrt{1+x^2}\, x^5\, dx.$

$= \int (\underbrace{1+x^2})^{1/2} x^5 dx$

Let $\boxed{1+x^2 = t} \Rightarrow x^2 = t-1$

$2x\,dx = dt$

$dx = \frac{dt}{2x}$

$= \int (t)^{1/2} x^5 \frac{dt}{2x}$

$= \frac{1}{2}\int t^{1/2} x^4 dt$

$= \frac{1}{2}\int t^{1/2}(t-1)^2 dt$

$= \frac{1}{2}\int t^{1/2}(1+t^2-2t)\, dt$

$= \frac{1}{2}\int t^{1/2} + t^{5/2} + 2t^{3/2} dt$

$= \frac{1}{2}\left[\frac{t^{3/2}}{\frac{3}{2}}\right] + \frac{1}{2}\frac{t^{7/2}}{\frac{7}{2}} - \frac{t^{5/2}}{\frac{5}{2}}$

$= \frac{1}{3}t^{3/2} + \frac{1}{7}t^{7/2} - \frac{2}{5}t^{5/2}$

$= \boxed{\frac{1}{3}(1+x^2)^{3/2} + \frac{1}{7}(1+x^2)^{7/2} - \frac{2}{5}(1+x^2)^{5/2} + C}$

Evaluate $\int x\sqrt{2x+1}\, dx.$ $\rightarrow \int (2x+1)^{1/2} \cdot x\, dx$

Let $2x+1 = t \Rightarrow \frac{t-1}{2} = x$

$2dx = dt$

$dx = \frac{dt}{2}$

$= \int t^{1/2} \cdot x \frac{dt}{2}$

$= \frac{1}{2}\int t^{1/2} \cdot x\, dt$

$= \frac{1}{2}\int t^{1/2} \cdot \frac{t-1}{2} dt$

$= \frac{1}{4}\int t^{3/2} - \frac{1}{4}\int t^{1/2} dt$

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INDEFINITE INTEGRATION
BY TRIGO SUBTITUTION

$$= \frac{1}{4}\frac{t^{\frac{5}{2}}}{\frac{5}{2}} - \frac{1}{4}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

If the integrand contains	Make this substitution
$\sqrt{a^2 - u^2}$	$u = a\sin\theta$ (and $du = a\cos\theta\, d\theta$)
$\sqrt{a^2 + u^2}$	$u = a\tan\theta$ (and $du = a\sec^2\theta\, d\theta$)
$\sqrt{u^2 - a^2}$	$u = a\sec\theta$ (and $du = a\sec\theta\tan\theta\, d\theta$)

INDEFINITE INTEGRATION
BY TRIGO SUBTITUTION

$$\int \frac{1}{x^2\sqrt{x^2+1}}\,dx$$

$$= \int \frac{1}{x^2\sqrt{(x)^2+(1)^2}}\,dx$$

Let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$$= \int \frac{1}{\tan^2\theta\sqrt{\tan^2\theta+1}}\sec^2\theta\, d\theta$$

$$= \int \frac{\sec^2\theta}{\tan^2\theta\sqrt{\sec^2\theta}}\,d\theta$$

$$= \int \frac{\sec^2\theta}{\tan^2\theta\sec\theta}\,d\theta$$

$$= \int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$= \int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$= \int \frac{1}{\cos\theta}\frac{\cos^2\theta}{\sin^2\theta}\,d\theta$$

$$= \int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$$= \int \sin^{-2}\theta\cos\theta\, d\theta$$

$$= \frac{\sin^{-1}\theta}{-1} + C$$

$$= -\frac{1}{\sin\theta} + C$$

$$\boxed{= -\operatorname{cosec}\theta + C}$$

" TRIGO ":

