DERIVATIVES

Definition

"Yeh bi limits taran ek hi point per define Hai"

for a real valued func. f of single variable has derivative f' whose value at x is

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Kam Ki Bata

It tells us "how fast a func. changing at specific point"

If a derivative exists then called differentiable func.

Relation with continous func.

Any func with sharp bend, turn or cusp can be continous but can't be differentiable!

So, every Differentiable func. is also

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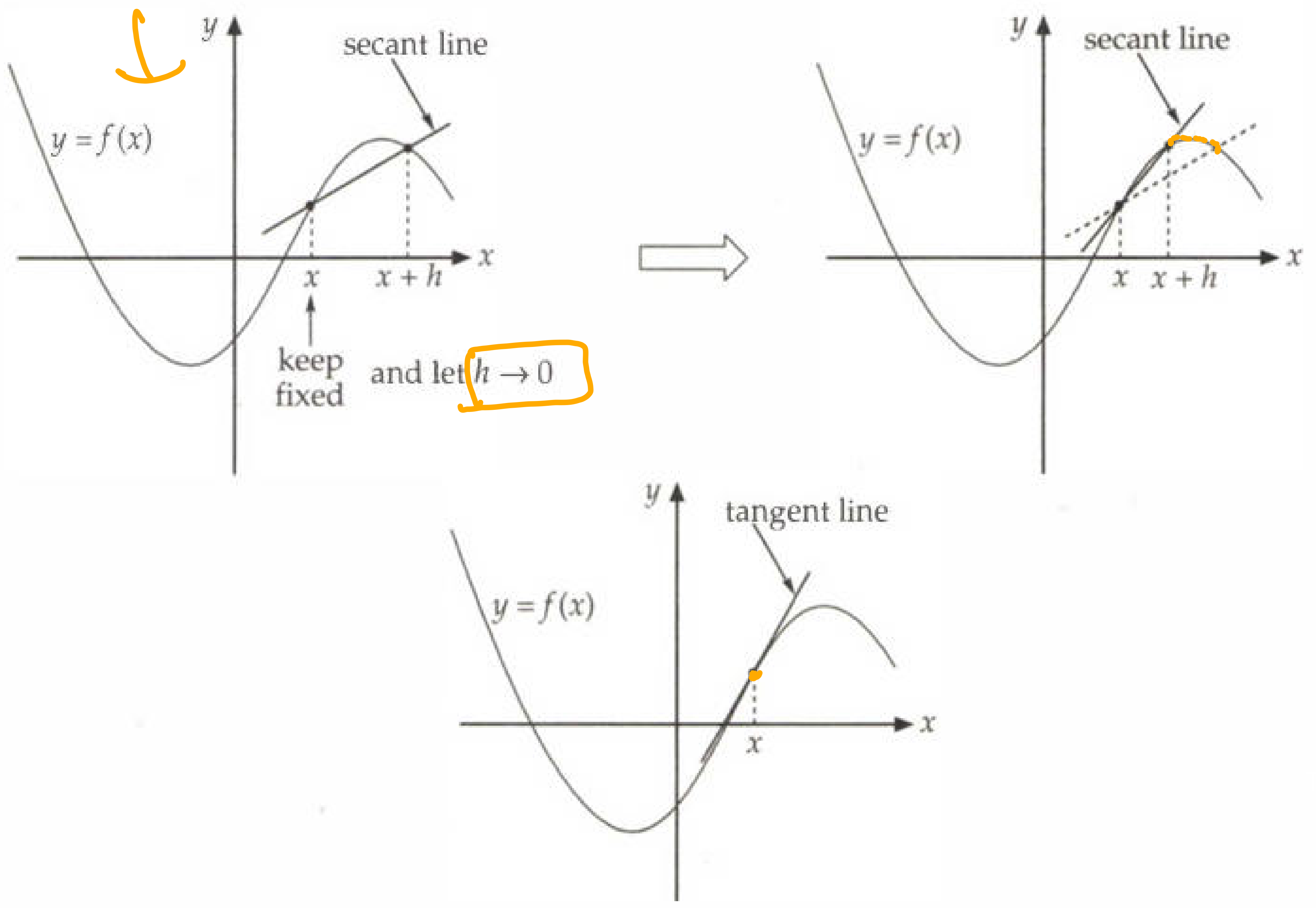
continuous func
But converse is not true



The value of the derivative, $f'(x)$, gives the slope of the tangent line, which is, by definition, the slope of the curve at x.

