University of Sargodha

M.A/M.Sc. Part-I/Composite, 1st A-Exam 2016

Mathematics: III **Complex Analysis & Differential Geometry**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: Objective part is compulsory. Attempt any **four questions from subjective part.**

Objective Part (Compulsory)

Q No. 1 Give short answers. [20]

(i) For any curve evaluate $\mathbf{n}'$.
(ii) Define Minimal surface.
(iii) Evaluat $\oint_c \frac{1}{z}\,dz$, c is unit circle.
(iv) Evaluate $Ln e^{1+\frac{3}{2}\pi i}$.
(v) Define edge of regression.
(vi) For any curve evaluate $\mathbf{r}''.\mathbf{r}'''$
(vii) Evaluate. $\oint_C \frac{z^2-4z+4}{z+i}\,dz$, where C is the circle $|z-2i|=4$
(viii) If $\bar{z}=z$ then explain the complex status of z.
(ix) What is Holomorphic or Regular?
(x) Define Curvature and Torsion.

(Subjective Part)

Note: Attempt any four questions.

Q2.

a) Find image of a triangle with vertices 0, 1 and i under the mapping $f(z)=e^{i\frac{\pi}{4}z}$, represent the linear mapping with a sequence of plots. [10]

b) Find all solutions to the equation $\sin z = 5$. [10]

Q3.

a) Find all the values of $\tan^{-1}(2i)$. [10]

b) Suppose that f is analytic in a simply connected domain D and C is any simple closed contour lying entirely within D. Then for any point z0 within C: $f''(z_0)=\frac{1}{\pi i}\oint\frac{f(z)}{(z-z_0)^3}dz$. [10]

Q4.

a) Prove that $\left|\frac{a+b}{1+\bar{a}b}\right|<1$ if $|a|<1, |b|<1$, when does the equality hold? [10]

b) State and prove Laurent's theorem. [10]

Q5.

a) Find the residues of $f(z)=\frac{e^z}{z^2(z-\pi i)^4}$. [10]

b) Prove that if K=0 at all points on a curve then curve will be straight line also show the behavior of the curve if $\tau=0$. [10]

Q6.

a) Prove that $\int_0^{2\pi}\frac{\cos 3\theta}{5-4\cos\theta}d\theta=\frac{\pi}{12}$. [10]

b) Consider the family of sphere with constant radius "b" having their centers on a fixed circle $x^2+y^2=a^2, z=0$, show that the equation of the envelope is the surface $(x^2+y^2+z^2+a^2-b^2)^2=4a^2(x^2+y^2)$. [10]

Q7.

a) Prove that if θ is the angle between the direction on a surface and the curve v = constant then $\cos\theta=\frac{1}{\sqrt{E}}\left(E\frac{du}{ds}+F\frac{dv}{ds}\right)$, and $\sin\theta=\frac{-H}{\sqrt{E}}\frac{dv}{ds}$. [10]

b) Define minimal surface and if the surface of revolution is minimal surface then show the following:

$$uf''(u)+f'(u)[1+(f'(u))^2]=0,$$ [10]