University of Sargodha

M.A/M.Sc Part-1 / Composite, 1st-A/2014

Mathematics: III Complex Analysis & Differential Geometry

Maximum Marks: 100 **Time Allowed: 3 Hours**

Objective Part Compulsory

Q. 1	Give short answers.	20
(i)	Define analytic function.	
(ii)	Differentiate between meromorphic function and integral function.	
(iii)	Differentiate between essential sigularity and pole.	
(iv)	Define Jardon curve.	
(v)	State Green's theorem.	
(vi)	Define simply and multiply connected domains.	
(vii)	Define helix and write the necessary condition for a curve to be helix.	
(viii)	Define tangent plane to a surface.	
(ix)	Evaluat $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d})$.	
(x)	Prove the sufficient condition for curves to be orthogonal	

(Subjective Part)

Note: Attempt any four questions.

Q. 2 (a) Find the locus of z where $z = at + \frac{b}{t}$, where t is a real parameter and a and b are complex constants. — 10

(b) Find the analytic function of which the real part is — 10

$$e^{-x}[(x^2 - y^2)\cos y + 2xy \sin y].$$

Q.3 (a) State and prove Cauchy integral formula. — 10

(b) Find the Laurents expansion of — 10

$$f(z) = \frac{1}{z^2 - z - 2}$$

in the domain $1 < |z| < 2$.

Q.4 (a) Find the residue at $z = 0$ for the function $\frac{\cot z}{z^4}$. — 10

(b) Prove taht an analytic function with constant modulus is constant. — 10

Q. 5 (a) Prove that — 10

$$\cot z = \frac{1}{z} + 2z \sum_{n=1}^{\infty} \frac{1}{z^2 - n^2\pi^2}.$$

(b) Find the curvature and torsion of locus of centre of curvature if the curve has a constant curvature. — 10

Q. 6 (a) Under the transformation $(W + 1)^2 Z = 4$, prove that if W describes a unit circle then Z describes a parabola. — 10

(b) Show that for any curve — 10

$$[\vec{r}', \vec{r}'', \vec{r}'''] = \kappa^5 \frac{d}{ds}\left(\frac{\tau}{\kappa}\right).$$

Q. 7 (a) Prove that the sum of squares of intercepts made by tangent plane to the surface — 10

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} + z^{\frac{2}{3}} = a^{\frac{2}{3}}$$

is constant.

(b) For the surface of revolution — 10

$$x = u\cos\phi, \quad y = u\sin\phi, \quad z = f(u),$$

Find first order and second order magnitudes.