University of Sargodha

M.A/M.Sc Part-1 / Composite, 2nd-A/2013

Mathematics: III **Complex Analysis**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part

Q.No.1. Write short answer of the following each in 2-3 lines only on your answer book. 2*10

I. Prove that $|Z_1 + Z_2| = |Z_1| + |Z_2|$.
II. Define orthogonal systems.
III. Write power series of e^Z .
IV. State Cauchy's fundamental Theorem.
V. Define locus of point.
VI. Find the singularities of $f(z) = \frac{z^2}{Z(Z^2+1)(Z^2+4)}$.
VII. Find the period of e^Z .
VIII. Define Rectifying Plane.
IX. Write down Serret-Fernet formulae.
X. Define Helix.

Subjective Part

Q. No. 2	a. Prove the necessary condition for a function $f(z) = U(x,y) + iV(x,y)$ to be analytic function.	10
	b. Derive Cauchy Riemann equations in polar form From Cartesian form.	10
Q .No. 3	a. Prove that $U(r,\theta) = r^3 \cos 3\theta$ is harmonic. Obtain its corresponding conjugate and the original function $f(z) = U(r,\theta) + iV(r,\theta)$.	10
	b. State and prove Taylors Theorem.	10
Q .No. 4	a. State and prove Cauchy's integral formula.	10
	b. Find the residue of $f(z) = \tanh z$.	10
Q .No. 5	a. State and prove the argument Principle theorem.	10
	b. Let $f(z)$ be analytic on and within the boundary of region C of a simply connected region D and let a be any point within C then $f'(a) = \frac{1}{2\pi i}\int_c \frac{f(z)}{(z-a)^2} dz$.	10
Q .No. 6	a. Find k and τ if $X = a(u - \sin(u))$, $Y = a(1 - \cos(u))$, $z = bu$	10
	b. Prove that the necessary and sufficient condition for a curve to be Helix is that the ratio of its curvature and torsion is constant i.e. $\frac{k}{\tau} = contant.$	10
Q .No. 7	a. Prove that $b'' = \tau(kt - \tau b) - \tau n$ $n'' = \tau b - (k^2 + \tau^2)n - k't$.	10
	b. For a curve $x = 4a\cos^3 u$, $y = 4a\sin^3 u$, $z = 3c\cos(2u)$ Prove that $n = (sin(u), \cos(u), 0)$, $k = \frac{a}{6(a^2+c^2)sin2u}$	10