University of Sargodha

M.A/M.Sc Part-1 / Composite, 1st-A/2014

Mathematics: IV **Mechanics**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part

Q.1. Write short answer of the following. (2*10)

i. If $\phi(x,y,z)=3x^2y-y^3z^2$ find $\nabla\phi$ at the point $(1,-2,-1)$

ii. Why the divergence theorem is often called Green's theorem in space?

iii. Prove that if u_1,u_2,u_3 are orthogonal curvilinear coordinates, then $\hat{e}_j=\hat{E}_j$, $j=1,2,3$.

iv. Express the plane $y=x$ in spherical polar coordinates.

v. Prove that $\delta_{ij}A_j=A_i$

vi. Define dummy and free indices.

vii. Differentiate between translational and rotational motion of a rigid body

viii. If no force is acting on a particle, then show that linear momentum of the particle is conserved.

ix. Define principal moment of inertia.

x. Show that for Euler's equation of motion kinetic energy in the absence of external forces is a constant of motion.

Subjective Part

Q.2. (a) If $\vec{A}$ and $\vec{B}$ are two vector point functions then prove that $(\vec{A}\times\nabla)\cdot\vec{B}=\vec{A}\cdot(\nabla\times\vec{B})$ **(10)**

(b) Verify the divergence theorem for $\vec{A}=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ where S is the surface of the cube bounded by $x=0,x=1,\ y=0,y=1,\ z=0,z=1$. **(10)**

Q.3. (a) Derive an expression for $\nabla\cdot\vec{A}$ in curvilinear coordinates system. **(10)**

(b) Show that differentiation of a tensor is also a tensor of higher rank. **(10)**

Q.4.(a) Using cylindrical polar coordinates, evaluate $\int_0^{2}\int_0^{\sqrt{4-x^2}}\int_0^{8}2yz\,dz\,dy\,dx$ **(10)**

(b) Show that finite rotation of the rigid body does not commute, but infinitesimal rotations commute. Also show that sum of the angular velocities is an angular velocity. **(10)**

Q.5. (a) A set of axis $Ox_1'x_2'x_3'$ is initially coincident with a set $Ox_1x_2x_3$ The set $Ox_1'x_2'x_3'$ is rotated so as to bring x_1' along the old x_2, x_2' along the old x_3, and x_3' along the old x_1 Find the transformation matrix T for this rotation and show that this transformation is orthogonal and right handed .Also, find the equations of transformation expressing the coordinates x_j' in the system $Ox_1'x_2'x_3'$ in terms of coordinates x_i of the system $Ox_1x_2x_3$. **(10)**

(b) Find the displacement and velocity of a particle moving horizontally in a resistive medium in which the retarding force is proportional to the velocity. **(10)**

Q6(a) A coordinate system OXYZ is rotating with angular velocity $\vec{\omega}=5\hat{i}-4\hat{j}-10\hat{k}$ relative to a fixed coordinate system $OX_0Y_0Z_0$, both systems having the same origin. Find the velocity of a particle at rest in the OXYZ system at the point $(3,1,-2)$ as seen by an observer in the fixed system. **(10)**

(b) Calculate the moment of inertia of a rigid body about a line. **(10)**

Q.7. (a) Four particles of masses m,2m,3m,4m,are located at the points (a, a, a), (a,-a,-a), (-a, a,-a), (-a,-a, a) respectively, and are rigidly connected to one another by a light framework. Calculate the principal moments of inertia of the system at the origin. **(10)**

(b) Derive Euler's dynamical equations of motion for a rigid body fixed at a point. **(10)**