University of Sargodha

M.A/M.Sc Part-1 / Composite, 2nd-A/2012

Mathematics: III Complex Analysis & Differential Geometry

Maximum Marks: 80 *Subjective Part* **Time Allowed: 2:40 Hours**

Note: **Question No.2 is compulsory. Attempt any three in all from remaining questions, Selecting at least one question from each section.**

2.	Give short answers. (i) Prove that $\|z_1 - z_2\| \geq \|\|z_1\| - \|z_2\|\|$. (ii) Solve the integral $I = \int_c \bar{z}\,dz$, where $c : z = z(t) = i + e^{it}$, $0 \leq t \leq 2\pi$. (iii) Prove that the function $u(r,\theta) = r^n \cos n\theta$ is harmonic.. (iv) State Cauchy residue therem. (v) Evaluate the integral $\int_{\|z\|=2} \frac{e^z}{z^2} dz$. (vi) For a curve $\vec{r} = \vec{r}(u)$, prove that $\|\dot{\vec{r}}\| = \|\dot{s}\|$, where '.' denotes the differentiation w.r.t. u. (vii) Prove that $\vec{r}_2 \times \hat{n} = \frac{G\vec{r}_1 - F\vec{r}_2}{H}$. (viii) Find the expression for the curvature of tangent indicatrix. (ix) For the surface given by $\vec{r} = (a\cos u \cos v, a\cos u \sin v, a\sin u)$, prove that the parametric curves are orthogonal. (x) If f=F=0 then prove that the lines of curvature are parametric curves.	20
	SECTION-I	
3(a)	Prove that if $f(z) = u(x,y) + iv(x,y)$ is differentiable at a point z_0, then its real and imaginary parts possess first partial derivatives and satisfy C.R.D. equations.	10
(b)	If f(z) is analytic at each point in a domain D of f(z) except at z=a which is a pole of order n, then prove that $\text{Res}[f,a] = \frac{1}{(n-1)!} \lim_{z\to a} \frac{d^{n-1}}{dz^{n-1}}\left[(z-a)^n f(z)\right]$.	10
4(a)	If the power series $\sum_{n=0}^{\infty} c_n (z-z_0)^n$ has a nonzero radius of convergence R, then for any circle $c : \|z - z_0\| = r$ where r<R, prove that the given power series represents a continuous function of z in the closed region bounded by the circle c.	10
(b)	State and prove maximum principle	10
5(a)	Show that $e^z = i$ if and only if $z = \left(\frac{1}{2} + 2n\right)\pi i$.	10
(b)	Evaluate the integral $I = \int_{-\infty}^{\infty} \frac{x^2 - x + 1}{x^4 + 10x^2 + 9} dx$, by residue method.	10
	SECTION-II	10
6(a)	Define radius of torsion and prove that $\frac{d\hat{b}}{ds} = -\tau\hat{n}$.	10
(b)	If c_1 denotes the locus of centre of spherical curvature of a curve c then prove that $\frac{\kappa}{\tau} = \frac{\tau_1}{\kappa_1}$.	10
7(a)	Find the envelope of the family of surfaces $x^2 + y^2 - 4az = -4a^2$.	10
(b)	Spheres of radius b are drawn in such a way that their centres lie on the circumference of the circle $x^2 + y^2 = a^2$ and $z = 0$. Prove that the envelope is $\left[x^2 + y^2 + z^2 + a^2 - b^2\right]^2 = 4a^2(x^2 + y^2)$.	10