University of Sargodha

M.A/M.Sc Part-1 / Composite, 1st -A/2013

Mathematics-IV **Mechanics**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

15658

Objective Part

Q.1 Write short answer of the following. 20

i. Show that the first partial derivative $\frac{\partial \phi}{\partial x}$ is the directional derivative in the direction of x-axis. **ii.** If $\vec{A}$ is a vector point function, then prove that $\nabla.(\nabla\times\vec{A})=0$. **iii.** Show that the vector field $\vec{A}=(\sin y+z)\hat{i}+(x\cos y-z)\hat{j}+(x-y)\hat{k}$ is conservative. **iv.** Prove that if u_1,u_2,u_3 are orthogonal curvilinear coordinates, then $|\nabla u_j|=h_j^{-1}\quad j=1,\ 2,\ 3$. **v.** Prove that Kronecker delta δ_{ij} is a tensor of rank 2. **vi.** Prove the work energy principle. **vii.** Find the degree of freedom of a rigid body free to move in a plane. **viii.** Show that using operators, the fixed and rotating coordinate systems can be related as $D_f=D_r+\bar{\omega}\times$ where D_f and D_r stand for $\frac{d}{dt}$ in the fixed and rotating coordinate systems. **ix.** Find the moment of inertia of a ring of radius a about an axis through its centre. **x.** Obtain an expression for the kinetic energy of rotation of a rigid body in terms of the Euler's angles.

Subjective Part

Q.2(a) If $\vec{A}$ and $\vec{B}$ are differentiable vector point functions, prove that 10

$$\nabla\times(\vec{A}\times\vec{B})=(\vec{B}.\nabla)\vec{A}-\vec{B}(\nabla.\vec{A})-(\vec{A}.\nabla)\vec{B}+\vec{A}(\nabla.\vec{B})$$

(b) Prove that a necessary and sufficient condition that a vector field $\vec{A}$ be conservative is that $\nabla\times\vec{A}=0$ (i.e $\vec{A}$ is irrotational) 10

Q.3(a) Using cylindrical polar coordinate. Evaluate $\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^8 2yz\,dz\,dy\,dx$. 10

(b) Show that general rigid body motion is a screw motion. 10

Q.4(a) Show that a tensor equation remains the same in every rectangular coordinate system. 10

(b) Calculate the moment of inertia of a rigid body about a given line. 10

Q.5(a) Prove that $\nabla\times\vec{A}$ is invariant vector field under the rotation of the coordinate axes. 10

(b) Express angular momentum in tensor notation. 10

Q.6(a) Prove that two systems are equimomental if and only if they have the same mass and same centriod. 10

(b) Solve Euler's torque free equation of motions for a symmetrical top. 10

Q.7. Derive Euler's geometrical equation of motions. 20