University of Sargodha

M.A/M.Sc. Part-I/Composite, 1st A-Exam 2016

Mathematics: IV Mechanics

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part (Compulsory)

Q.1. Write short answers of the followings on your answer sheet: (2*10)

i. Prove that $\nabla r^n = nr^{n-2}\vec{r}$, where n is any real number.

ii. Define directional derivative.

iii. Define scalar point function.

iv. Prove that if u_1, u_2, u_3 are orthogonal curvilinear coordinates, then $|\nabla u_j| = h_j^{-1} \quad j = 1,2,3$

v. If $y_i = a_{ij}x_j$, express the quadratic form $Q = g_{ij}y_i y_j$ in terms of x – variables.

vi. Derive transformation equations for unit vectors.

vii. Differentiate between translation and rotation.

viii. Prove that the rotational kinetic energy of a rigid body is $T = \frac{1}{2}\vec{\omega}\cdot\vec{L}$.

ix. What is a spherical top?

x. Show that kinetic energy of a rigid body in the absence of external force is constant.

Subjective Part

Q.2. **(a)** Find the angle between the normals to the surface $xy = z^2$ at the points $(1,4,2)$ and $(-3,-3,3)$. (10)

(b) If $\vec{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}$, evaluate $\iint_S \vec{A}\cdot\hat{n}\, dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ lying in the first octant. (10)

Q.3. **(a)** Derive an expressions for arc length,area, and volume elements in orthogonal curvilinear coordinates system. (10)

(b) The system $Ox_1'x_2'x_3'$ is initially coincident with a set $Ox_1x_2x_3$. The set $Ox_1'x_2'x_3'$ is rotated so as to bring x_1' along the old x_2, x_2' along the old x_3, and x_3' along the old x_1. Find the transformation matrix T for this rotation and show that this transformation is orthogonal and right handed. Also find equations of transformation expressing the coordinates x_j' in the system $Ox_1'x_2'x_3'$ in terms of coordinate x_i of the system $Ox_1x_2x_3$. (10)

Q.4. **(a)** If ϕ is a continuous scalar function in a region R bounded by a closed surface S, then prove that $\iiint_R \nabla\phi\, dV = \iint_S \phi\hat{n}\, dS$ (10)

(b) If $A_{ij}B_j$ is a vector, and A_{ij} is an arbitrary tensor of order 2, then prove that B_i is a vector. (10)

Q.5. **(a)** A particle moves in an elliptical part with constant angular speed. At what points the magnitude of the acceleration (a) maximum (b)minimum? If the major and minor axes of the elliptical path are 4 and 2 feet respectively, determine the magnitudes of these accelerations. (10)

(b) Discuss the motion of projectile in the presence of the air resistance and calculate the horizontal and vertical distance of the projectile. (10)

Q.6. **(a)** Express angular momentum of a rigid body in terms of moment and product of inertia. (10)

(b) For a uniform rectangular lamina ABCD with sides of lengths 2a, 2b, (b>a), find the directions of the principal axes at the corner A. (10)

Q.7. **(a)** Discuss torque-free motion of a symmetrical top. (10)

(b) A rectangular plate spins with constant angular velocity $\vec{\omega}$ about a diagonal. Find the couple which must act on the plate in order to maintain this motion (10)