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

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INDEFINITE INTEGRATION

INTEGRATION BY PARTS

$$\int \underset{\mathrm{I}}{u}\,\underset{\mathrm{II}}{v}$$

$$= u\int v - \int\left(\int v\right)\frac{du}{dx} + C$$

I → inverse

L → log

A → algebraic

T → Trigo

E → exponent

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INDEFINITE INTEGRATION
INTEGRATION BY PARTS

$\int \underset{\mathrm{II}}{x} \underset{\mathrm{I}}{\log x}\, dx$

$= \log x \int x - \int \left(\int x \frac{d}{dx} \log x \right) + C$

$= \frac{x^2}{2} \log x - \int \frac{x^2}{2} \left(\frac{1}{x} \right)$

$= \frac{x^2}{2} \log x - \frac{1}{2} \int x \, dx$

$= \frac{x^2}{2} \log x - \frac{1}{2} \cdot \frac{x^2}{2}$

$= \frac{x^2}{2} \log x - \frac{1}{4} x^2 + C$

$\int \underset{\mathrm{II}}{1} \cdot \underset{\mathrm{I}}{\ln x}\, dx.$

$= \ln x \int 1 - \int \left(\int 1 \frac{d}{dx} \ln x \right) + C$

$= x \ln x - \int x \left(\frac{1}{x} \right) dx$

$= x \ln x - \int 1 \, dx$

$\boxed{= x \ln x - x + C}$

$\int \underset{\mathrm{I}}{t^2} \underset{\mathrm{II}}{e^t}\, dt.$

ILATE

$= t^2 \int e^t - \int \left(\int e^t \frac{d}{dt} t^2 \right) dt$

$= t^2 e^t - \int e^t (2t)\, dt$

$= t^2 e^t - 2 \int t e^t \, dt$

$= t^2 e^t - 2 \left[t e^t - \int e^t \cdot 1 \cdot dt \right]$

$= t^2 e^t - 2 \left[t e^t - \int e^t \, dt \right]$

$\boxed{= t^2 e^t - 2 \left[t e^t - e^t \right] + C}$

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INDEFINITE INTEGRATION

TRIGO FORMULAS

$\int \sin x \, dx = -\cos x + C$

$\int \cos x \, dx = \sin x + C$

$\int \tan x \, dx = \ln|\sec x| + C$

$\int \cot x \, dx = \ln|\sin x| + c$

$\int \sec^2 x \, dx = \tan x + C$ ←

$\int \operatorname{cosec}^2 x \, dx = -\cot x + C$

$\int \sec x \tan x \, dx = \sec x + C$

$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$

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INDEFINITE INTEGRATION

TRIGO FORMULAS

Find $\int \sec^2(5x + 1) \cdot 5\, dx$ $= \boxed{\tan(5x+1) + C}$

Find $\int \cos(7\theta + 3)\, d\theta.$ $= \frac{1}{7}\int \cos(7\theta+3)(7)\, d\theta$

$= \boxed{\frac{1}{7}\sin(7\theta+3) + C}$

$\int x^2 \cos x^3\, dx$ $= \int \cos x^3 \cdot x^2 = \frac{1}{3}\int \cos x^3 \cdot 3x^2 dx$

$\boxed{= \frac{1}{3}\sin x^3 + C}$

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INDEFINITE INTEGRATION

TRIGO FORMULAS

Find $\int x^3 \cos(x^4 + 2)\, dx$.

$= \int \cos(x^4+2) \cdot x^3\, dx$

$= \frac{1}{4} \int \cos(x^4+2) \cdot 4x^3\, dx$

$$\boxed{= \frac{1}{4} \sin(x^4+2) + C}$$

Trigo kay Half Angle formulas $\Rightarrow$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\sin^2\theta = \frac{1-\cos 2\theta}{}$

INDEFINITE INTEGRATION

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TRIGO SQUARE FORMULAS

$$\int \sin^2 x \, dx$$

$$= \int \frac{1-\cos 2x}{2} dx$$

$$= \frac{1}{2}\left[\int 1 \, dx - \int \cos 2x \, dx\right]$$

$$= \frac{1}{2}\left[x - \frac{1}{2}\int \cos 2x (2) \, dx\right]$$

$$\boxed{= \frac{1}{2}x - \frac{1}{4}\sin 2x + C}$$

$$\int \cos^2 x \, d\theta = \boxed{\frac{1}{2}x + \frac{1}{4}\sin 2x + C}$$

$$\int \operatorname{cosec}^2 x \, dx = \boxed{-\cot x + C}$$

$$\int \sec^2 x \, dx = \boxed{\tan x + C}$$

$$\int \cot^2 x \, dx =$$

$$= \int \operatorname{cosec}^2 x - 1 \, dx$$

$$= \boxed{-\cot x - x + C}$$

$$\int \tan^2 \theta \, d\theta$$

$$= \int (\sec^2 \theta - 1) \, d\theta$$

$$= \int \sec^2 \theta \, d\theta - \int 1 \, d\theta$$

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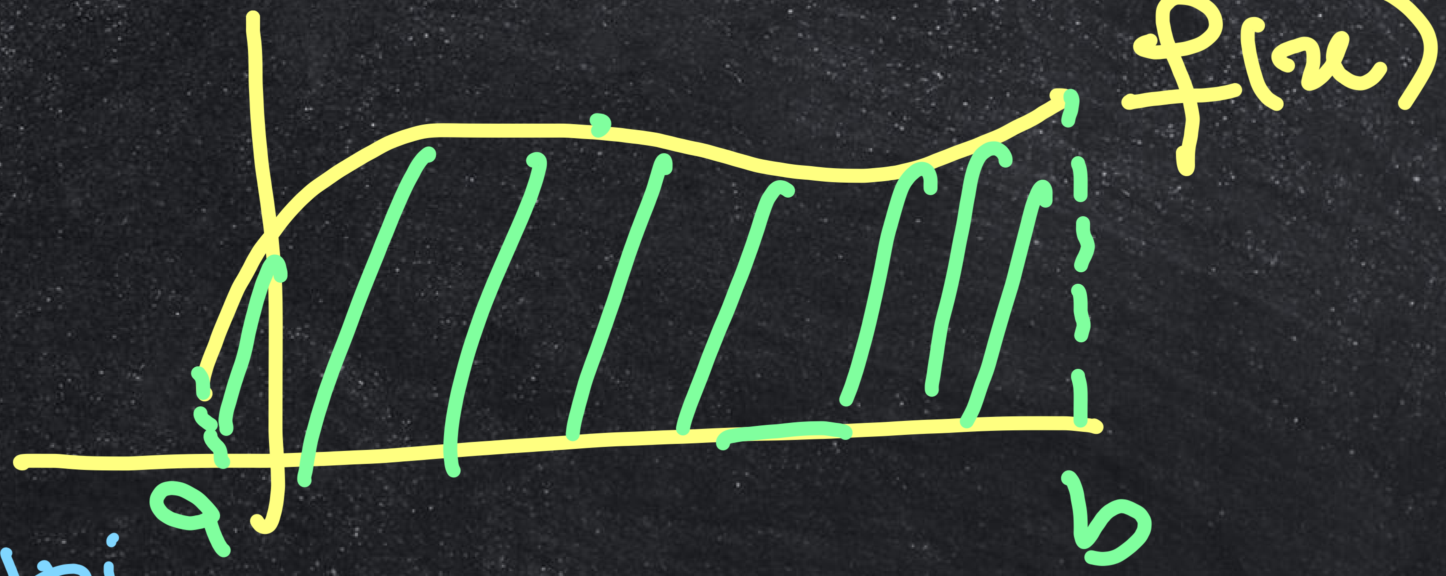
$= \boxed{\tan\theta - \theta + C}$

