University of Sargodha

M.A/M.Sc Part- 1/Composite, 2nd-A/2014

Mathematics: III Complex Analysis & Differential Geometry

Maximum Marks: 100 **Time Allowed: 3 Hours**

Objective Part Compulsory

Q. 1	Give short answers.	20
(i)	Define level curves.	
(ii)	Define period of a function.	
(iii)	Evaluate $Log(-1+i)$.	
(iv)	Evaluate the integral $\int_c \frac{\cosh z}{(z+5)(z+3)}$ where C : $\|z\| = 1$.	
(v)	State Liouville's theorem.	
(vi)	Discuss the nature of singularity for $f(Z) = e^{\frac{1}{2z}}$.	
(vii)	Define a devlopable surface.	
(viii)	Along the helix $x = (a\cos t, a\sin t, bt)$, find the unit tangent vector independent of x'.	
(ix)	iv. Define line of curvature and write down its equation.	
(x)	Prove that $H[\vec{n}\ \vec{n_2}\ \vec{r_1}] = EN - FM$.	
	(Subjective Part)	
	Note: Attempt any four questions.	
Q. 2	(a) Prove that a harmonic function satisfies the differential equation $\frac{\partial^2 U}{\partial Z \partial \bar{Z}} = 0$.	10
	(b) Prove that $U(r,\theta) = r^n \cos n\theta$ is harmonic.Find $V(r,\theta)$ and the original function $f(Z)$	10
Q.3	(a) State and prove Moreras Theorem.	10
	(b) Expand $log(1+z)$ in a Taylor's series about $z = 0$ and determine the region of convergence for the resulting series. Find the Laurents expansion of $$f(z) = \frac{1}{(z^2+1)(z^2+2)}$$ in the domain $1 < \|z\| < \sqrt{2}$.	10

Q. No.	Questions	Marks
Q.4	(a) Find the nature and location of singularities of the function $f(Z) = \frac{1}{Z(e^z)-1}$ and prove that $f(Z)$ can be expanded as $$\frac{1}{Z^2} + \frac{1}{2Z} + \frac{1}{12} + \frac{Z^2}{360} + \dots\dots$$	10
	(b) Find the residues of the function $$f(Z) = \frac{e^z}{(Z - \pi i)^4}.$$	10
Q. 5	(a) Prove that $$\cot z = \frac{1}{2} + 2z \sum_{n=1}^{\infty} \frac{1}{z^2 - n^2\pi^2}.$$	10
	(b) Prove that for any curve $$[\vec{b}', \vec{b}'', \vec{b}'''] = \tau^3(\kappa'\tau - \kappa\tau') = \tau^5 \frac{d}{ds}(\frac{\kappa}{\tau}).$$	10
Q. 6	(a) Prove that Under the transformation $W = \frac{1}{Z-1}$, the circle $\|Z\| = 2$ in Z-plane is mapped into the circle $$3(U^2 + V^2) = 2U + 1$$ in w-plane.	10
	(b) Derive an equation of involute and also find the curvature of involute.	10
Q. 7	(a) Prove that the edge of regression of osculating developable is curve itself.	10
	(b) For the surface $$x = u\cos\phi, \quad y = u\sin\phi, \quad z = c\phi,$$ Find first order and second order magnitudes.	10

State and prove Lagrange's mean value theorem.