University of Sargodha

M.A/M. Sc. Part-1/Composite, 2nd-A/2015

Mathematics: III **Complex Analysis & Differential Geometry**

Maximum Marks: 100 **Time Allowed: 3 Hours**

	Objective Part	
Q. 1	Give short answers.	20
(i)	Write Cauchy Riemann equations in polar form.	
(ii)	Verify that $\cos z$ has primitive period 2π.	
(iii)	Evaluate $Log(-1+i)$.	
(iv)	Define inverse of the curve.	
(v)	Write Maclaurin's series.	
(vi)	Discuss the nature of singularity for $f(Z) = \frac{e^z}{(z-1)^3}$.	
(vii)	Define one parameter family of surfaces.	
(viii)	Define curvilinear coordinates.	
(ix)	If $\kappa = 0$ then prove that curve is a straight line.	
(x)	Define minimal surface.	
	(Subjective Part)	
	Note: Attempt any four questions.	
Q. 2	(a) Derive Cauchy Riemann equations in polar form.	12
	(b) Calculate Z for which $\sin Z = 2i$.	08
Q.3	(a) Evaluate $\lvert \int_c \overline{Z} dZ \rvert$ where C is a semi unit circle.	10
	(b) Let $f(Z)$ be analytic on and within boundary of C of a simply connected region D and let "a" be any point within C then prove that $$f^{(n)}(a) = \frac{n!}{2\pi i}\int_c \frac{f(Z)}{(Z-a)^{n+1}}dZ.$$	10

Q. No.	Questions	Marks
Q.4	(a) Show that the function $f(Z) = Exp(c(Z+\frac{1}{z}))$ can be expanded as Lauret's series $\sum_{-\infty}^{\infty} a_n Z^n$ for $\|Z\| > 0$ where $$a_n = \frac{1}{2\pi}\int_0^{2\pi} e^{2c\cos\theta}\cos n\theta d\theta$$	10
	(b) Evaluate the following integral by Residue theorem $$\int_c \frac{Z^2 - Z + 1}{(Z-1)(Z-4)(Z+3)} dZ$$ where C is the circle $\|Z\| = 5$.	10
Q. 5	(a) Prove that $$\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$$ where $a > b > 0$.	10
	(b) find κ and τ for the curve $$x = e^t, \; y = e^{-t}, \; z = -\sqrt{2}\, t.$$	10
Q. 6	(a) Prove that Under the transformation $W = \frac{1}{z}$, the line $y = x - 1$ is mapped into a circle $$U^2 + V^2 - U - V = 0.$$	10
	(b) If $\rho\tau + \frac{d}{ds}(\frac{\rho'}{\tau}) = 0$ then prove that curve is spherical.	10
Q. 7	(a) Show that the tangent plane at the point common to the surface $a(xy + yz + zx) = xyz$ and sphere $x^2 + y^2 + z^2 = b^2$ makes intercepts on the axis whose sum is constant.	10
	(b) For the surface $$\vec{r} = (x, y, c\tan^{-1}\frac{y}{x})$$ Find first order and second order magnitudes.	10