This is taken from BOOK Cracking the GRE Subject Mathematics

Take

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Now

$\boxed{\Delta x = x_{new} - x}$, $y = f(x)$

$\Delta x + x = x_{new}$

$$= \lim_{\Delta x \to 0} \frac{f(x_{new}) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

[redacted]

$$\boxed{f'(x) = \frac{dy}{dx}}$$

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Geometrically:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

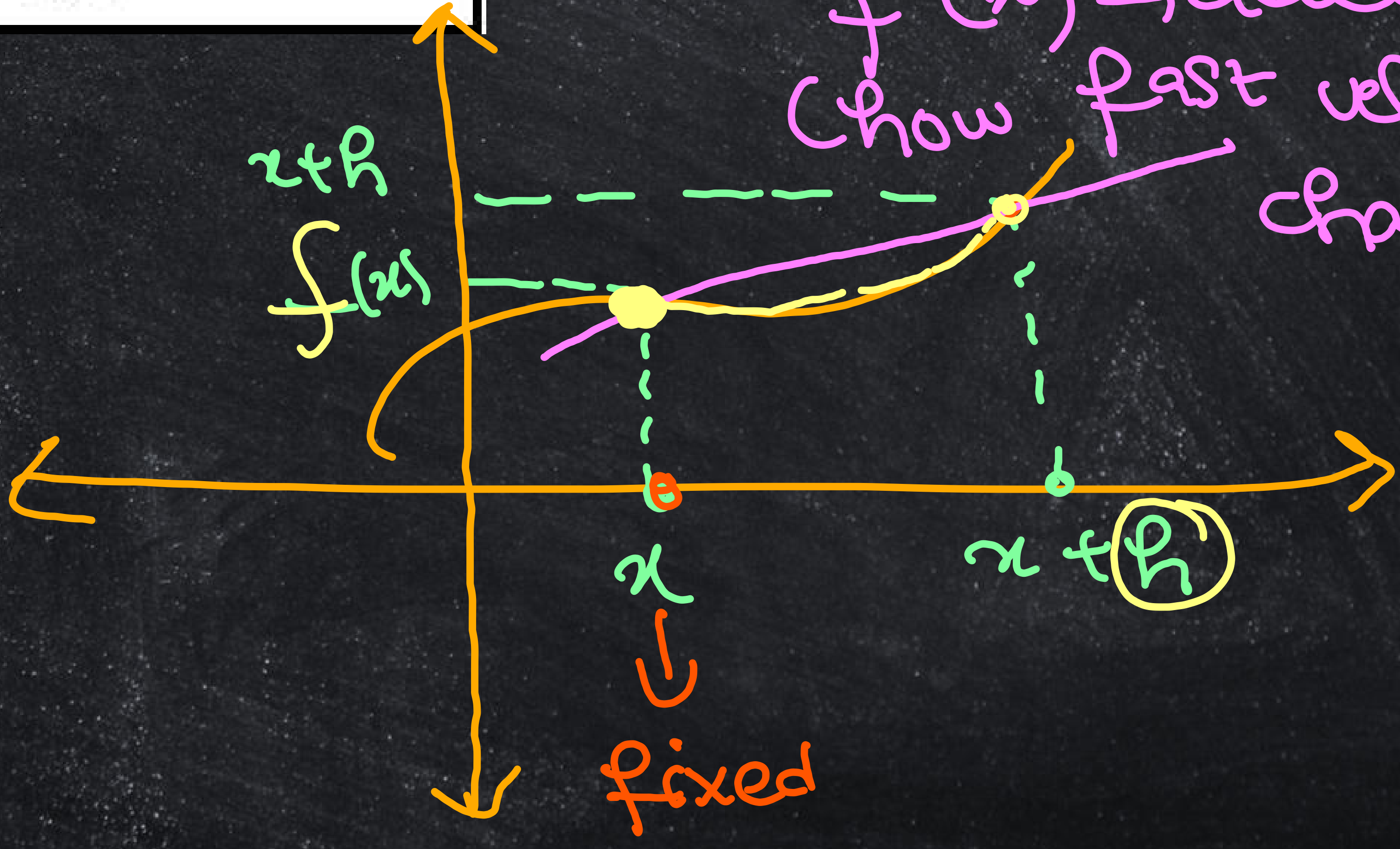
→ This formula gives the slope of tangent line at $(x, f(x))$

→ Instantaneous Rate of change of time x

physically:

$f(x)$ represent position at x

- $f'(x) \to$ velocity at x (how fast position change)
- $f''(x) \to$ acceleration (how fast velocity change)



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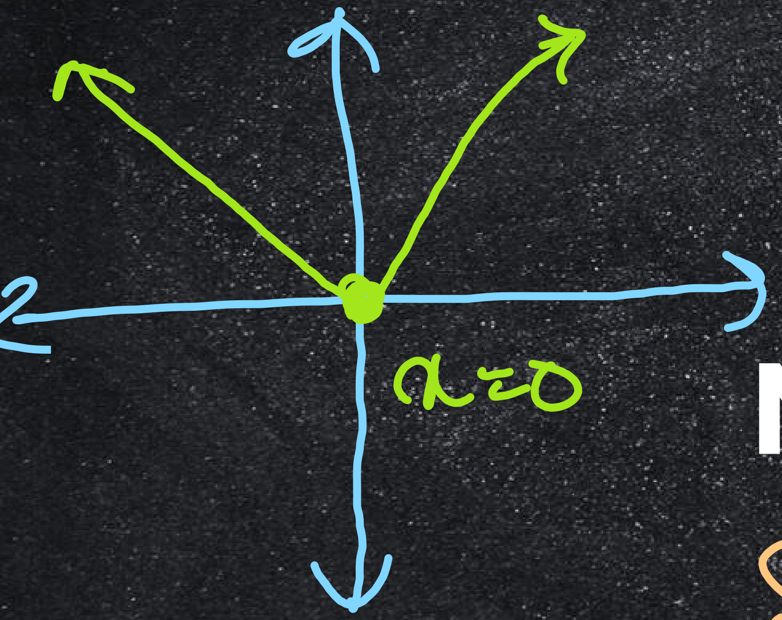

DERIVATIVES

NOT ALL FUNCTIONS HAVE DERIVATIVES...