"AREA UNDER CURVE"

DEFINITE INTEGRATION

$$\int_{a=}^{b=} \boxed{f(x)}\, dx$$



Solve wasey hi karain gy as he done in I.I but in then we must put values of limits!

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DEFINITE INTEGRATION
FUNDAMENTAL THEOREM OF CALCULUS

the area under a curve. The method is based on the **fundamental theorem of calculus**, which links the apparently unrelated concepts of the slope of a curve and the area under it.

The area bounded by the x-axis and the curve $y = f(x)$, from $x = a$ to $x = b$, is denoted by

$$\int_a^b f(x)dx$$

which is called the **definite integral of f from a to b**. The numbers a and b are called the **limits of integration**. The fundamental theorem of calculus tells us that the definite integral of f can be computed by first determining an indefinite integral (antiderivative) of f. Let $f(x)$ be a continuous function on the interval $[a, b]$, and let $F(x)$ be an antiderivative of $f(x)$; then:

$$\int_a^b f(x)dx = F(b) - F(a)$$

The expression $F(b) - F(a)$ is written more compactly as $F(x)\Big]_a^b$, so this equation becomes:

$$\int_a^b f(x)dx = F(x)\Big]_a^b$$

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DEFINITE INTEGRATION
QUESTIONS

$$\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx. = \int (x^3+1)^{1/2}(3x^2)\,dx = \left[\frac{(x^3+1)^{3/2}}{\frac{3}{2}}\right]_{-1}^{1}$$

$$= \frac{2}{3}\left\{(1)^3+1)^{3/2} - ((-1)^3+1)\right\}$$

$$\boxed{= \frac{2}{3}(2)^{3/2}}$$

Find the area of the region bounded by the x-axis, the line $x = 4$, and the curve $y = \sqrt{x}$.

$$F(x) = \int \sqrt{x}\,dx \qquad F(x) = \frac{2}{3}x^{3/2}$$

$$F(x) = \frac{(x)^{3/2}}{\frac{3}{2}}$$

$$\boxed{F(4) = \frac{2}{3}(4)^{3/2}}$$

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DEFINITE INTEGRATION

QUESTIONS

$$\int_1^e \frac{\ln x}{x}\,dx.$$

$$\int_1^2 \frac{dx}{(3-5x)^2}.$$

$$\int_{\pi/4}^{\pi/2} \cot\theta \csc^2\theta \, d\theta$$

$$\int_1^e (\ln x)^1 \left(\frac{1}{x}\right) dx$$

$$= \left[\frac{(\ln x)^2}{2}\right]_1^e$$

$$= \frac{1}{2}\left[(\ln \overset{1}{e})^2 - (\ln \overset{0}{1})^2\right]$$

$$\boxed{= \frac{1}{2}}$$

$$= \frac{1}{-5}\int_1^2 (3-5x)^{-2}(-5)\,dx$$

$$= \frac{-1}{5}\left[\frac{[3-5x]^{-1}}{-1}\right]_1^2$$

$$= \frac{1}{5}\left[3-5(2)-3+5(1)\right]^{-1}$$

$$= \frac{1}{5}\left[\cancel{3}-10-\cancel{3}+5\right]^{-1} = \frac{1}{25}$$

$$= -\int_{\pi/4}^{\pi/2} \operatorname{cosec}^2\theta(-\cot\theta)\, d\theta$$

$$= -\left[\frac{\operatorname{cosec}^3\theta}{3}\right]_{\pi/4}^{\pi/2}$$

$$= -\frac{1}{3}\left[\operatorname{cosec}^3\left(\frac{\pi}{2}\right) - \operatorname{cosec}^3\left(\frac{\pi}{4}\right)\right].$$

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