Subject: Math- III M.A/M.Sc: Part- I / Composite, 2nd -A/10

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# University of Sargodha

**M.A/M.Sc Part-1 / Composite, 2nd -A/2010**

**Math- III Complex Analysis & Differential Geometry**

**Maximum Marks: 40** ***Objective Part*** **Fictitious #:** __________

**Time Allowed: 45 Min.** **Signature of CSO:** __________

**Note:** **Cutting, Erasing, overwriting and use of Lead Pencil are strictly prohibited. Only first attempt will be considered.**

**Q.1.A: Choose the correct option.** (5)

i. The function $f(z) = e^{1/3z}$ has _______ singularity at $z = 0$.
a. No b. Essential c. Removable d. Isolated

ii. A curve traced by a function $z = f(t)$ such that the intial point and the terminal point are the same then the curve is called a ______:
a. Jordan Curve b. Simple Curve c. Closed Curve d. Piecewise Curve

iii. The value of the integral usually depends upon the ______:
a. Path of Integration b. Indefinite Integral c. Line Integral d. Definite Integral

iv. The equation $x = a\cos\theta, y = a\sin\theta, z = 0$ is called the equation of ______:
a. Osculating Plane b. Tangent c. Normal d. Rectifying Plane

v. $\left[\vec{R}-\vec{r} \quad \vec{t} \quad \vec{n}\right]=0$ is the equation of:
a. Principal Normal b. Rectifying Plane c. Osculating Plane d. Normal Plane

**B: Fill in the Blanks.** (5)

i. A set of complex numbers form a __________________. ii. A complex function $f$ is continuous at $Z = Z_0$, if $\lim\limits_{Z\to Z_0} f(Z) =$ __________________. iii. If $f(Z) = Z - \overline{Z}$ then the range of $f$ is contained in the __________________ axis. iv. If there is a hole in the region, then the region is called __________________ region. v. Spherical indicatrix lies on the surface of a ________________.

**C: Write True or False.** (10)

i. The domain of the function $f(Z) = \dfrac{1}{Z^2 + i}$ is all complex numbers.

ii. $\text{Log } Z = \text{Log } |Z| + i\theta$ is a single valued function.

iii. The function $f(Z) = Z^2 + \overline{Z}$ is nowhere analytic.

iv. $\ell n i = \dfrac{1}{2}\pi i$

v. There are complex Z such that $|\sin Z| > 1$.

vi. There are three types of singularities.

vii. A curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is an ellipse if $h^2 - ab > 0$

viii. $(\vec{R}-\vec{r})\cdot\vec{t} = 0$ is called the equation of the Normal Plane.

x. A curve traced on the surface of a cylinder and cutting the generator at the constant angle is called spherical indicatrix.

. The equation $\underline{c} = \underline{r} + p\underline{n}$ is called the equation of principal normal.

**Q.2. Give short answers.** **(20)**

Define Piecewise smooth curve. ii. Define Isolated singularity.

ii. Investigate zeros, Poles and singularities of $f(Z) = e^{2Z}$ at $Z = \infty$. iv. Evaluate the integral $\int \frac{\cosh Z + Z^2}{(Z+5)(Z+3)} dz$ where $C : |Z| = 2$ v. State Cauchy's Integral formula. vi. Find the radius of convergence of the following Power Series $\sum (\log n)^n Z^n$ vii. Define Multi valued function.

iii. Define one parametric family of surface. ix. Prove that $\vec{r}''.\vec{r}''' = K(K'' - K\tau^2 - K^3)$

. Define equation of Normal Plane.

# University of Sargodha

**M.A/M.Sc Part-1 / Composite, 2nd -A/2010**

**Math- III Complex Analysis & Differential Geometry**

**Maximum Marks: 60** **Time Allowed: 2:15 Hours**

***Subjective Part***

Note: **Attempt any three questions in all. Selecting at least one question from each section.**

## Section- I

**Q.3.** a. Represent graphically all points Z s.t. $\left|\frac{Z+1}{Z-1}\right| = 4$ Find the center and radius of the locus of $Z$. (10)

b. Prove that $\operatorname{Re}\left[(1+i)^{\log(1+i)}\right] = 2^{\frac{1}{4}\log 2}\ e^{-\pi^2/16}\ Cos\left(\frac{\pi}{4}\log 2\right)$ (10)

**Q.4.** a. Prove that the value of $\int_C \frac{1}{Z}dZ$ where C is a semi circular arc $|Z| = 1$ from -1 to -1, is $-\pi i$ *or* $\pi i$ according to the arc lies above or below the real axis. (10)

b. State and prove the Argument Principal Theorem. (10)

**Q.5.** a. Prove that a bilinear transformation maps circles or straight lines into circles or straight lines. (10)

b. Prove that $\int_{-\infty}^{\infty}\frac{x^2dx}{(x^2+a^2)^3} = \frac{\pi}{8a^3}$ Provided that $R(a)$ is positive. What is the value of this integral when $R(a)$ is negative. (10)

## Section- II

**Q.6.** a. State and prove Meunier's Theorem. (10)

b. Find the envelope of the plane (10)

$$\frac{x}{a+u}+\frac{y}{b+u}+\frac{Z}{c+u}=1$$

where u is a parameter. Also determine the edge of regression.

**Q.7.** a. Prove that the normal at a point P of the ellipsoid (10)

$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{Z^2}{c^2}=1$ meets the coordinate planes in $G_1$, $G_2$, $G_3$. Prove that the ratios $PG_1 : PG_2 : PG_3$ are constant.

b. Prove that the shortest distance between the principal normal at consecutive points, distant $S$ apart, is $\frac{S\rho}{\sqrt{\rho^2+\sigma^2}}$ and that it divides the curvature in the ratio $\rho^2 : \sigma^2$. (10)

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