So, absolute value func at $x=0$ is not differentiable.

$$f(x) = |x|$$

$$f'(x) = \lim_{h \to 0} \frac{|x+h| - |x|}{h} \qquad \boxed{x=0}$$

By def:

$$f'(0) = \lim_{h \to 0} \frac{|h| - 0}{h} = \lim_{h \to 0} \frac{|h|}{h}$$

$$|h| = \begin{cases} -h, & h<0 \\ h, & h>0 \end{cases}$$

$$\lim_{h \to 0^-} \frac{-h}{h} = -1, \quad \lim_{h \to 0^+} \frac{h}{h} = 1$$

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So, limit DNE Hence $f(x) = |x|$ is not differentiable $x=0$

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$d(k) = 0$

$d(u^k) = ku^{k-1}du$

$d(e^u) = e^u du$

$\boxed{d(a^u) = (\log a)a^u du}$

$d(\log u) = \frac{1}{u}du$

$d(\log_a u) = \frac{1}{(u \log a)}du \ (a \neq 1)$

$d(\sin u) = \cos u \ du$

$d(\cos u) = -\sin u \ du$

$d(\tan u) = \sec^2 u \ du$

$d(\cot u) = -\csc^2 u \ du$

$d(\sec u) = \sec u \tan u \ du$

$d(\csc u) = -\csc u \cot u \ du$

$d(\arcsin u) = \frac{du}{\sqrt{1-u^2}}$

$d(\arctan u) = \frac{du}{1+u^2} \quad \frac{du}{du}$

$\frac{d}{dx}x^2 = 2x^1 = 2x$ (power rule)

$\frac{d}{dx}e^{2x} = e^{2x} \cdot \frac{d}{dx}2x = e^{2x} \cdot 2 \cdot \frac{dx}{dx} = 2 \cdot e^{2x}$

$\frac{d}{dx}\log x = \frac{1}{x}\frac{dx}{dx}$

$\frac{d}{dx}2^x = (\log 2) \cdot 2^x \cdot dx$

$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}\frac{dx}{dx}$

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}\frac{dx}{dx}$

DERIVATIVES

$f(x) = x^2 + x^3$

$f'(x) = 2x + 3x^2$

$f(x) = 3x^2$

$f'(x) = 3 \cdot 2x$
$= 6x$

1. *Derivative of a sum*

   The derivative of a sum is the sum of the derivatives: $(f+g)'(x) = f'(x) + g'(x)$

2. *Derivative of a constant times a function* $\boxed{(kf)'(x) = kf'(x)}$

3. *Derivative of a product*

   The **product rule** says that: $\boxed{(fg)'(x) = f(x)g'(x) + f'(x)g(x)}$

4. *Derivative of a quotient*

   The **quotient rule** says that: $\left(\frac{f}{g}\right)'(x) = \boxed{\frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}}$

5. *Derivative of a composite function*

   The **chain rule** says that: $\boxed{(f \circ u)'(x) = f'(u(x)) \cdot u'(x)}$

6. *Derivative of an inverse function*

   The **inverse-function rule** says that if f^{-1} is the inverse of f, and f has a nonzero derivative at x_0, then f^{-1} has a derivative at $y_0 = f(x_0)$, and $(f^{-1})'(y_0) = \frac{1}{f'(x_0)}$.

$f(x) = e^x \cdot x^2$

$f'(x) = e^x \cdot 2x + x^2 e^x$

$f(x) = \frac{x^2}{e^x}$

$f'(x) = \frac{e^x(2x) - x^2 e^x}{(e^x)^2}$

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Find the derivative of each of the following:

(a) $f(x) = x^3 e^{-x^3} - x - 3$ (b) $g(x) = \dfrac{\log(\sin^2 x)}{\cos x}$ (c) $h(x) = \arctan\sqrt{x}$

$$f'(x) = x^3 e^{-x^3}(-3x^2) + e^{-x^3}(3x^2) - 1 - 0$$

$$g'(x) = \frac{\cos x\left(\frac{1}{\sin^2 x}\right)(2\sin x\cos x) - \log(\sin^2 x)(-\sin x)}{(\cos x)^2}$$

$$h'(x) = \frac{1}{1+(\sqrt{x})^2}\frac{d}{dx}\sqrt{x} = \frac{1}{1+x}\,\frac{1}{2\sqrt{x}}$$

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Derivatives in Real Life

- In Economics: $f'(x)$ can represent marginal cost or marginal revenue.
- In Physics: describes velocity, acceleration, force relationships.
- In Biology: growth rate of population models.

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